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On Intuitionistic Fuzzy β Generalized α Connectedness

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Abstract

In this paper we have introduced the intuitionistic fuzzy β generalized α connected space, intuitionistic fuzzy β generalized α super connected space and intuitionistic fuzzy β generalized α extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy β generalized α super connected space.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy β generalized α closed set, intuitionistic fuzzy β generalized α continuous mapping, intuitionistic fuzzy β generalized α connected space and intuitionistic fuzzy β generalized α super connected space.

INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Connectedness in intuitionistic fuzzy special topological spaces was introduced by ozcag and coker [7]. In this paper we have introduced intuitionistic fuzzy β generalized α connectedness in intuitionistic fuzzy topological spaces. Also we have provided some characterizations of intuitionistic fuzzy β generalized α connectedness.

PRELIMINARIES

Definition 1: **[1]** An intuitionistic fuzzy set (IFS for short) A is an object having the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x ∈ X to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2: **[1]** Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ∈ X,

(b) A = B if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle: x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 3: **[2]** An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i: i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 4: [6] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy β closed set (IFβCS in short) if $int(cl(int(A))) \subseteq A$

(ii) intuitionistic fuzzy β open set (IFβOS in short) if $A \subseteq cl(int(cl(A)))$

Definition 5: [11] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi pre closed set(IFSPCS in short) if $int(B) \subseteq A \subseteq B$

(ii) intuitionistic fuzzy semi pre open set(IFSPOS in short) if $B \subseteq A \subseteq cl(B)$

Remark 6: [6] Every IFSPCS is an IFβCS in (X, τ) but not conversely in general.

Definition 7: [3] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy β generalized α closed set (IFβGαCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFαOS in (X, τ). The complement A^c is called an intuitionistic fuzzy β generalized α open set (IFβGαOS for short) in X.

Definition 8: [4] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy β generalized α continuous (IFβGα continuous for short) mapping if $f^{-1}(V)$ is an IFβGαCS in (X, τ) for every IFCS V of (Y, σ).

Definition 9: [3] An IFTS (X, τ) is an intuitionistic fuzzy $\beta_{g\alpha\beta}T_{1/2}$ ($IF\beta_{g\alpha\beta}$ $T_{1/2}$ in short) space if every IFβGαCS is an IFβCS in X.

Definition 10: [10] Two IFSs A and B are said to be q-coincident ($A_q B$ in short) if and only if there exits an element x ∈X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 11: [2] An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5-connected space if the only IFSs which are both an IFOS and an IFCS are $0_\sim$ and $1_\sim$.

Definition 12: [10] An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-connected space if the only IFSs which are both an IFGOS and an IFGCS are $0_\sim$ and $1_\sim$.

Definition 13: [8] An IFTS (X, τ) is an intuitionistic fuzzy C_5-connected between two IFSs A and B if there is no IFOS E in (X, τ) such that A ⊆ E and $E\ {}_q^c\ B$.

Definition 14: [5] If an IFS A in an IFTS (X, τ) is an IFβGαCS in X, then βgαcl(A) = A. But the converse may not be true in general, since intersection of two non-trivial IFβGαCS is not an IFβGαCS.

Definition 15: [5] If an IFS A in an IFTS (X, τ) is an IFβGαOS in X, then βgαint(A) = A. But the converse may not be true in general, since union of two non-trivial IFβGαOS is not an IFβGαOS.

INTUITIONISTIC FUZZY β GENERALIZED α CONNECTED SPACES

In this we have introduced intuitionistic fuzzy β generalized α connected space and intuitionistic fuzzy β generalized α super connected space. We have investigated some of their properties and provided a characterization theorem for an intuitionistic fuzzy β generalized α super connected space.

Definition 16: An IFTS (X, τ) is said to be an intuitionistic fuzzy β generalized α (IFβGα for short) connected space if the only IFSs which are both IFβGαCS and IFβGαOS are $0_\sim$ and $1_\sim$

Theorem 17: Every IFβGα connected space is an IFC_5-connected space but not conversely in general.

Proof: Let (X, τ) be an IFβGα connected space. Suppose (X, τ) is not an IFC_5-connected space, then there exists a proper IFS B which is both IFO and IFC in (X, τ). That is B is both an IFβGαOS and an IFβGαCS in (X, τ). This implies that (X, τ) is not an IFβGα connected space. This is a contradiction. Therefore (X, τ) is an IFC_5-connected space.

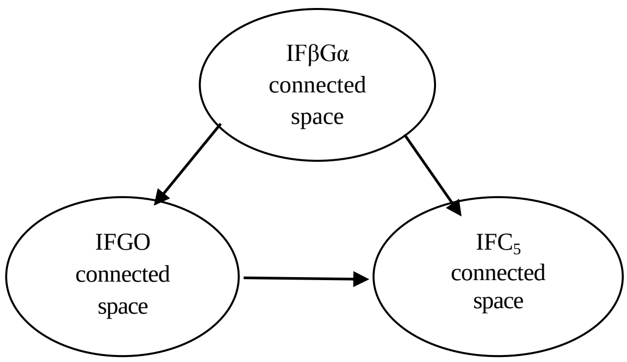
Example 18: Let X = {a, b} and τ = {$0_\sim$, B, $1_\sim$} be an IFT on X, where $B = (x, (0.5_a, 0.4_b), (0.5_a, 0.6_b))$. Then (X, τ) is an IFC_5-connected space but not an IFβGα connected space, since the IFS B in τ is both IFβGαO and IFβGαC in (X, τ).

Theorem 19: Every IFβGα connected space is an IFGO-connected space but not conversely in general.

Proof: Let (X, τ) be an IFβGα connected space. Suppose (X, τ) is not an IFGO-connected space, then there exists a proper IFS B which both an IFGOS and an IFGCS in (X, τ). That is B is both an IFβGαOS and an IFβGαCS in (X, τ). This implies that (X, τ) is not an IFβGα connected space. This is a contradiction. Therefore (X, τ) is an IFGO-connected space.

Example 20: In Example 18, (X, τ) is an IFGO-connected space but not an IFβGα connected space.

The relation among various types of intuitionistic fuzzy connectedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 21: An IFTS (X, τ) is an IFβGα connected space if and only if there exist no non-zero IFβGαOSs A and B in (X, τ) such that $A = B^c$.

Proof: Necessity: Let A and B be two IFβGαOSs in (X, τ) such that A ≠ 0~ ≠ B and A = B^c. Therefore B^c is an IFβGαCS. Since B ≠ 0~, A = B^c ≠ 1~. This implies A is a proper IFS which is both IFβGαOS and IFβGαCS in (X, τ). Hence (X, τ) is not an IFβGα connected space. But this is a contradiction to our hypothesis. Thus there exist no non-zero IFβGαOSs A and B in (X, τ) such that A = B^c.

Sufficiency: Let A be both an IFβGαOS and IFβGαCS in (X, τ) such that 1~ ≠ A ≠ 0~. Now let B = A^c. Then B is an IFβGαOS and B ≠ 1~. This implies B^c = A ≠ 0~, which is a contradiction to our hypothesis. Therefore (X, τ) is an IFβGα connected space.

Theorem 22: An IFTS (X,τ) is an IFβGα connected space if and only if there exists no non-zero IFβGαOSs A and B in (X, τ) such that B = A^c, B = $(\beta cl(A))^c$ and A = $(\beta cl(B))^c$.

Proof: Necessity: Assume that there exist IFSs A and B such that A ≠ 0~ ≠ B, B = A^c, B = $(\beta cl(A))^c$ and A = $(\beta cl(B))^c$. Since $(\beta cl(A))^c$ and $(\beta cl(B))^c$ are IFβGαOSs in (X, τ), A and B are IFβGαOSs in (X, τ). This implies (X, τ) is not an IFβGα connected space, which is a contradiction. Therefore there exists no non-zero IFβGαOSs A and B in (X, τ) such that B = A^c, B = $(\beta cl(A))^c$ and A = $(\beta cl(B))^c$.

Sufficiency: Let A be both an IFβGαOS and IFβGαCS in (X, τ) such that 1~ ≠ A ≠ 0~. Now by taking B = A^c, we obtain a contradiction to our hypothesis. Hence (X, τ) is an IFβGα connected space.

Theorem 23: Let (X, τ) be an $IF\beta_{g\alpha c}T_{1/2}$ space, then the following are equivalent.

(i) (X,τ) is an IFβGα connected space.

(ii) (X, τ) is an IFGO-connected space.

(iii) (X, τ) is an IFC_5-connected space.

Proof:(i) ⇒(ii) is obvious from Theorem 19.

(ii) ⇒ (iii) is obvious.

(iii) ⇒ (i) Let (X, τ) be an IFC_5-connected space. Suppose (X, τ) is not an IFβGα connected space, then there exists a proper IFS A in (X, τ) which is both an IFβGαOS and IFβGαCS in (X, τ). But since (X, τ) is an $IF\beta_{g\alpha c}T_{1/2}$ space, A is both IFOS and IFCS in (X, τ). This implies that (X, τ) is not an IFC_5-connected space, which is a contradiction to our hypothesis. Therefore (X,τ) is IFβGα connected space.

Theorem 24: If f : (X,τ) → (Y, σ) is an IFβGα continuous mapping and (X,τ) is an IFβGα connected space, then (Y, σ) is an IFC_5-connected space.

Proof: Let (X, τ) be an IFβGα connected space. Suppose (Y, σ) is not an IFC_5-connected space, then there exists a proper IFS A which is both IFOS and IFCS in (Y, σ). Since f is an IFβGα continuous mapping, $f^{-1}(A)$ is both IFβGαOS and IFβGαCS in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an IFC_5-connected space.

Theorem 25: If f : (X, τ) → (Y, σ) is an IFβGα irresolute surjection mapping and (X, τ) is an IFβGα connected space, then (Y, σ) is also an IFβGα connected space.

Proof: Suppose (Y, σ) is not an IFβGα connected space, then there exists a proper IFS B which is both IFβGαOS and IFβGαCS in (Y, σ). Since f is an IFβGα irresolute mapping, $f^{-1}(B)$ is both IFβGαOS and IFβGαCS in (X, τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an IFβGα connected space.

Definition 26: An IFTS (X, τ) is called IFβGα connected between two IFSs A and B if there is no IFβGαOS E in (X, τ) such that A ⊆ E and E $_q{}^c$ B.

Example 27: Let X = {a, b} and τ = {0~, G, 1~} be an IFT on X, where G = (x, (0.5a, 0.4_b), (0.5a, 0.6_b)). Then,

IFβO(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0, 1]$ / $0 \le \mu_a + \nu_a \le 1$, $0 \le \mu_b + \nu_b \le 1$}.

The IFTS (X, τ) is an IFβGα connected between the two IFSs A = ⟨x, (0.3, 0.2), (0.7, 0.8)⟩ and B = ⟨x, (0.8, 0.8), (0.2, 0.2)⟩ as there exists no IFβGαOS E such that A ⊆ E and E $_q{}^c$ B.

Theorem 28: If an IFTS (X, τ) is IFβGα connected between two IFSs A and B, then it is IFC_5-connected between two IFSs A and B but the converse my not be true in general.

Proof: Suppose (X, τ) is not IFC_5-connected between A and B, then there exists an IFOS E in (X, τ) such that A ⊆ E and E $_q{}^c$ B. Since every IFOS is an IFβGαOS, there exists an IFβGαOS E in (X, τ) such that A ⊆ E and E $_q{}^c$ B. This implies (X, τ) is not IFβGα connected between A and B, a contradiction to our hypothesis. Therefore (X, τ) is IFC_5-connected between A and B.

Example 29: Let X = {a, b} and τ = {0~, A, 1~} be an IFT on X, where M = (x, (0.5a, 0.4_b), (0.5a, 0.6_b)).

IFβO(X) = {0~, 1~ , $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1$}.

Then (X, τ) is an IFC_5-connected between the IFSs A = (x, (0.3a, 0.2_b), (0.7a, 0.8_b)) and B = (x, (0.6a, 0.6_b), (0.4a, 0.4_b)). But (X, τ) is not an IFβGα connected between

A and B, since the IFS $E = (x, (0.3_a, 0.3_b), (0.7_a, 0.7_b))$ is an IFβGαOS such that $A \subseteq E$ and $E \subseteq B^c$.

Theorem 30: An IFTS (X, τ) is IFβGα connected between two IFSs A and B if and only if there is no IFβGαOS and IFβGαCS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Proof: Necessity: Let (X, τ) be IFβGα connected between two IFSs A and B. Suppose that there exists an IFβGαOS and IFβGαCS E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E\ {}_q{}^c\ B$ and $A \subseteq E$. This implies (X, τ) is not IFβGα connected between A and B, by Definition 26. A contradiction to our hypothesis. Therefore there is no IFβGαOS and IFβGαCS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not IFβGα connected between A and B. Then there exists an IFβGαOS E in (X, τ) such that $A \subseteq E$ and $E\ {}_q{}^c\ B$. This implies that there is no IFβGαOS E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) is IFβGα connected between A and B.

Theorem 31: If an IFTS (X, τ) is IFβGα connected between two IFSs A and B, $A \subseteq A_1$ and $B \subseteq B_1$, then (X, τ) is IFβGα connected between A_1 and B_1.

Proof: Suppose that (X, τ) is not IFβGα connected between A_1 and B_1, then by Definition, there exists an IFβGαOS E in (X, τ) such that $A_1 \subseteq E$ and $E\ {}_q{}^c\ B_1$. This implies $E \subseteq B_1{}^c$ and $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1{}^c$, $B_1 \subseteq E^c$, $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X, τ) is not IFβGα connected between A and B, which is a contradiction to our hypothesis. Thus (X, τ) is IFβGα connected between A_1 and B_1.

Theorem 32: Let (X, τ) be an IFTS and A and B be IFSs in (X, τ). If $A\ {}_q\ B$, then (X, τ) is IFβGα connected between A and B.

Proof: Suppose (X, τ) is not IFβGα connected between A and B. Then there exists an IFβGαOS E in (X, τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is $A\ {}_q{}^c\ B$. But this is a contradiction to our hypothesis. Therefore (X, τ) is IFβGα connected between A and B.

Definition 33: An IFS A is called an intuitionistic fuzzy regular β generalized α open set (IFRβGαOS for short) if A = βgαint(βgαcl(A)). The complement of an IFRβGαOS is called an intuitionistic fuzzy regular β generalized α closed set (IFRβGαCS for short).

Definition 34: An IFTS (X, τ) is called an intuitionistic fuzzy β generalized α super (IFβGα super for short) connected space if there exists no proper intuitionistic fuzzy regular β generalized α open set in (X, τ).

Theorem 35: Let (X, τ) be an IFTS, then the following are equivalent.

(i) (X, τ) is an IFβGα super connected space

(ii) For every non-zero IFRβGαOS A, $\beta g\alpha cl(A) = 1_\sim$

(iii) For every IFRβGαCS A with $A \neq 1_\sim$, $\beta g\alpha int(A) = 0_\sim$

(iv) There exists no IFRβGαOSs A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$

(v) There exists no IFRβGαOSs A and B in (X, τ) such that $A \neq 0_\sim \neq B$, $B = (\beta g\alpha cl(A))^c$, $A = (\beta g\alpha cl(B))^c$

(vi) There exists no IFRβGαCSs A and B in (X, τ) such that $A \neq 1_\sim \neq B$, $B = (\beta g\alpha int(A))^c$, $A = (\beta g\alpha int(B))^c$

Proof:

(i)⇒(ii) Assume that there exists an IFRβGαOS A in (X, τ) such that $A \neq 0_\sim$ and $\beta g\alpha cl(A) \neq 1_\sim$. Now let $B = \beta g\alpha int(\beta g\alpha cl(A))^c$. Then B is a proper IFRβGαOS in (X, τ). But this is a contradiction to the fact that (X, τ) is an IFβGα super connected space. Therefore $\beta g\alpha cl(A) = 1_\sim$.

(ii) ⇒ (iii) Let $A \neq 1_\sim$ be an IFRβGαCS in (X, τ). If $B = A^c$, then B is an IFRβGαOS in (X, τ) with $B \neq 0_\sim$. Hence $\beta g\alpha cl(B) = 1_\sim$, by hypothesis. This implies $(\beta g\alpha cl(B))^c = 0_\sim$. That is $\beta g\alpha int(B^c) = 0_\sim$. Hence $\beta g\alpha int(A) = 0_\sim$.

(iii) ⇒ (iv) Suppose A and B be two IFRβGαOSs in (X, τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$. Since B^c is an IFRβGαCS in (X, τ) and $B \neq 0_\sim$ implies $B^c \neq 1_\sim$, $B^c = \beta g\alpha cl(\beta g\alpha int(B^c))$ and we have $\beta g\alpha int(B^c) = 0_\sim$. But $A \subseteq B^c$. Therefore $0_\sim \neq A = \beta g\alpha int(\beta g\alpha cl(A)) \subseteq \beta g\alpha int(\beta g\alpha cl(B^c)) = \beta g\alpha int(\beta g\alpha cl(\beta g\alpha cl(\beta g\alpha int(B^c)))) = \beta g\alpha int(\beta g\alpha cl(\beta g\alpha int(B^c))) = \beta g\alpha int(B^c) = 0_\sim$. A contradiction arises. Therefore (iv) is true.

(iv)⇒(i) Suppose $0_\sim \neq A \neq 1_\sim$ be an IFRβGαOS in (X, τ). If we take $B = (\beta g\alpha cl(A))^c$, then B is an IFRβGαOS, since $\beta g\alpha int(\beta g\alpha cl(B)) = \beta g\alpha int(\beta g\alpha cl(\beta g\alpha cl(A))^c) = \beta g\alpha int(\beta g\alpha int(\beta g\alpha cl(A)))^c = \beta g\alpha int(A^c) = (\beta g\alpha cl(A))^c = B$. Also we get $B \neq 0_\sim$, since otherwise, if $B = 0_\sim$, this implies $(\beta g\alpha cl(A))^c = 0_\sim$. That is $\beta g\alpha cl(A) = 1_\sim$. Hence $A = \beta g\alpha int(\beta g\alpha cl(A)) = \beta g\alpha int(1_\sim) = 1_\sim$, which is a contradiction. Therefore $B \neq 0_\sim$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) is an IFβGα super connected space.

(i) ⇒ (v) Suppose A and B are any two IFRβGαOSs in (X, τ) such that $A \neq 0_\sim \neq B$, $B = (\beta g\alpha cl(A))^c$ and $A = (\beta g\alpha cl(B))^c$. Now we have $\beta g\alpha int(\beta g\alpha cl(A)) = \beta g\alpha int(B^c) = (\beta g\alpha cl(B))^c = A$, $A \neq 0_\sim$ and $A \neq 1_\sim$, since if $A = 1_\sim$, then $1_\sim = (\beta g\alpha cl(B))^c \Rightarrow \beta g\alpha cl(B) = 0_\sim \Rightarrow B = 0_\sim$. But $B \neq 0_\sim$. Therefore $A \neq 1_\sim \Rightarrow A$ is a proper IFRβGαOS in (X, τ), which is a contradiction to (i). Hence (v) is true.

(v) ⇒ (i) Suppose A is an IFRβGαOS in (X, τ) such that $0_\sim \neq A \neq 1_\sim$. Now take $B = (\beta g\alpha cl(A))^c$. In this case we get $B \neq 0_\sim$ and B is an IFRβGαOS in (X, τ), $B = (\beta g\alpha cl(A))^c$ and $(\beta g\alpha cl(B))^c = (\beta g\alpha cl((\beta g\alpha cl(A))^c)^c = \beta g\alpha int(\beta g\alpha cl(A)^c)^c = \beta g\alpha int(\beta g\alpha cl(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) is an IFβGαS connected space.

(v) ⇒ (vi) Suppose A and B be two IFRβGαCSs in (X, τ) such that $A \neq 1_\sim \neq B$, $B = (\beta g\alpha int(A))^c$ and $A = (\beta g\alpha int(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become IFRβGαOSs in (X, τ) with $C \neq 0_\sim \neq D$, $D = (\beta g\alpha cl(C))^c = (\beta g\alpha cl(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi) ⇒ (v) It can be proved easily by the similar way as in (v) ⇒ (vi).

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