

Numerical Investigations of Hydraulic Transients in Pipelines having Centrifugal Pumps

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Abstract:

This paper focus on the different between the diagnostic results of hydraulic grade line system. Basic fluid equations solved either, in the time domain, using classical method of characteristics (MOC) and compare the results with Laplace transformations method and Fourier transformations method. Laplace transform solution approach overcomes this difficulty accordingly, the results for the pipeline system having varying demand showed that the Laplace transformation sense to wave pressure occur due to suddenly increase in flow rate either than Fourier transformations method. By applying these method on assumed network having suddenly change in flowrate.

Keywords: Hydraulic Transients.

INTRODUCTION

The transients of hydraulic is time varying system. The flow in the pipe transients is defined by set of hyperbolic calculations consequent after the preservation of mass and Newton’s law in the motion [1, 2, 3, 10, 11, 12], and Mathematically the method of characteristics (MOC) is used for the calculations and other numerical methods [3,4,5,6] with reasonable success.

Theoretical analysis (Characteristic Equations)

Generally, the spatial variation is fewer important in defining the performance than the time-varying, two independent partial differential equations [7]

$$\frac{\partial V}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + \frac{f}{2D} V|V| = 0 \dots\dots\dots(1)$$

$$a^2 \frac{\partial V}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial t} = 0 \dots\dots\dots(2)$$

λ is a constant and named a Lagrange multiplier

$$\lambda \left(\frac{\partial V}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + \frac{f}{2D} V|V| \right) + \left(a^2 \frac{\partial V}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial t} \right) = 0 \dots\dots\dots(3)$$

Regrouping terms,

$$\left(\lambda \frac{\partial V}{\partial t} + a^2 \frac{\partial V}{\partial s} \right) + \left(\frac{1}{\rho} \frac{\partial p}{\partial t} + \frac{\lambda}{\rho} \frac{\partial p}{\partial s} \right) + \lambda g \frac{dz}{ds} + \frac{\lambda f}{2D} V|V| = 0 \dots\dots\dots(4)$$

The partial equations are changed by two pair of ordinary differential equations as shown

$$\frac{dV}{dt} - \frac{g}{a} \frac{dH}{dt} + \frac{f}{2D} V|V| = 0 \text{ only when } \frac{ds}{dt} = -a \dots\dots\dots(5)$$

$$\frac{dV}{dt} - \frac{g}{a} \frac{dH}{dt} + \frac{f}{2D} V|V| = 0 \text{ only when } \frac{ds}{dt} = -a \dots\dots\dots(6)$$

Finite Difference

The equations (5) and (6) have new representation

$$\frac{V_P - V_{Le}}{t_P - 0} + \frac{g}{a} \frac{H_P - H_{Le}}{t_P - 0} + \frac{f}{2D} V_{Le} |V_{Le}| = 0 \dots\dots\dots(7)$$

$$\frac{V_P - V_{Ri}}{t_P - 0} - \frac{g}{a} \frac{H_P - H_{Ri}}{t_P - 0} + \frac{f}{2D} V_{Ri} |V_{Ri}| = 0 \dots\dots\dots(8)$$

The term (tp - 0) is replaced with (Δt) then the new form gives

$$C^+ : (V_P - V_{Le}) + \frac{g}{a} (H_P - H_{Le}) + \frac{f\Delta t}{2D} V_{Le} |V_{Le}| = 0 \dots\dots\dots(9)$$

And

$$C^- : (V_P - V_{Ri}) - \frac{g}{a} (H_P - H_{Ri}) + \frac{f\Delta t}{2D} V_{Ri} |V_{Ri}| = 0 \dots\dots\dots(10)$$

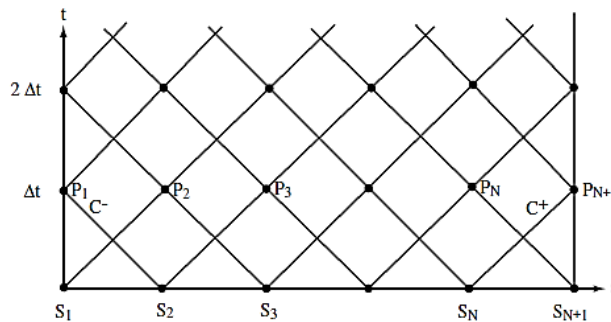


Figure 1. The characteristic grid for a single pipe

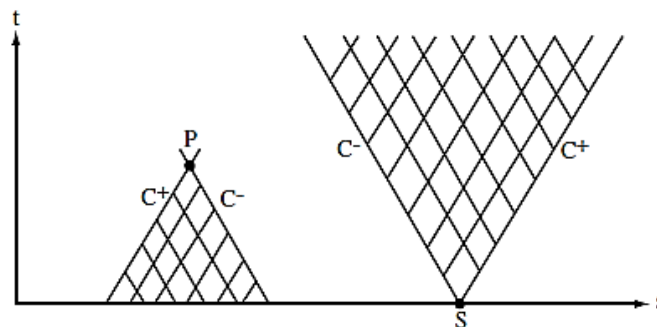


Figure 2: Disturbance propagation in the s-t plane

Numerical Process

The (H) value and (V) value which placed in the ends of pipe are founded by boundary conditions. The equations are developed to calculate (H) and (V) at the inside. Solving (9) and (10) to get

$$V_P = \frac{1}{2} \left[(V_{Le} + V_{Ri}) + \frac{g}{a} (H_{Le} - H_{Ri}) - \frac{f\Delta t}{2D} (V_{Le} |V_{Le}| + V_{Ri} |V_{Ri}|) \right] \dots\dots\dots(11)$$

$$H_P = \frac{1}{2} \left[\frac{a}{g} (V_{Le} - V_{Ri}) + (H_{Le} + H_{Ri}) - \frac{a}{g} \frac{f \Delta t}{2D} (V_{Le} |V_{Le}| - V_{Ri} |V_{Ri}|) \right] \dots\dots\dots(12)$$

The Pipeline with varying demand discharge

For this case the pipeline can be represented and divided by the intakes gives a new demand discharge with very small neighborhood (2ϵ) as explained in Figure (3) [3]

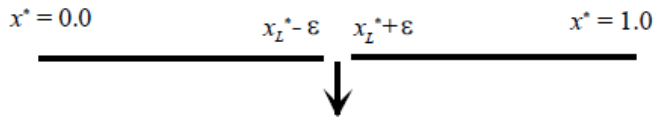


Figure 3: A pipeline with varying discharge

Figure 4 shows the block diagram of surge phenomenon.

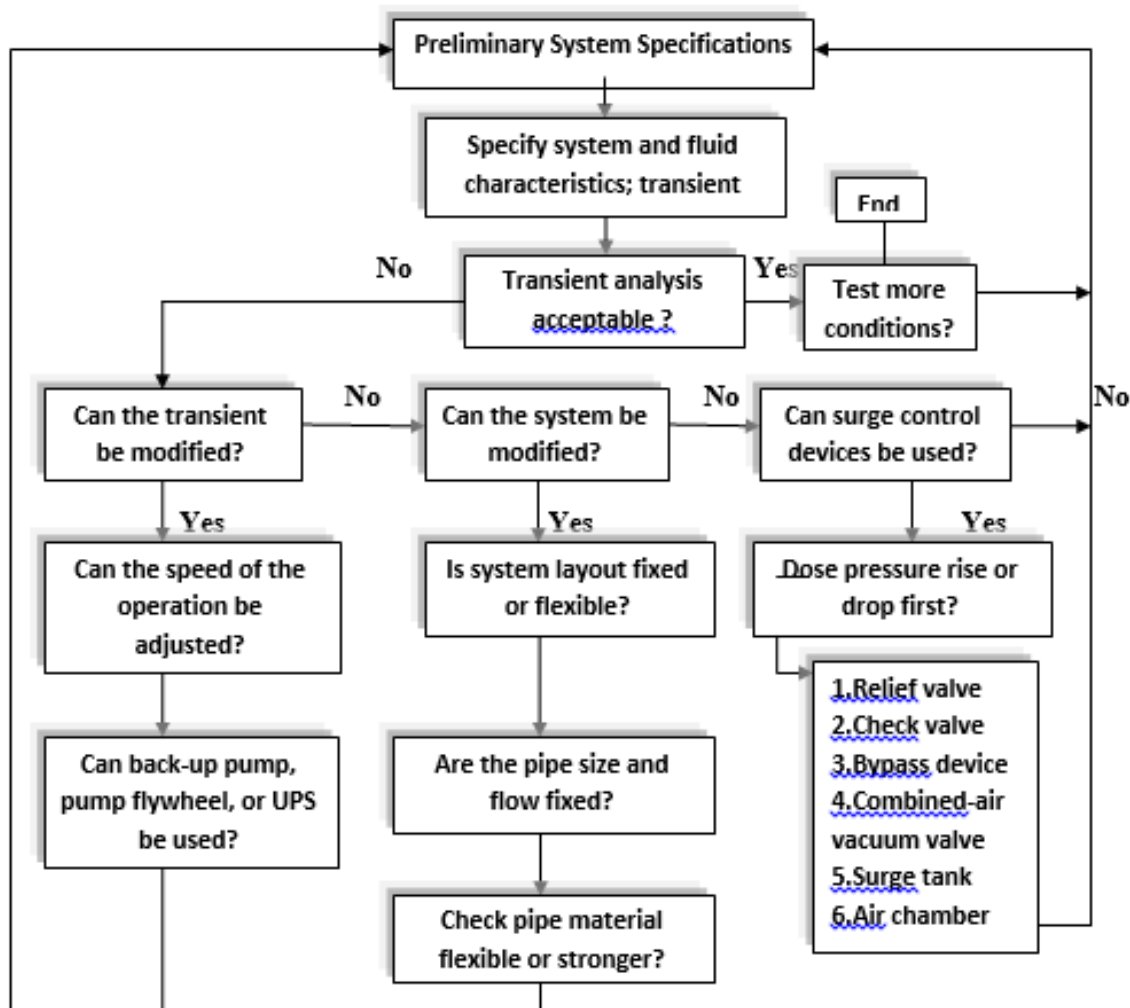


Figure 4: Block diagram of transient flow analysis in pipeline systems

HYDRAULIC APPLICATIONS

Figure 5 explain one of the most case used to study to analysis the system, the pipeline model contains six pipes, the characteristics of this system is shown in table (1). The six nodes data about are shown in table (2), the pump is combined

into a net and situated at one of 3 reservoirs. The analyzing take up the demand release is rapidly increased from (50 gpm) to (790 gpm) unpaid pump on pipe six revive to operate at (20 sec.) and the transient occur down the pump, the wave speed is about 3000 ft. /s also the friction factor (0.02) for all pipes, [7].

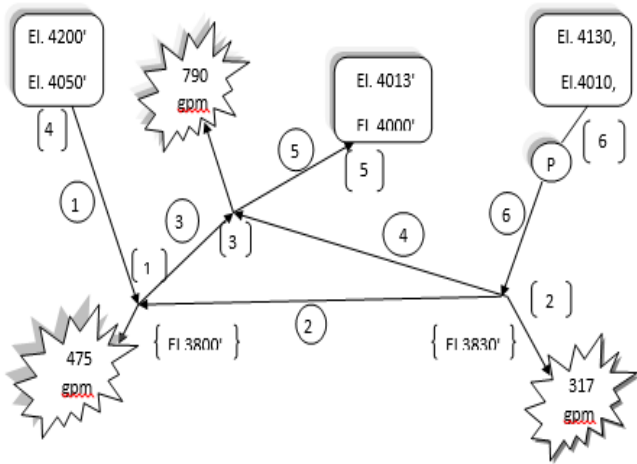


Figure 5: Pipeline system

Table 1: Pipeline data

Pipe No.	Nodes From	to	Length Ft.	Dia. In.	f	Q gpm	Vel. Ft/s	H _L ft.
1	4	1	3300	12.0	0.02	340.1	0.96	0.40
2	2	1	8200	8.0	0.02	273.0	1.47	1.91
3	1	3	3300	8.0	0.02	138.1	0.88	0.54
4	2	3	4900	12.0	0.02	1110.0	3.15	3.57
5	3	5	3300	6.0	0.02	458.1	5.20	20.27
6	6	2	2600	14.0	0.02	1700.0	3.54	3.71

Table 2: Nodes data

Node	Demand gpm	El. Ft.	Head Ft.	Pressure Psi.	HGL El. Ft.
1	475	3800	398.68	172.76	4198.68
2	317	3830	384.38	166.57	4214.38
3	790	3770	426.89	184.99	4196.89
6	-1700	4010	214.03	92.74	4224.03
4	-340	4050	150.00	65.00	4200.00
5	458	4000	130.00	56.33	4130.00

CONCLUSIONS

In the analysis and observations of the method used in this paper, which depend on Laplace transform method for accurate sensing the pressure wave when suddenly change in the pumping rate network systems. Figure. (5) obviously exposed the wave pressure accure in pipe number six in the proposed work after change in the rate of flow. The Laplace provides the pressure better than others methods. Same results are got when applying the model on pipe number two.

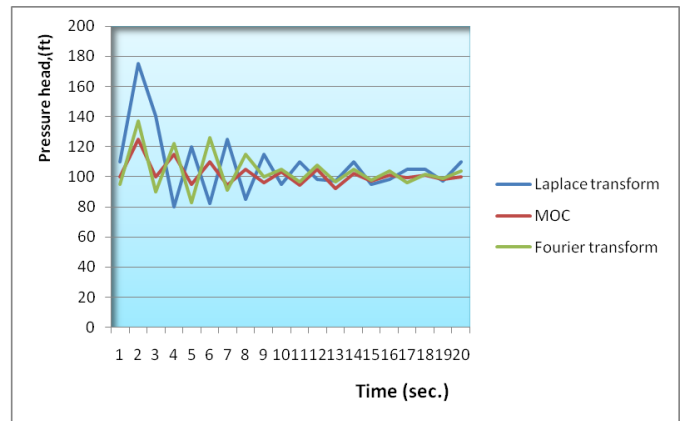


Figure 5: Pressure head downstream the pump in pipe six.

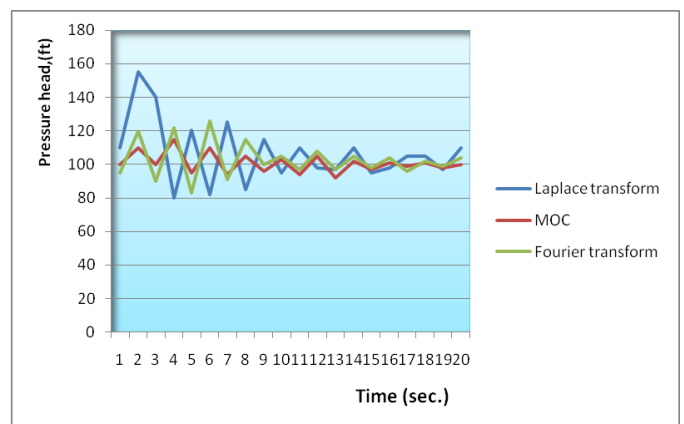


Figure 6: Pressure head downstream the pump in pipe two .

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NOMENCLATURE

Symbol	Quantity	Unit		Dimension
		SI	English	
A	Cross section area of pipe	m ²	ft ²	L ²
A _n	Fourier coefficients	Dimensionless		
A _p	Polynomial constants	Dimensionless		
a	Wave celerity	m/s	ft/s	L/t
a _{IC} (x*) and b _{IC} (x*)	Piecewise continuous functions	Dimensionless		
C _d A _L	Effective intake area	m ²	ft ²	L ²
D	Pipe diameter	m	ft	L
F _i = C _d A _L a/ A√2Gh _{L0}	Intake parameter	Dimensionless		
f	Friction factor	Dimensionless		
g	Gravitational acceleration	m/s ²	ft/s ²	L/t ²
H	Transient head	m	ft	L
H ₀	Steady state head	m	ft	L
H ₁	Reference head in a pipe	m	ft	L
H _{L0}	Steady state head at the intake	m	ft	L
h _p	Pump head	m	ft	L
I	Scale factor	Dimensionless		
P	Pressure	Pa	lb/ft ²	ML ⁻¹ t ⁻²
Q ₀	Steady state flow rate	m ³ /s	ft ³ /s	L ³ t ⁻¹
R= fLQ ₀ /2DAa	Resistance term	Dimensionless		
s	Space	m	ft	L
t* = t / (L/a)	Dimensionless time	Dimensionless		
x* = x/L	Dimensionless distance	Dimensionless		
V	Average velocity	m/s	ft/s	Lt ⁻¹
x _L * = x _L /L	Dimensionless intake function	Dimensionless		
ρ	Mass density	Kg/m ³	lb/ft ³	ML ⁻³
λ	Lagrange multiplier	Dimensionless		
sub scribt) _P	Node value for finite deference			
sub scribt) _{Le}	Left value for finite deference			
sub scribt) _{Ri} :	Right value for finite deference			
δ(x* - x _L)*	Dirac delta function	Dimensionless		
ε	small neighborhood	m	ft	L