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Design and Evaluation of Integer Dual Tree Complex Wavelet Transform Filter Coefficients for Image processing

A. Hazarathaiah, and Dr. B. Prabhakar Rao

Jawaharlal Nehru Technological University, -JNTU, Kakinada, India

Abstract

In this paper, integer Dual Tree Complex Wavelet Transform (DTCWT) filters are derived from basic DTCWT filters proposed by Kingsbury, Selesnick and Iulian. The integer filters are designed considering hardware complexity and they are evaluated to support three properties of shift invariance, directionality and phase information. The energy levels of proposed integer filter demonstrate less than 2% variation compared with their counter parts. The similarity index measured in terms of joint histogram shows 16% improvement as compared with Kingsbury DTCWT filters. The integer DTCWT filters are represented using 8-bit signed representation and is used in image registration process. The procedures for deriving integer filter coefficients from basic wavelet filters serve as reference model.

Keywords: DTCWT, Integer Wavelet Filters, Image Registration, Filter Evaluation, Joint Histogram

INTRODUCTION

Wavelet Transform is a mathematical tool for analysis of non-stationary or transient signals and is oscillatory in nature and time limited so as to describe the time-frequency representation of input signal. The Mother Wavelet represented by Ψ(t) from which family of wavelets can be constructed by scaling and translation of the mother wavelet. The advantage of wavelet is that a set of wavelet family can be constructed to make it compatible to the required resolution in both time-frequency plane representing input signal characteristics. The first wavelet was introduced by Haar in 1909, further in 1946 Denis Gabor introduced Gabor function for signal representation [1,2]. In 1975, George introduced the continuous wavelets for neurobiology, Morlet also worked on shorter duration functions for study of seismology. Grossmann and Morlet together in 1982 introduced continuous wavelet transforms. Meyer, Daubechies together in 1988 and Stephen Mallat [3.] introduced the biorthogonal wavelets with compact support and filter bank structure for Discrete Wavelet Transform. The work carried out by Mallat [4.] for wavelet implementation is very much associated with wavelet theory and multiresolution analysis. Limitations of wavelets such as shift sensitivity and poor directionality have been reported by Kingsbury in 1998 [5]. The input signal if it is translated and wavelet transform is computed the output coefficients are found to have unpredictable change in energy as compared with the energy computed on input signal without translation demonstrating failure of shift insensitive property. The wavelet filters are not able to resolve between diagonal features in the HH band but are effective in capturing the horizontal features and vertical features in the LH and HL band respectively. For analysis of image signals singularities such as lines and edges which are oriented and curved are not captured by separable transforms and with introduction of Complex Wavelets Transforms these limitations were addressed. Fourier Transforms do not have these limitations as the transform is based on complex valued oscillating sinusoid basis functions of cosine and sine components that form Hilbert transform pair. Kingsbury in 1999[6] and Selesnick [7] introduced the Kingsbury Dual Tree Complex Wavelet Transform and Selesnick's Dual Tree Complex Wavelet Transform to overcome limitations of shift sensitivity and directionality by having real and imaginary filter banks with filter coefficients being Hilbert transform pair similar to Fourier transforms pairs. Kingsbury in 2001[8] presented Q-shift filters that are of odd/even length for different levels is arrived based on approximations and is a complicated procedure. The filters introduced by Kingsbury achieve approximate shift invariance with redundancy of 2^m: 1. The filters were shown to be successful in analysis of images for motion estimation, denoising, texture synthesis and retrieval applications. Medical image registration is one of the prime image processing applications and with advances in technology 3D imaging is in demand for medical diagnosis. In image registration, extraction of features from reference image and the image to be registered is carried out using wavelet transform. The selected features are compared to measure similarity index and transformation of input image is carried out to register with reference image. Using Kingsbury dual tree wavelet filters supporting Q-shift property for image registration is presented that uses motion estimation approach for image registration. The computation complexity of image registration process limits its use for real time applications. In this paper, new filter coefficients that support shift invariance and directional property are arrived from biorthogonal wavelet filters and extended to be used for dual tree complex wavelet transform. Section II discusses wavelet filter evaluation for image registration, section III discusses design of new wavelet filters for image registration algorithm, section IV discusses the results and section V is conclusion.

WAVELET FILTER EVALUATION

The input signal x(n) is processed by the low pass filter $h_0(n)$ and high pass filter $h_1(n)$ to generate $r_1(n)$ the reference signal and the details signal $d_1(n)$ respectively that are obtained after discarding the odd samples by a factor of 2.

The reference signal $r_1(n)$ is further filtered by the $h_0(n)$ and $h_1(n)$ filter pair by the second level. This process is iteratively repeated and after L levels of decomposition the reference signal $r_L(n)$ with resolution reduced by factor of 2^L is obtained along with the detail signal d(n) at levels L, L-1, …..1. The input signal can be reconstructed by considering the reference signal and the detail signal at each level to obtain the next higher level. The L level decomposition of input signal to obtain $r_L(n)$ signal is represented by the filter using cascaded form as in Eq. (1), whose length is $(2^L-1)(N+1)+1$ [9].

$$h^{(L)}(n) = h(n) * h\left(\frac{n}{2}\right) * h\left(\frac{n}{4}\right) * \dots * h\left(\frac{n}{2^L}\right) \tag{1}$$

The wavelet transform scaling functions $\phi_A(x)$ and $\phi_s(x)$ need to satisfy the difference equations as in Eq. (2), and the continuous wavelets $\psi_A(x)$ and $\psi_s(x)$ defined in terms of scaling and coefficients of high pass filter as in Eq. (3).

$$\phi_A(x) = \sum_n 2^{1/2} h_0(n) \phi_A(2x-n)$$

$$\phi_s(x) = \sum_n 2^{1/2} g_0(n) \phi_A(2x-n) \tag{2}$$

$$\psi_s(x) = \sum_n g_1(n) \psi_s(2x-n) \tag{3}$$

Determining $\phi_A(x)$ and $\phi_s(x)$ are very important in selection of filter coefficients for suitable image processing applications. Design of filter for wavelet transforms needs to meet the constraints of perfect reconstruction, filter length, regularity and linear phase. In order to eliminate aliasing effects it is required to meet the conditions in Eq. (4), and for perfect reconstruction Eq. (5) need to be satisfied.

$$g_1(n) = (-1)^{n+1} h_0(n)$$

$$h_1(n) = (-1)^n g_0(n) \tag{4}$$

$$p(2n+1) = \delta_n M \tag{5}$$

Where, $g_0(n)$ and $g_1(n)$ represent the synthesis low pass and high pass filters, p(n) = $h_0(n) * g_0(n)$ M is the index of p(n). In addition to the above constraints the filter bank should also be shift invariants, which are measured by analyzing filters to be minimum shift variance and evaluating side lobe strengths. Strang [10] and Mallat [11] have analyzed wavelet transform for its shift variance property and have reported that the wavelet filters are disadvantageous for signal and image processing applications. In addition to shift variant limitations Simoncelli et al [12] has also mentioned the scaling and orientation as limitations of wavelets. Selection of filter depends upon constraints such as shift variant, impulse response peak to side lobe ratio, step response oscillations and regularity is an open ended challenge. Villasenor et al. in 1995 have reported that the 6/10 biorthogonal filter is minimum shift invariant as compared with all other biorthogonal filters including 9/7 filter. The even length filter is evaluated for its PSNR, impulse response peak to side lobe ratio and step response second side lobe strength by considering more than eight test images. From the results obtained it is reported by Villasenor et al. that the even length biorthogonal filter with H_0 ={0.788486, 0.047699, -0.129078} and G_0={0.615051, 0.133389, -0.067237, 0.006989, 0.081914} with lengths of 6 & 10 respectively is less shift variant and also preserves features such as shape, location and intensity of the input signal being analyzed. Table 1 shows the filter coefficients proposed by Villasenor et al.

FILTER DESIGN FOR DTCWT

For design of filters for dual tree complex wavelet transform the filter need to satisfy the desired properties such as half-sample delay, perfect reconstruction, finite support, vanishing moment, good stop band and linear phase. The half sample delay condition [13] in terms of magnitude and phase is given as in Eq. (6),

$$|G_0(e^{j\omega})| = |H_0(e^{j\omega})|,$$

$$\angle G_0(e^{j\omega}) = \angle H_0(e^{j\omega}) - 0.5\omega \tag{6}$$

The dual tree filter is designed by considering existing wavelet filter, in this work the existing wavelet filter is the 6/10 even filter. There are three methods for design of dual tree filter: linear phase biorthogonal solution, q-shift solution and common factor solution. In this paper, common factor solution is used for design of dual tree filter.

Common factor solution

The common factor solution was introduced by Selesnick in 2002 [14] for design of dual tree CWT filters from orthogonal and biorthogonal filters. The biorthogonal filters h_0 and g_0 from which dual tree filters are designed need to be approximately linear and form Hilbert transform pair (Ψ represents the corresponding wavelet function) and satisfy Eq. (7) and (8) respectively.

$$g_0(n) = h_0(N-1-n) \tag{7}$$

$$\psi_g(t) = \mathcal{H}\{\psi_h(t)\}$$

where,

$$\psi_g(\omega) = \begin{cases} -j\psi_h(\omega), & \omega > 0 \\ j\psi_h(\omega), & \omega < 0 \end{cases} \tag{8}$$

Figure 1 shows the eight filters of dual tree CWT that need to be designed from basic filters. The dual scaling functions and wavelets are denoted by $\bar{\phi}_h(t)$, $\bar{\psi}_h(t)$, $\bar{\phi}_g(t)$ and $\bar{\psi}_g(t)$, and. The dual scaling function $\bar{\phi}_h(t)$ and the wavelet $\bar{\psi}_h(t)$ are expressed as in Eq. (9).

$$\bar{\phi}_h(t) = \sqrt{2} \sum_n \bar{h}_0(n) \bar{\phi}_h(2t-n)$$

$$\bar{\psi}_h(t) = \sqrt{2} \sum_n \bar{h}_1(n) \bar{\phi}_h(2t-n) \tag{9}$$

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 5796-5808

For arriving at dual tree filters it is required that the synthesis and analysis wavelet filters form Hilbert transform pair as in Eq. (10).

$$\psi_g(t) = \mathcal{H}\{\psi_h(t)\}$$
$$\text{and} \qquad \bar{\psi}_g(t) = \mathcal{H}\{\bar{\psi}_h(t)\} \qquad (10)$$

The product of biorthogonal filters of dual tree are defined as in Eq. (11) for H-tree and G-tree, then the biorthogonal conditions for the product filters are as in Eq. (12), which implies that both the product filter p_h and p_g is half band.

$$p_h(n) \coloneqq \bar{h}_0(n) * h_0(n)$$
$$p(2n + n_0) = \delta(n)_g(n) \coloneqq \bar{g}_0(n) * g_0(n) \qquad (11)$$

$$p_g(2n + n_0) = \delta(n) \qquad (12)$$

From equations 11 and 12 without loss of generality the expression for high pass filters can be written [15] as in Eq. (13)

$$h_1(n) \coloneqq (-1)^n \bar{h}_0(n)$$
$$\bar{h}_1(n) \coloneqq -(-1)^n h_0(n)$$
$$g_1(n) \coloneqq (-1)^n \bar{g}_0(n)$$
$$\bar{g}_1(n) \coloneqq -(-1)^n g_0(n) \qquad (13)$$

Based on the expression presented in Eq. (13) it is required to find the solutions for the terms in Eq. (14).

$$h_0(n) = f(n) * d(n)$$
$$\bar{h}_0(n) = \bar{f}(n) * d(L - n)$$
$$g_0(n) = f(n) * d(L - n)$$
$$\bar{g}_0(n) = \bar{f}(n) * d(n) \qquad (14)$$

Expressing $h_0(n)$ and $g_0(n)$ in Eq. (14) in terms of Z-transform, Eq. (15) is obtained.

$$H_0(z) = F(z)D(z)\,,$$
$$G_0(z) = F(z)z^{-L}D\left(\frac{1}{z}\right) \qquad (15)$$

From the solutions presented in [16], the half sample delay conditions as presented in Eq. (6) is obtained for Eq. (15) and is presented in Eq. (16). From the equations it is found that the magnitude conditions are satisfied but the phase conditions are not met which is also a mandatory condition.

$$\left|G_0(e^{j\omega})\right| = \left|H_0(e^{j\omega})\right|,$$
$$\angle G_0(e^{j\omega}) \neq \angle H_0(e^{j\omega}) - 0.5\omega \qquad (16)$$

Solution for design of dual tree filter is a two-step process, first the term D(z) is determined and second the F(z) is determined so as to satisfy perfect reconstruction property of $h_0(n)$ and $g_0(n)$.

Computing D(Z)

Substituting for F(z) in Eq. 15, $G_0(z)$ is expressed as in Eq. (17).

$$G_0(z) = H_0(z)A(z) \qquad (17)$$

Where

$A(z) \coloneqq \frac{z^{-L}D\left(\frac{1}{z}\right)}{D(z)}$ is a all pass transfer function and has the property such that $\left|A(e^{j\omega})\right| = 1$

and D(z) is given by Eq. (18).

$$D(z) = 1 + \sum_{n=1}^{L} d(n)z^{-n} \qquad (18)$$

The terms $d(n)$ is given by Eq. (19) with the rising factorial term (x)n given by Eq. (20).

$$d(n) = (-1)^n \binom{L}{n} \frac{(\tau - L)_n}{(\tau + L)_n} \qquad (19)$$

$$(x)_n \coloneqq (x)(x+1)\ldots(x+n-1) \qquad (20)$$

The coefficients of $d(n)$ given in Eq. (19) can be computed by using the ratio term in Eq. (21)

$$\frac{d(n+1)}{d(n)} = -\frac{\binom{L}{n+1}}{\binom{L}{n}} \cdot \frac{(\tau - L)_{n+1}}{(\tau - L)_n} \cdot \frac{(\tau + L)_{n1}}{(\tau + L)_{n+1}}$$
$$= \frac{(L-n)(L-n-\tau)}{(n+1)(n+1+\tau)} \qquad (21)$$

From the ratio expression in Eq. (21), the filter d(n) can be generated as in Eq. (22).

d(0) =1

$$d(n+1) = d(n)\,\frac{(L-n)(L-n-\tau)}{(n+1)(n+1+\tau)}\,, 0 \leq n \leq L-1 \qquad (22)$$

The expressions presented for the maximally flat delay all-pass filter is adapted from Thiran's formula for the maximally flat delay all-pole filter [17].

The MATLAB code for generation of filter coefficients is presented in Eq. (22).

```
n=0: L-1;
     d= cumprod([1, (L-n) .*(L-n-t ) ./(n+1) ./(n+1+t)]
```

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 5796-5808

Computing F(z)

To determine F(z) it is required to satisfy perfect reconstruction property and it is required for the system to be linear for which arbitrary number of vanishing moments are specified for the wavelets. The terms $F(z)$ and $\bar{F}(z)$ to have zero moment properties it is required to express them in the form as in Eq. (23).

$$F(z) = Q(z)(1 + z^{-1})^{K}$$
$$\bar{F}(z) = \bar{Q}(z)(1 + z^{-1})^{\bar{K}} \qquad (23)$$

K and $\bar{K}$are the number of zero moments of synthesis and analysis filters respectively and they are odd. The product filter terms presented in Eq. (11) are expressed by considering the terms in Eq. (23) as in Eq. (24).

$$P(z) \coloneqq P_h(z) = P_g(z)$$
$$= Q(z)\bar{Q}(z)(1 + Z^{-1})^{K+\tilde{K}} D(z) D\left(\frac{1}{z}\right) z^{-L} \qquad (24)$$

The half band property which is required for filter design is obtained by solving the symmetric odd length sequence r(n) meeting the conditions as in Eq. (25) to be half band.

$$R(z)(1 + z^{-1})^{K+\tilde{K}} D(z) D\left(\frac{1}{z}\right) z^{-L} \qquad (25)$$

The symmetric sequence r(n) is obtained by solving linear system equations as given in the following steps:

(a) Find r(n) of minimal length such that
   a. r(n) = r(-n)
   b. R(z)$(z + 2 + z^{-1})^{k} D(z) D(\frac{1}{z})$ is halfband.

Note that r(n) of minimal length will be supported on the range (1-K-L) $\leq n \leq$(K+L-1). The terms $Q(z)$ and $\bar{Q}(z)$ are obtained by factorizing R(z) as in step 2,

2. Set Q(z) to be a spectral factor of R(z),

$$R(z) = Q(z)Q(1/z).$$

The terms q(n) and $\bar{q}(n)$ are both symmetric. Since these terms are symmetric the terms f(n) and $\bar{f}(n)$ are also symmetric. The terms $h_0(n)$ and $g_0(n)$ are given as in Eq. (26) based on reversal property.

$$g_o(n) = h_o(N - 1 - n)$$
$$\bar{g}_0(n) = \bar{h}_0(N - 1 - n) \qquad (26)$$

Based on the procedures discussed above the filter terms $h_0(n)$, $\bar{h}_0(n)$, $g_o(n)$and $\bar{g}_0(n)$ have linear phase and also satisfy Hilbert transform pair conditions. Table 2 shows the DTCWT filters proposed by Selesnick derived using the method discussed above.

New method for DTCWT filters

The DTCWT filters proposed by Kingsbury demonstrated that the shift invariant property can be achieved at higher levels of decomposition by ensuring that the low pass filter in tree B need to have half sample delay over low pass filters in tree A [18]. Selesnick has demonstrated that if low pass filters from two channels are perfect reconstruction filter banks with half sample delay form Hilbert pair. The algorithms proposed by Kingsbury were not complete in designing DTCWT filters for higher levels. The filters proposed by Selesnick were more appropriate but are approximately linear phase for biorthogonal filters. To overcome the limitations of approximate linear phase Voicu & Borda in 2005 [19] have presented a new method for design of DTCWT filter coefficients. The method proposed designs the filter coefficients for tree-A from which filter coefficients for tree-B are arrived considering Hilbert pair functions. Lagrange Multiplier Method (LMM) is used to minimize two different subjective functions to design filters for tree-A and tree-B. The DTCWT filters obtained by Voicu & Borda method is shown in Table 3. There are 12 coefficients that are symmetry and support biorthogonal property.

Table 1. Villasendor filter coefficients

	h_v	g_v
1	-0.129079	0.018914
2	0.047699	0.006989
3	0.788486	-0.067237
4	0.788486	0.133398
5	0.047699	0.615051
6	-0.129078	0.615051

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 5796-5808

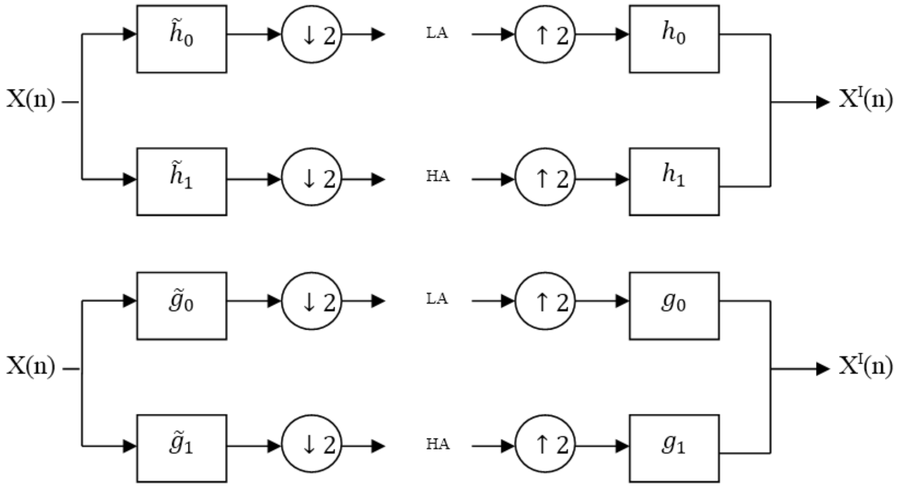


Figure 1. Filter bank for biorthogonal DTCWT

Table 2. DTCWT filters by Selesnick

S.No	h	g
1	-0.105512996386194	-0.021102599277239
2	-0.497787919664400	-0.268378378150790
3	-0.764270772339708	-0.712965714025906
4	-0.294105321868032	-0.605543425035451
5	0.231826057852836	0.047563788370878
6	0.094497819046390	0.214427465912253
7	-0.086267604737123	-0.034799514500489
8	-0.006340085364500	-0.052591871682228
9	0.017300957911461	0.016656062380724
10	-0.003409940503397	0.004786091932740
11	-0.000182423487826	-0.002458804134522
12	0.000038667167385	0.000193335836924

Table 3. DTCWT filters by Voicu & Borda method

S.No	h_i	g_i
1	-0.004872385	0.003335733
2	0.000312740	0.009883406
3	-0.005156990	-0.045254562
4	-0.050058580	-0.118989972
5	0.099268431	0.151650898
6	0.455588330	0.680863020
7	0.455588330	0.680863020
8	0.099268431	0.151650898
9	-0.050058580	-0.118989972
10	-0.005156990	-0.045254562
11	0.000312740	0.009883406
12	-0.004872385	0.003335733

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 5796-5808

Table 4. Integer DTCWT filters

	Villasenor Filters		Selesnick Filters		Voicu & Borda	
S.No	Integer hv	Integer gv	Integer hv	Integer gv	Integer hi	Integer gi
1	-17	2	-108	2	-1	1
2	6	1	-510	1	0	3
3	101	-9	-483	-9	-1	-12
4	101	17	-301	17	-13	-30
5	6	79	237	79	25	39
6	-17	79	97	79	117	174
7	-	17	-88	17	117	174
8	-	-9	-6	-9	25	39
9	-	1	18	1	-13	-30
10	-	2	-3	2	-1	-12
11	-	-	-	-	0	3
12	-	-	-	-	-1	1

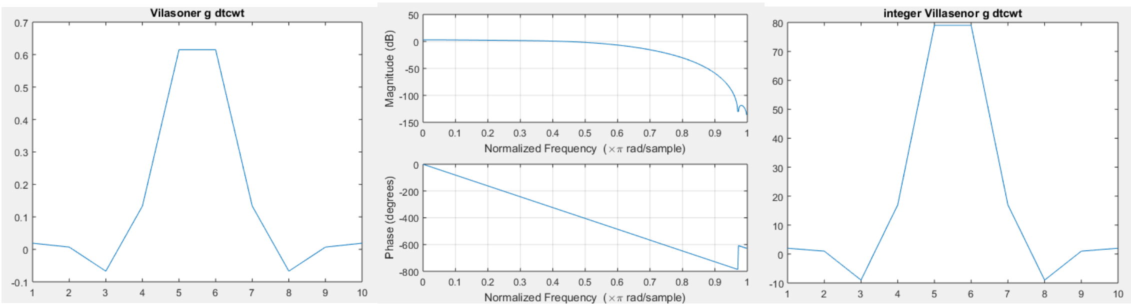


Figure 2. Villasenor G tree filter (a) Time domain plot (b) Frequency response (c) Integer G filter

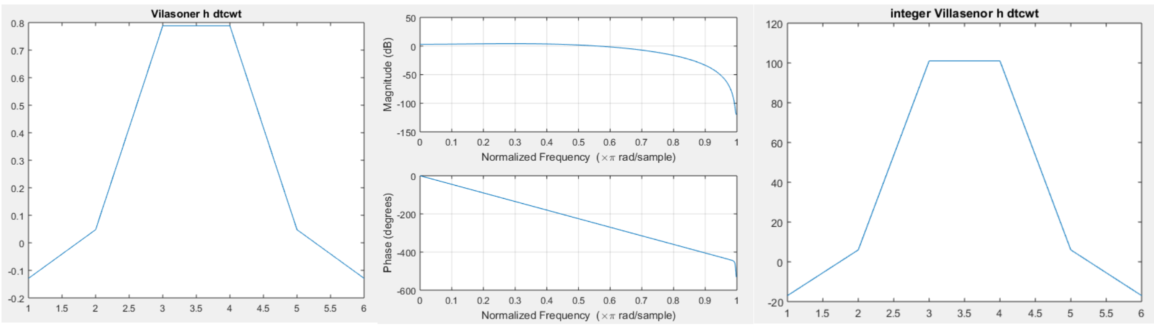


Figure 3. Villasenor H tree filter (a) Time domain plot (b) Frequency response (c) Integer H filter

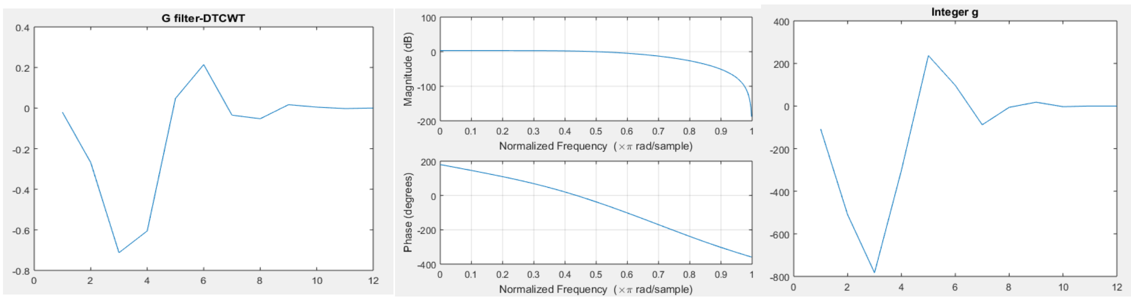


Figure 4. Selesnick G tree filter (a) Time domain plot (b) Frequency response (c) Integer G filter

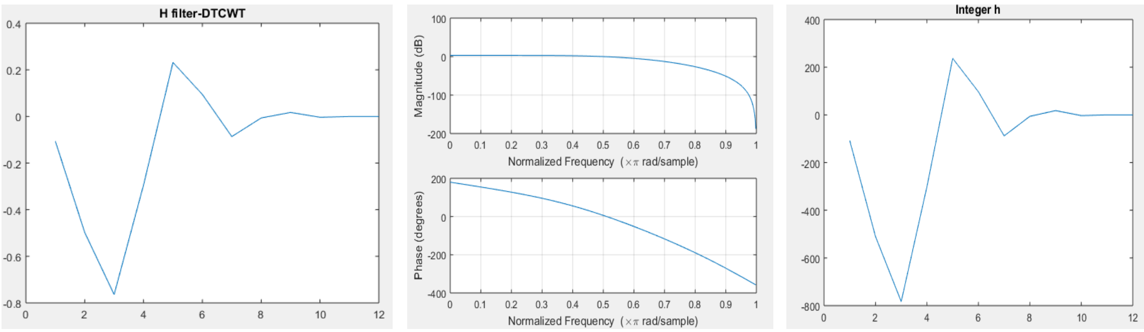


Figure 5. Selesnick H tree filter (a) Time domain plot (b) Frequency response (c) Integer H filter

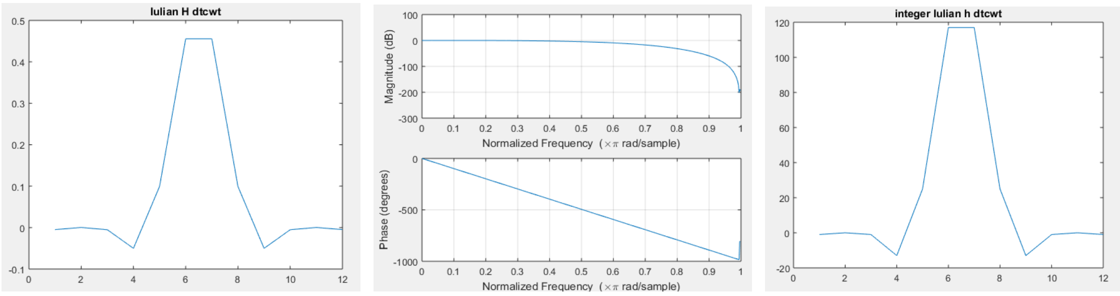


Figure 6. Iulian H tree filter (a) Time domain plot (b) Frequency response (c) Integer H filter

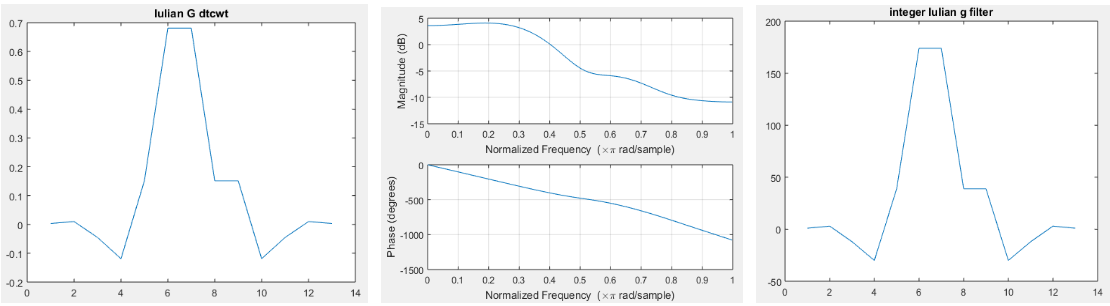


Figure 7. Iulian G tree filter (a) Time domain plot (b) Frequency response (c) Integer G filter

Table 5. Energy levels (in dB) of Ra measure for wavelet and scaling functions of DTCWT

Level	Kingsbury	Integer Equivalent DTCWT	Selesnick	Integer Equivalent DTCWT	Iulian	Integer Equivalent DTCWT
Wavelet						
1	-∞	-∞	-∞	-∞	-∞	-∞
2	-28.25	-29.12	-20.51	-21.11	-08.15	-08.78
3	-23.62	-23.81	-16.86	-17.12	-04.64	-05.01
4	-22.96	-22.01	-18.69	-19.32	-03.63	-04.34
5	-22.81	-23.31	-06.78	-07.43	-03.52	-03.88
Scaling						
1	-∞	-∞	-∞	-∞	-∞	-∞
2	-29.37	-29.89	-21.90	-22.61	-11.06	-11.67
3	-28.17	-28.76	-24.35	-24.69	-08.97	-09.56
4	-27.88	-28.32	-11.72	-12.33	-08.87	-09.77
5	-27.75	-28.41	-09.42	-09.86	-08.86	-09.91

Integer DTCWT Filters

DTCWT filters supporting shift invariant property proposed by Villasenor et al [20], Selesnick and Voicu & Borda have been demonstrated to be suitable for image compression, texture analysis and signal processing applications respectively. One of the major challenges in use of the filters is the complexity in terms of number of bits required to represent the filter coefficients for hardware implementation. Each of the filter require minimum of 16 bits for representation and with 16-bit representation, computation complexity of arithmetic units increase leading to delay, power dissipation and area requirement. In software environment developing image or signal processing algorithms that use DTCWT filters does not account for number of bits required for representation. As DTCWT based signal and image processing requires additional filter banks compared with DWT based processing, in this work integer DTCWT filters are designed that are appropriate for hardware implementation. The integer filters derived are based on the filters proposed by Villasenor et al, Selesnick and Voicu & Borda. The integer DTCWT filters will support all three properties such as shift invariant, directionality and phase information. The integer DTCWT coefficients are derived by modifying Eq. (14) by scaling each of the parameters f and d by a factor of N and M respectively. $N = 2^n$ and $M=2^m$, n and m are integers with the constraint such that $N*M \leq 2^{10}$. The scaled DTCWT coefficients derived from Selesnick filters are represented in Eq. (27).

$$h_0(n) = (N * f(n)) * (M * d(n))$$
$$g_0(n) = (N * f(n)) * (M * d(L - n)) \qquad (27)$$

The integer DTCWT filters that are derived require maximum of 10 bits for its representation. The integer DTCWT filters derived are presented in Table 4.

The maximum value of filter coefficient is 510 which requires 9 bits for representation, considering sign bit the DTCWT filters presented in table 4 requires maximum of 10 bits for representation. The integer filters derived from Voicu & Borda has a maximum value of 174 and there are 12 filters for each filter structure. There are filter coefficients that or of numerical value of 1, by further scaling these coefficients by a factor of 2 and rounding to nearest integer most of the filter coefficients become zero and the number of bits required to represent will be 6 bits. The derived integer filter coefficients are modeled in MATLAB and are analyzed for their properties. Figure 4 and Figure 5 presents the time domain and frequency spectrum of Selesnick filters and integer DTCWT filters derived from Selesnick.

RESULTS AND DISCUSSION

In this section, the integer DTCWT filter coefficients derived are used for image registration process and are used to extract features for similarity measures. In image registration process, the input image which is scaled, rotated and translated is compared with reference image and needs to be registered. With DTCWT filters supporting shift invariant property and is directional selective, it is appropriate to consider image registration as a evaluation procedure to evaluate the performances of DTCWT filters. To the authors knowledge this is the first paper reporting evaluation of DTCWT filters for image registration.

Comparison of DTCWT filters

Time domain plot, magnitude & phase response of three filters are obtained in MATLAB and are compared with the Integer DTCWT filter coefficients derived in this work. For the integer DTCTW filters the time domain plot is obtained. Figure 2 and Figure 3 presents time domain plot and frequency response of Villasenor G tree and H tree low pass filters. In Figure 2(c) and Figure 3(c) the derived integer DTCWT filter response from Villasenor is presented. The time domain plot of both the filters match the only difference is the scaling of filter coefficients.

Figure 6 and 7 presents the time domain and frequency response of Iulian filters and integer DTCWT filters derived from Iulian filters. Frequency responses of integer DTCWT filters exhibit similar behavior of their original filters. The integer DTCWT filters can be used for signal and image processing applications and will introduce minimum loss during reconstruction process.

Based on the Kingsbury method (Eq. 28) [21] for estimating the shift invariance property of DTCWT filters, the energy levels are computed for the proposed integer filters that are derived from Villasenor, Selesnick and Iulian filters and are compared in Table 5.

$$R_a = \frac{\sum_{k=1}^{M-1} \varepsilon\left[A(W^k z)C(z) + B(W^k z)D(z)\right]}{\varepsilon\left[A(z)C(z) + B(z)D(z)\right]} \qquad (28)$$

From the energy levels obtained using Eq. (28), the energy levels of integer DTCWT filters almost match the energy levels of their original counterparts. The Iulian filters energy levels are less than 10dB, indicating that the aliases energy levels are lesser than signal energy levels and hence is found to be better in supporting shift invariance property. In this work, integer DTCWT filters derived from Iulian filters are further evaluated for their performances in extracting features in medical images for performing image registration.

Feature extraction using DTCWT wavelet filter

Medical images that are obtained at different intervals and with different sensors are registered to obtain maximum information for diagnostic purpose. One of the most popular image registration processes is registering MRI data and CT data. In this work MRI and CT data of human brain is considered as test data sets for evaluation. The MRI (fixed data) and CT (moving data) is transformed into DTCWT sub bands considering integer DTCWT filter coefficients and the low pass sub bands of fixed image and moving image are presented in Figure 8 and Figure 9 respectively. Three different images are considered for analysis. The evaluation of DTCWT filter properties these three images are considered to perform image registration, it is observed that the intensity, orientation, position and size of objects and regions vary frame to frame. DTCWT sub bands need to capture these variations between two images, for which mutual information and joint histogram are computed as measure of similarity index for image registration. For this purpose the joint histogram is computed from the DTCWT sub bands obtained by using Kingsbury filter and integer DTCWT filter derived from Iulian filters. Considering image 1 of MRI and CT and computing joint histogram (shown in Figure 10), it is found that the gray shades are scattered indicating limited redundancy between these frames and clearly indicates the limitations in use of Kingsbury DTCWT filter. Figure 11 to Figure 13 presents the joint histogram results of image registration process obtained by considering Kingsbury DTCWT filters and the derived integer DTCWT filters from Iulian filters. The figure on the left presents the joint histogram before registration of MRI and CT data. The figure at the center presents the histogram results of reference image and registered image. The figure on right presents the histogram results of input image and registered image. The joint histogram obtained using integer Iulian filters are able to extract more features both before registration and after registration process for all three images (frame 1, frame 4 and frame 8 shown in Figure 8 and Figure 9) considered in this work.

From the results obtained, it is observed that the Integer Iulian filters are able to extract features and hence the joint histogram plots provide information on joint entropy indicate more gray areas and the spread is in all directions. The shift invariance property of Iulian filters being superior to Kingsbury filters are demonstrated by the results obtained and presented in Table 6.

For all the three images considered as references for evaluation of feature extraction the joint entropy is found to be improved by a factor of 16% as compared with the results of Kingsbury filters. During the decomposition process carried out using DTCWT filters for feature extraction, the decomposition levels are restricted to level-2. It is observed that from level-1 and level-2 decomposed sub bands the features obtained from the DTCWT sub bands are more visible and sufficient for image registration process. With the sub bands obtained in level 3 and above the features are not adequate in DTCWT bands obtained using Iulian filters, there is need for deriving appropriate Iulian DTCWT filters for level 3 and above. With the complexity of DTCWT decomposition twice greater than DWT decomposition, most of the work reported in literature restricts DTCWT decomposition to level-2. Further to the integer Iulian filter coefficients derived in this work which are represented using 8-bit representation, the metrics of filter can be evaluated with less than 8-bit representation. For hardware implementation it is appropriate for representing derived Iulian DTCWT filters using 8-bit signed representation.

Table 6. Joint histogram comparison

Frame	Data Sets	Joint Entropy (Kingsbury Filter)	Joint Entropy (Integer Iulian Filters)
Frame 1	CT-MRI	9.525	9.757
	MRI-REG	7.370	7.753
	CT-REG	8.058	8.644
Frame 4	CT-MRI	11.609	11.744
	MRI-REG	6.391	6.591
	CT-REG	5.243	5.397
Frame 8	CT-MRI	8.625	8.925
	MRI-REG	4.147	4.585
	CT-REG	3.120	3.718

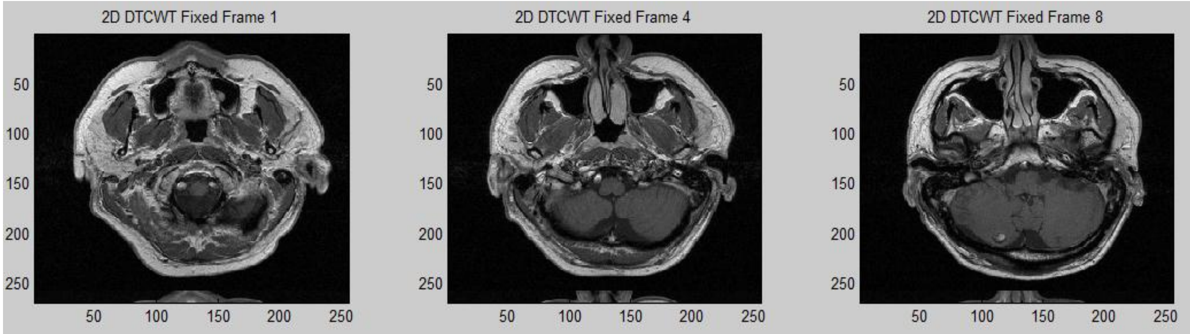


Figure 8. DTCWT sub bands of MRI data

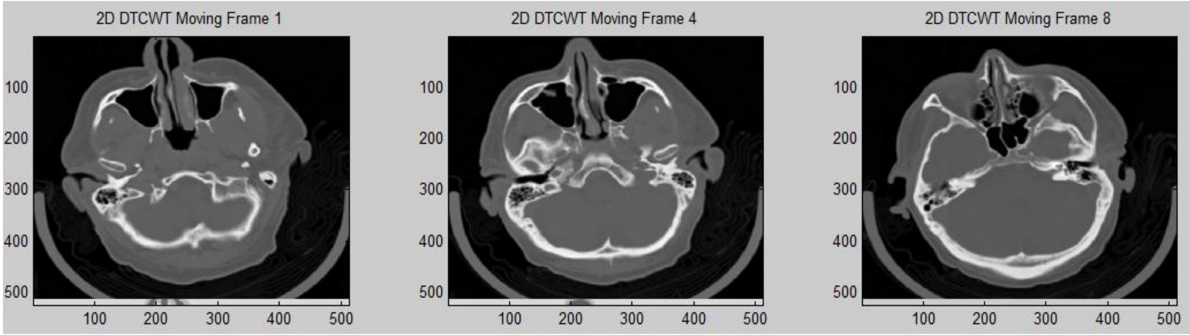


Figure 9. DTCWT sub bands of CT data of frame 1, 4 and 8

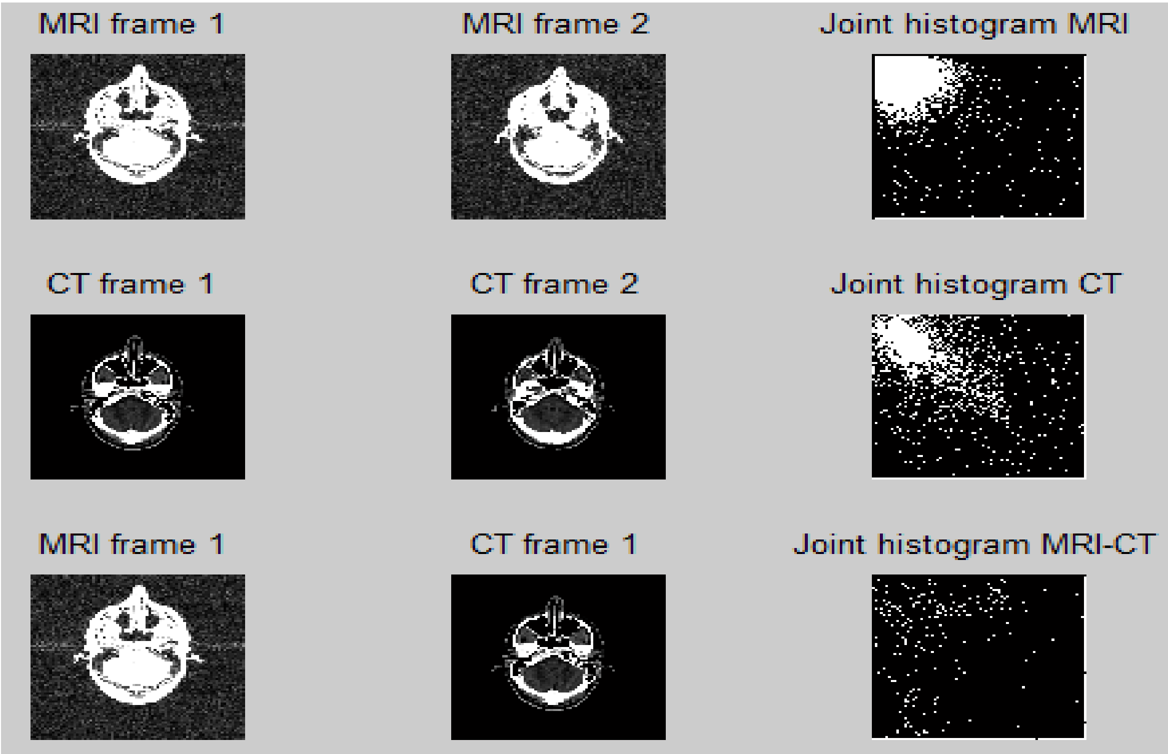
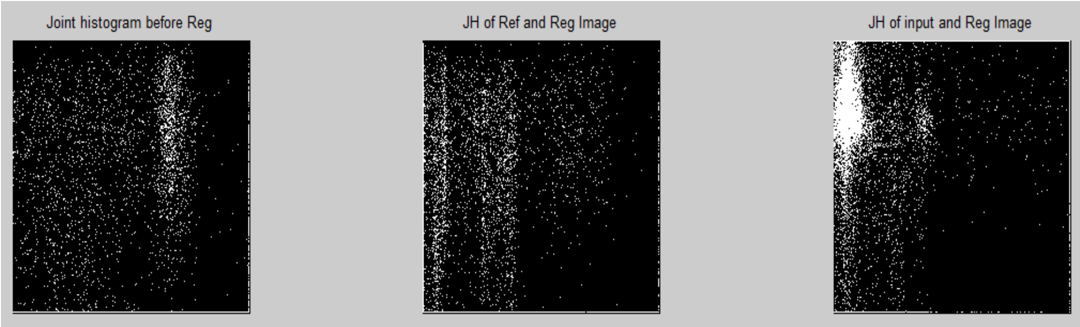


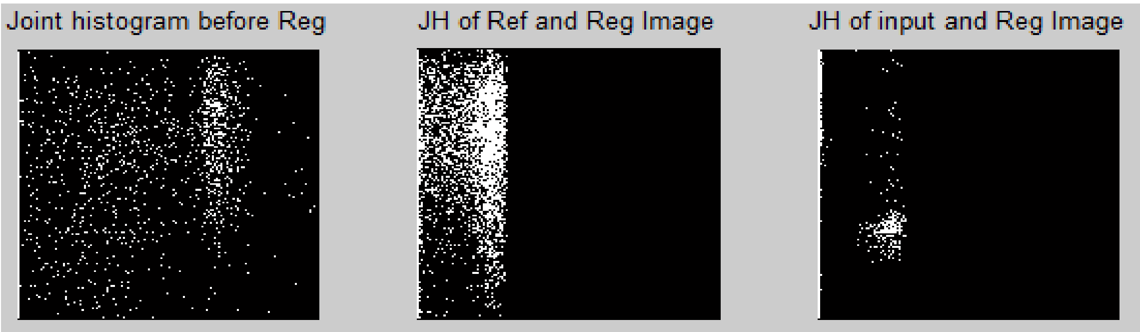
Figure 10. Joint histogram of CT and MRI images using Kingsbury filters



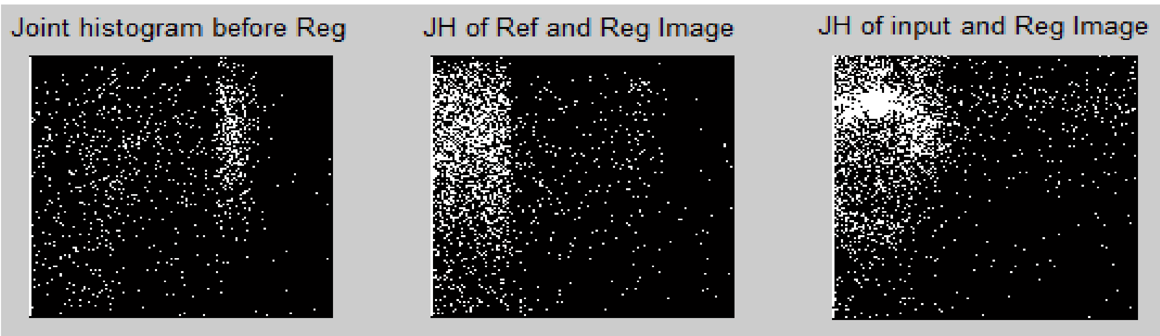
(a)

(b)

Figure 11. Joint histogram of frame 1(a) Kingsbury filter (b) Integer Iulian filter



(a)



(b)

Figure 12. Joint histogram of frame 4 (a) Kingsbury filter (b) Integer Iulian filter

(a)

(b)

Figure 13. Joint histogram of frame 8 (a) Kingsbury filter (b) Integer Iulian filter

CONCLUSION

With DTCWT filter supporting three major properties (shift invariance, directionality and phase information) compared with DWT filters, there is demand for use of DTCWT filters for both signal and image processing applications. One of the major requirements in use of DTCWT filters is to develop hardware efficient architectures for DTCWT computation. DTCWT filters have both real and imaginary tree with each filter coefficient requiring more than 16-bits for representation increases computation complexity. In this work, new DTCWT filters that are derived from three different filters (Villasenor, Selesnick and Iulian) are presented and represented in integer format using 8-bit signed representation. The derived filters are evaluated for its shift invariance property and feature extraction activity measured in terms of joint histogram. From the results obtained it is found that the proposed filters are superior in terms of energy levels and measurement of similarity index and hence is suitable for hardware implementation of DTCWT filter banks.

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