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Fuzzy Supra b-Open Sets

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Abstract

In this paper fuzzy supra b-open sets and fuzzy supra b-closed sets are introduced and certain properties and relations of fuzzy supra b-open sets and fuzzy supra b-closed sets are investigated.

Keywords: Fuzzy supra topological space, fuzzy supra b-open sets, fuzzy supra b-closed sets.

INTRODUCTION

In 1983, Mashhour, et al., [7] introduced the concept of supra topological spaces and in 1987, Abd

El-Monsef, et al., [1] introduced the concept of fuzzy supra topological spaces. In 1996, Andrijevic,[4] introduced and studied a class of generalized open sets in a topological space called b-open sets. In 2010, Benchalli, and Jenifer J. Karnel [10] introduced the concept of fuzzy b-open sets. In 2010, Sayed, and Takashi Noiri [9] introduced supra b-open sets.

In this paper fuzzy supra b-open sets and fuzzy supra b-closed sets are introduced. In section 2 of this paper preliminary definitions and properties regarding fuzzy sets and fuzzy supra sets are given. In section 3 of this paper the concept of fuzzy supra b-open sets and fuzzy supra b-closed sets are introduced and certain properties and relations of fuzzy supra b-open and fuzzy supra b-closed sets are investigated.

Preliminary Definitions

Throughout the paper X denotes a non empty set.

Definition: 2.1 [3]

A fuzzy set in X is a map $f: X \rightarrow [0, 1] = I$. The family of fuzzy sets of X is denoted by I^X.

Following are some basic operations of fuzzy sets in X. For the fuzzy sets f and g in X,

1) $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$
2) $f \leq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$
3) $(f \vee g)(x) = \max \{ f(x), g(x) \}$ for all $x \in X$
4) $(f \wedge g)(x) = \min \{ f(x), g(x) \}$ for all $x \in X$
5) $f^c(x) = 1 - f(x)$ for all $x \in X$ here f^c denotes the complement of f.
6) For a family $\{ f_\lambda / \lambda \in \Lambda \}$ of fuzzy sets defined on a set X

$$(\bigvee_{\lambda \in \Lambda} f_\lambda)(x) = \bigvee_{\lambda \in \Lambda} (f_\lambda(x))$$

$$(\bigwedge_{\lambda \in \Lambda} f_\lambda)(x) = \bigwedge_{\lambda \in \Lambda} (f_\lambda(x))$$

7) For any $\alpha \in I$, the constant fuzzy set $\boldsymbol{\alpha}$ in X is a fuzzy set in X defined by $\boldsymbol{\alpha}(x) = \alpha$ for all $x \in X$.

0 denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

Definition: 2.2 [3]

A fuzzy topological space is a pair (X, δ) where X is a nonempty set and δ is a family of fuzzy set on X satisfying the following properties:

1) The constant fuzzy sets **0** and **1** belong to δ.
2) $f, g \in \delta$ implies $f \wedge g \in \delta$.
3) $f_\lambda \in \delta$ for each $\lambda \in \Lambda$ implies $(\bigvee_{\lambda \in \Lambda} f_\lambda) \in \delta$.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set is called fuzzy closed set.

Definition: 2.3 [3]

The closure and interior of a fuzzy set $f \in I^X$ are defined respectively as

$$cl(f) = \wedge \{ g / g \text{ is a fuzzy closed set in X and } f \leq g \}$$

$$int(f) = \vee \{ g / g \text{ is a fuzzy open set in X and } g \leq f \}$$

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

Definition: 2.4 [9]

A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) The constant fuzzy sets **0** and **1** belong to δ^*.
2) $f_\lambda \in \delta^*$ for each $\lambda \in \Lambda$ implies $(\bigvee_{\lambda \in \Lambda} f_\lambda) \in \delta^*$.

The pair (X, δ^*) is called a fuzzy supra topological space. The elements of $\delta^*$are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition: 2.5 [9]

Let (X, δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f) = \wedge$ { g / g is a fuzzy supra closed set in X and $f \leq g$}

$int^*(f) = \vee$ { g / g is a fuzzy supra open set in X and $g \leq f$}

Definition: 2.6 [9]

Let (X, δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X .

We call δ^* a fuzzy supra topology associated with δ if $\delta \leq \delta^*$

Remark: 2.7 [9]

1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f.
2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.
3) If (X, δ^*)is an associated fuzzy supra topological space with the fuzzy topological space (X, δ) and f is any fuzzy set in X, then

$$int (f) \leq int^*(f) \leq f \leq cl^*(f) \leq cl (f)$$

Theorem: 2.8 [9]

For any two fuzzy sets f and g in a fuzzy supra topological space(X, δ^*),

1) f is a fuzzy supra closed set if and only if $cl^*(f) = f$.
2) f is a fuzzy supra open set if and only if $int^*(f) = (f)$.
3) $f \leq g$ implies $int^*(f) \leq int^*(g)$ and

   $cl^*(f) \leq cl^*(g)$
4) $cl^*(cl^*(f)) = cl^*(f)$ and $int^*(int^*(f)) = int^*(f)$.
5) $cl^*(f \vee g) \geq cl^*(f) \vee cl^*(g)$
6) $cl^*(f \wedge g) \leq cl^*(f) \wedge cl^*(g)$
7) $int^*(f \vee g) \geq int^*(f) \vee int^*(g)$
8) $int^*(f \wedge g) \leq int^*(f) \wedge int^*(g)$
9) $cl^*(f^c) = [int^*(f)]^c$
10) $int^*(f^c) = [cl^*(f)]^c$

Definition: 2.9 [9]

Let (X, δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if

$$f \leq cl^*(int^*(f))$$

The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

Definition: 2.10 [5]

Let (X, δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra preopen set if

$$f \leq int^*(cl^*(f))$$

The complement of a fuzzy supra preopen set is called a fuzzy supra preclosed set.

FUZZY SUPRA b-OPEN SETS

Definition:3.1

Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra b-open set if

$$f \leq cl^* [int^*(f)] \vee int^*[cl^*(f)]$$

The complement of a fuzzy supra b-open set is called a fuzzy supra b-closed set.

Theorem: 3.2

1) Arbitrary union of fuzzy supra b-open sets is a fuzzy supra b-open set.
2) Arbitrary intersection of fuzzy supra b-closed sets is a fuzzy supra b-closed set.

Proof:

1) Let $\{f_\lambda / \lambda \in \wedge\}$ be a collection of fuzzy supra b-open sets.

Then for each λ,

$$f_\lambda \leq cl^* [int^*(f_\lambda)] \vee int^*[cl^*(f_\lambda)]$$

$$\therefore \vee f_\lambda \leq \vee\{ cl^* [int^*(f_\lambda)] \vee int^*[cl^*(f_\lambda)]\}$$

$$\leq \{\vee cl^* [int^*(f_\lambda)]\} \vee \{\vee int^*[cl^*(f_\lambda)]\}$$

$$\leq cl^* [\vee int^*(f_\lambda)] \vee int^*[\vee cl^*(f_\lambda)]$$

$$\vee f_\lambda \leq cl^* [int^*(\vee f_\lambda)] \vee int^*[cl^*(\vee f_\lambda)] \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

∴ An Arbitrary union of fuzzy supra b-open sets is a fuzzy supra b-open set.

2) From (1)

$$\vee f_\lambda \leq cl^* [int^*(\vee f_\lambda)]\} \vee \{ int^*[cl^*(\vee f_\lambda)]$$

By taking Complement

$$[\vee f_\lambda]^c \geq [\{cl^* [int^*(\vee f_\lambda)]\} \wedge \{ int^*[cl^*(\vee f_\lambda)]\}]^c$$

$$\wedge f_\lambda^{\,c} \geq \{cl^* [int^*(\vee f_\lambda)]\}^c \wedge \{int^*[cl^*(\vee f_\lambda)]\}^c$$

$$\geq int^*[(int^*(\vee f_\lambda))^c] \wedge cl^*[(cl^*(\vee f_\lambda))^c]$$

$\geq \text{int}^*(\text{cl}^*[(\vee f_\lambda)^c]) \wedge \text{cl}^*(\text{int}^*[(\vee f_\lambda)^c])$

$\geq \text{int}^*(\text{cl}^*(\wedge f_\lambda{}^c)) \wedge \text{cl}^*(\text{int}^*(\wedge f_\lambda{}^c))$

∴ An Arbitrary intersection of fuzzy supra b-closed sets is a fuzzy supra b-closed set.

Definition: 3.3

Let (X, δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra b-closure and fuzzy supra b-interior of f is defined respectively as

$\text{bcl}^*(f) = \wedge$ { g / g is a fuzzy supra b-closed set in X and $f \leq g$}

$\text{bint}^*(f) = \vee$ { g / g is a fuzzy supra b-open set in X and $g \leq f$}

Remark 3.4:

For a fuzzy set f in a fuzzy supra topological space (X, δ^*),

1)$\text{bcl}^*(f)$ is the smallest fuzzy supra b-closed set containing f.

2) $\text{bint}^*(f)$ is the largest fuzzy supra b-open set contained in f.

3) $\text{int}^*(f) \leq \text{bint}^*(f) \leq f \leq \text{bcl}^*(f) \leq \text{cl}^*(f)$.

Theorem: 3.5

For any fuzzy set f in a fuzzy supra topological space(X, δ^*),

1) $[\text{bint}^*(f)]^c = \text{bcl}^*(f^c)$
2) $[\text{bcl}^*(f)]^c = \text{bint}^*(f^c)$

Proof:

1) Consider

$\text{bint}^*(f) = \vee$ { g / g is a fuzzy supra b-open set in X and $g \leq f$}

$[\text{bint}^*(f)]^c = 1 - \vee$ { g / g is a fuzzy supra b-open set in X and $g \leq f$}

$= \wedge$ { g^c / g^c is a fuzzy supra b-closed set in X and $g^c \geq f^c$}

$= \text{bcl}^*(f^c)$

2) $\text{bcl}^*(f) = \wedge$ { g / g is a fuzzy supra b-closed set in X and $f \leq g$}

$[\text{bcl}^*(f)]^c = 1 - \wedge$ { g / g is a fuzzy supra b-closed set in X and $f \leq g$}

$= \vee$ {g^c / g^c is a fuzzy supra b-open set in X and $f^c \geq g^c$}

$= \text{bint}^*(f^c)$

Theorem: 3.6

For any fuzzy set f in a fuzzy supra topological space(X, δ^*),

1) $f \leq \text{bcl}^*(f)$

2) $\text{bint}^*(f) \leq f$

Proof:

1) Let f be a fuzzy set. Then

$\text{cl}^*(f) = \wedge$ { g / g is a fuzzy supra closed set in X and $f \leq g$}

$\text{bcl}^*(f) = \wedge$ { g / g is a fuzzy supra b-closed set in X and $f \leq g$}

obviously $\text{bcl}^*(f)$ is the smallest fuzzy supra b-closed set which contains f.

$\therefore f \leq \text{bcl}^*(f) \leq \text{cl}^*(f)$

$\therefore f \leq \text{bcl}^*(f)$

2) Let f be a fuzzy set. Then

$\text{int}^*(f) = \vee$ { g / g is a fuzzy supra open set in X and $g \leq f$}

$\text{bint}^*(f) = \vee$ { g / g is a fuzzy supra b-open set in X and $g \leq f$}

obviously $\text{bint}^*(f)$ is the largest fuzzy supra b-open set which is contained in f.

$\therefore f \geq \text{bint}^*(f) \geq \text{int}^*(f)$

$\therefore \text{bint}^*(f) \leq f$

Theorem: 3.7

For any two fuzzy sets f and g in fuzzy supra topological space(X, δ^*), if $f \leq g$ then

1) $\text{bint}^*(f) \leq \text{bint}^*(g)$

2) $\text{bcl}^*(f) \leq \text{bcl}^*(g)$

Proof:

1) By theorem 3.6, $\text{bint}^*(f) \leq f$

Since $f \leq g$ and $\text{bint}^*(g)$ is the largest fuzzy supra b-open set which is contained in g.

$\therefore \text{bint}^*(f) \leq f \leq \text{bint}^*(g) \leq g$

$\therefore \text{bint}^*(f) \leq \text{bint}^*(g)$…………(1)

2) From (1)

$f \leq g \Longrightarrow \text{bint}^*(f) \leq \text{bint}^*(g)$

By taking Complement

$[\text{bint}^*(f)]^c \geq [\text{bint}^*(g)]^c$

$\text{bcl}^*(f^c) \geq \text{bcl}^*(g^c)$

$\Longrightarrow \text{bcl}^*(f) \leq \text{bcl}^*(g)$.

Theorem: 3.8

For any two fuzzy sets f and g in fuzzy supra topological space(X, δ^*)

1) $\text{bint}^*(f) \vee \text{bint}^*(g) \leq \text{bint}^*(f \vee g)$
2) $\text{bcl}^*(f \wedge g) \leq \text{bcl}^*(f) \wedge \text{bcl}^*(g)$

Proof:

1) $f \leq f \vee g$

$g \leq f \vee g$

$\Rightarrow bint^{*}(f) \leq bint^{*}(f \vee g)$

$bint^{*}(g) \leq bint^{*}(f \vee g)$

$\therefore bint^{*}(f) \vee bint^{*}(g) \leq bint^{*}(f \vee g)$

2) $f \wedge g \leq f$

$f \wedge g \leq g$

$\Rightarrow bcl^{*}(f \wedge g) \leq bcl^{*}(f)$

$bcl^{*}(f \wedge g) \leq bcl^{*}(g)$

$\therefore bcl^{*}(f \wedge g) \leq bcl^{*}(f) \wedge bcl^{*}(g)$

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