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Mathematical Model of a Synchronous Machine under Complicated Fault Conditions

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Abstract

This paper presents a mathematical model of a synchronous machine when a complicated fault occurs in the external network. The equations of stator and rotor circuits are written in space vector using Clarke's transformation. The equations are subjected to mathematical manipulations based on phasor concepts, and series expansion of algebraic and differential equations, neglecting high order harmonics, in order to obtain a final model of the machine. This model of the machine is an appropriate one to obtain the negative sequence impedance of the synchronous machine.

Keywords: synchronous machine; Clarke's transformation; stator and rotor equations.

INTRODUCTION

At present time short circuit calculations are performed on the basis of IEC 60909, where initial symmetrical short circuit current is calculated for fundamental frequency, and consequently, all other quantities, such as, asymmetrical, peak, making and breaking currents are obtained for the evaluation of thermal and mechanical effects of short circuit [1].

Symmetrical components transformation has been used for decades in calculation of short circuit currents in power systems. The application of time dependent symmetrical components makes sense, as the network parameters are available in this form [2].

Usually, these calculations are limited to short period of time as the protection will operate, and in a fast manner, clear the short circuit. In these situations, the assumption of constant emf of synchronous machines is accepted and doesn't lead to appreciable errors. At the same time, in evaluation of electromechanical transient processes, positive sequence emf is implemented in calculations, which in turn is considered constant during the time of fault.

There are cases when the protection fails to clear the fault, and the fault duration is extended, therefore, the assumption of constant emf will lead to errors. In such cases, it required to take into consideration the electromechanical transient processes in combination with short circuit calculations [3].

Mathematical model of a synchronous machine is considered to be of prime factor among all models of network elements, especially, when complicated fault conditions occurred in the external network. Under these conditions, pulsating magnetic field in rotor circuit will be created, which in turns will produce full spectrum of higher harmonics.

When a complicated short circuit occurs in external network, asymmetrical current will flow in stator and rotor circuits of the machine, and emf of odd harmonics will be induced in stator circuits, while even harmonics in terms of rotor position will be created.

When asymmetrical current of frequency ω flows through stator winding, in the rotor circuit rotating with ω_r, emf and current will appear, rotational speed of which can be determined by the following formula:

$$\omega_{ir} = |\omega \pm (2i-1)\omega_r|$$

Where $i = 0,1,2,....$ harmonic order.

At the same time, in stator circuit emf and current will be created with rotational speed calculated by the following formula:

$$\omega_{ist} = |\omega \pm 2i\omega_r|$$

Mathematical model of a synchronous machine

The following model of a synchronous machine is used to perform calculations of any complicated fault conditions in power system [4]. This model is based on transforming equations from time domain phase quantities [5] to space vector by using Clarke transformation:

$$\frac{d\psi_\alpha}{dt} = -\omega_s(V_\alpha + r_a i_\alpha) \quad (1)$$

$$\frac{d\psi_\beta}{dt} = -\omega_s(V_\beta + r_a i_\beta) \quad (2)$$

$$\frac{d\psi_r}{dt} = \omega_s \rho_r (e_r - E_q) \quad (3)$$

$$\frac{d\psi_{rd}}{dt} = -\omega_s \rho_{rd} E_{rq} \quad (4)$$

$$\frac{d\psi_{rq}}{dt} = -\omega_s \rho_{rq} E_{rd} \quad (5)$$

$$\psi_\alpha = (X - X_a \cos 2\gamma) i_\alpha - X_a \sin 2\gamma . i_\beta - (E_q + E_{rq}) \sin\delta - \cos\gamma . E_{rd} \quad (6)$$

$$\psi_\beta = (X + X_a \cos 2\gamma) i_\beta - X_a \sin 2\gamma . i_\alpha + (E_q + E_{rq}) \cos\gamma - \sin\gamma . E_{rd} \quad (7)$$

$$\psi_r = \frac{X_{ad}^2}{X_r}(i_\beta . \cos\gamma - i_{\alpha .} \sin\gamma) + E_q + \frac{X_{ad}}{X_r}. E_{rq} \quad (8)$$

$$\psi_{rd} = \frac{X_{aq}^2}{X_{rd}}(i_\beta . \cos\gamma - i_{\alpha .} \sin\gamma) + \frac{X_{ad}}{X_{rd}} E_q + E_{rq} \quad (9)$$

$$\psi_{rq} = \frac{X_{aq}^2}{X_{rq}}(i_\alpha . \cos\gamma + i_\beta \sin\gamma) - E_{rd} \quad (10)$$

Where: $X = \frac{X_d + X_q}{2}$

And $X_a = \frac{X_d - X_q}{2}$

$\gamma = \omega t + \gamma_0$ Angle between magnetic axis of phase A and quadrature rotor axis.

$i_\alpha, i_\beta, V_\alpha, V_\beta, \psi_\alpha, \psi_\beta$ are currents, flux linkages, and voltages in Clarke's coordinate system $\alpha, \beta, 0$.

r_a is the active resistance of stator circuit.

ω_s is the synchronous rotational speed.

E_q, E_{rq}, E_{rd} are emf proportional to field current, and emf of amortisseur winding in q,d coordinate system.

$\rho_r, \rho_{rd}, \rho_{rq}$ decrements of field winding and amortisseur winding in q,d coordinate system.

The above mentioned equations are written in stationary space vectors, so the relationship between quantities $\psi_\alpha, \psi_\beta, \psi_r, \psi_{rd}, \psi_{rq}$ and currents i_α, i_β will contain components with periodic coefficients.

Model evaluation

In evaluation of the mathematical model of a synchronous machine, the following assumptions shall be made:

1. We will ignore the transformation emf $\frac{d\psi}{dt}$ and rotational emf $\frac{d\theta}{dt}$.
2. As the time constant of rotor circuits is greater than the period of fundamental frequency, so differentiation of these circuits will be kept in our discussion.

Using phasor notation for sinusoids, we can write the following for any damped sinusoid [6]:

$$f = F_m \cos(\omega t + \gamma_0)$$

Where:

F_m is the peak value of the function,

γ_0 is the phase angle.

Using Euler's identity, the above-mentioned equation will take the following form:

$$f = Re\left[\left(F_m e^{j\gamma_0}\right). e^{j\omega t}\right]$$

Or

$$f = Re\left[\dot{F}_m . e^{j\omega t}\right]$$

Therefore, $\psi_\alpha, \psi_\beta, \psi_r, \psi_{rd}, \psi_{rq}, i_\alpha, i_\beta, V_\alpha, V_\beta$ may be written in the following form:

$$f = Re \sum_{k=0}^{k=\infty} \dot{F}_{\gamma k} . e^{jk\gamma}$$

Where:

$$\dot{F}_{\gamma k} = F'_{\nu k} + jF''_{\nu k};$$

$\nu = \alpha, \beta - and\ q, d$ Coordinate systems.

Also, taking into consideration the following identities:

$\sin k\gamma = -\frac{e^{jk\gamma} - e^{-jk\gamma}}{2}$ and

$\cos k\gamma = \frac{e^{jk\gamma} + e^{-jk\gamma}}{2}$

Equation (6) will take the following form:

$$Re \sum_{k=0}^{k=\infty} \dot{\psi}_{\alpha k} . e^{jk\gamma} = Re \sum_{k=0}^{k=\infty} \Big(X . \dot{I}_{\alpha k} . e^{jk\gamma} - \frac{X_a}{2} . \dot{I}_{\alpha k} . e^{j(k+2)\gamma} - \frac{X_a}{2} . \dot{I}_{\alpha k} . e^{j(k-2)\gamma} + j\frac{X_a}{2} . \dot{I}_{\beta k} . e^{j(k+2)\gamma} - j\frac{X_a}{2} \dot{I}_{\beta k} . e^{j(k+2)\gamma} - j\frac{X_a}{2} . \dot{I}_{\beta k} . e^{j(k+2)} + \frac{\dot{E}_{qk} + \dot{E}_{rqk}}{2} . e^{j(k+1)\gamma} - j\frac{\dot{E}_{qk} + \dot{E}_{rqk}}{2} . e^{j(k+1)\gamma} - \frac{\dot{E}_{rdk}}{2} . e^{j(k+1)\gamma} - \frac{\dot{E}_{rdk}}{2} . e^{j(k-1)\gamma} \Big) \quad (11)$$

Consider the following identities:

$$Re\left(\dot{A}.e^{-jk\gamma}\right) = Re\left(\breve{A}.e^{jk\gamma}\right)$$

And $Re\left(j\dot{A}.e^{-jk\gamma}\right) = Re\left(-j\breve{A}e^{jk\gamma}\right)$.

We obtain the following equation for first order harmonics when k=1:

$$\dot{\psi}_{\alpha 1} = X.\dot{I}_{\alpha 1} - \frac{X_a}{2}\left(\breve{I}_{\alpha 1} - j\breve{I}_{\beta 1}\right) - \frac{X_a}{2}\left(\dot{I}_{\beta 3} + j\dot{I}_{\alpha 3}\right) - j\frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2} - \frac{\dot{E}_{rd2}}{2} + j\frac{\dot{E}_{q0}+\dot{E}_{rq0}}{2} - \frac{\dot{E}_{rd0}}{2} + j\frac{\breve{E}_{q0}+\breve{E}_{rq0}}{2} - \frac{\breve{E}_{rd0}}{2} \quad (12)$$

Similarly, we obtain:

$$\dot{\psi}_{\beta 1} = X.\dot{I}_{\beta 1} + \frac{X_a}{2}\left(\breve{I}_{\beta 1} + j\breve{I}_{\alpha 1}\right) + \frac{X_a}{2}\left(\dot{I}_{\beta 3} - j\dot{I}_{\alpha 3}\right) + \frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2} - j\frac{\dot{E}_{rd2}}{2} + \frac{\dot{E}_{q0}+\dot{E}_{rq0}}{2} + j\frac{\dot{E}_{rd0}}{2} + \frac{\breve{E}_{q0}+\breve{E}_{rq0}}{2} + j\frac{\breve{E}_{rd0}}{2} \quad (13)$$

The abovementioned discussions will be applied to rotor and amortisseur winding,

so finally we obtain:

$$\dot{\psi}_{r0} = \frac{X_{ad}}{2X_r}\left(I'_{\beta 1} + I''_{\alpha 1}\right) + \dot{E}_{q0} + \frac{X_{ad}}{X_r}.\dot{E}_{rq0} \quad (14)$$

$$\dot{\psi}_{rd0} = \frac{X_{ad}^2}{2X_{rd}}\left(I'_{\beta 1} + I''_{\alpha 1}\right) + \frac{X_{ad}}{X_{rd}}.\dot{E}_{q0} + \dot{E}_{rq0} \quad (15)$$

$$\dot{\psi}_{rq0} = \frac{X_{aq}^2}{2X_{rq}}\left(I'_{\alpha 1} - I''_{\beta 1}\right) - \dot{E}_{rd0} \quad (16)$$

Negative sequence currents in stator circuits will generate in rotor circuits double frequency currents. So for k=2, Eqs. (14-16) will take the following form:

$$\dot{\psi}_{r2} = \frac{X_{ad}^2}{2X_r}\left(\dot{I}_{\beta 1} + j\dot{I}_{\alpha 1}\right) + \frac{X_{ad}^2}{2X_r}\left(\dot{I}_{\beta 3} - j\dot{I}_{\alpha 3}\right) + \dot{E}_{q2} + \frac{X_{ad}}{X_r}.\dot{E}_{rq2} \quad (17)$$

$$\dot{\psi}_{rd2} = \frac{X_{ad}^2}{2X_{rd}}\left(\dot{I}_{\beta 1} + j\dot{I}_{\alpha 1}\right) + \frac{X_{ad}^2}{2X_{rd}}\left(\dot{I}_{\beta 3} - j\dot{I}_{\alpha 3}\right) + \dot{E}_{rq2} + \frac{X_{ad}}{X_r}.\dot{E}_{q2} \quad (18)$$

$$\dot{\psi}_{rq2} = \frac{X_{aq}^2}{2X_{rq}}\left(\dot{I}_{\alpha 1} - \dot{I}_{\beta 1}\right) + \frac{X_{aq}^2}{2X_{rq}}\left(\dot{I}_{\alpha 3} + j\dot{I}_{\beta 3}\right) - \dot{E}_{rd2} \quad (19)$$

For differential equations (1-5), and taking into consideration the following expression:

$$\frac{d\left(Re\sum \dot{F}_k.e^{jk\gamma}\right)}{dt} = Re\sum_{k=0}^{k=\infty} e^{jk\gamma}.\frac{d\dot{F}_k}{dt} + Re\sum_{k=0}^{k=\infty} jk\omega_s.e^{jk\gamma}.\dot{F}_k \quad (20)$$

And apply it to Eq.(1), when $k = 1$ and $\omega = \omega_s$, yields:

$$\frac{d\dot{\psi}_{\alpha 1}}{dt} + j\omega_s.\dot{\psi}_{\alpha 1} = -\omega_s\left(\dot{V}_{\alpha 1} + R_a.\dot{I}_{\alpha 1}\right) \quad (21)$$

If we ignore transformation emf $\frac{d\dot{\psi}_{\alpha 1}}{dt} = 0$, finally we obtain:

$$\dot{\psi}_{\alpha 1} = j\left(\dot{V}_{\alpha 1} + R_a.\dot{I}_{\alpha 1}\right) \quad (22)$$

Similarly, we obtain:

$$\dot{\psi}_{\beta 1} = j\left(\dot{V}_{\beta 1} + R_a . \dot{I}_{\beta 1}\right) \tag{23}$$

Now, multiply Eq. (13) by j and add it to Eq. (12), we obtain:

$$\dot{\psi}_{\alpha 1} + j\dot{\psi}_{\beta 1} = X.\dot{I}_{\alpha 1} - \frac{X_a}{2}\left(\check{I}_{\alpha 1} - j\check{I}_{\beta 1}\right) - \frac{X_a}{2}\left(\dot{I}_{\beta 3} + j\dot{I}_{\alpha 3}\right) - j\frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2} - \frac{\dot{E}_{rd2}}{2} + j\frac{\dot{E}_{q0}+\dot{E}_{rq0}}{2} - \frac{\dot{E}_{rd0}}{2} + j\frac{\check{E}_{q0}+\check{E}_{rq0}}{2} - \frac{\check{E}_{rd0}}{2} + jX\dot{I}_{\beta 1} + j\frac{X_a}{2}\left(\check{I}_{\beta 1} + j\check{I}_{\alpha 1}\right) + j\frac{X_a}{2}\left(\dot{I}_{\beta 3} - j\dot{I}_{\alpha 3}\right) + j\frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2} - \frac{\dot{E}_{rd2}}{2} + j\frac{\dot{E}_{q0}+\dot{E}_{rq0}}{2} - \frac{\dot{E}_{rd0}}{2} + j\frac{\check{E}_{q0}+\check{E}_{rq0}}{2} - \frac{\check{E}_{rd0}}{2} \tag{24}$$

If we neglect second harmonic and higher, Eq. (24) may be transformed back to q,d coordinate system and will take the following form:

$$\dot{\psi}_1 = X\dot{I}_1 + 2j\left(\dot{E}_{q0} + \dot{E}_{rq0}\right) - 2\dot{E}_{rd0} \tag{25}$$

Similarly, we obtain:

$$\dot{\psi}_2 = X\dot{I}_2 - j\frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2} - \frac{\dot{E}_{rq2}}{2} \tag{26}$$

Multiply Eq. (22) one time by j and add it to Eq. (23), and second time by –j and add it another time to Eq. (23), we will get the following two equations after transforming the results to coordinate q,d system:

$$\dot{\psi}_1 = j\left(\dot{V}_1 + r_a . \dot{I}_1\right) \tag{27}$$

$$\dot{\psi}_2 = j\left(\dot{V}_2 + r_a . \dot{I}_2\right) \tag{28}$$

Equate the right parts of Eqs. (25) and (27), we get:

$$X\dot{I}_1 + 2j\left(\dot{E}_{q0} + \dot{E}_{rq0}\right) - 2\dot{E}_{rd0} = j\left(\dot{V}_1 + r_a . \dot{I}_1\right) \tag{29}$$

Similarly, we obtain:

$$X\dot{I}_1 + 2j\left(\frac{\dot{E}_{q2}+\dot{E}_{rq2}}{2}\right) - \frac{\dot{E}_{rd2}}{2} = j\left(\dot{V}_2 + r_a . \dot{I}_2\right) \tag{30}$$

For rotor and amortisseur circuits when k=0, we obtain the following equations:

$$\frac{d\psi_{r0}}{dt} = \omega_s . \rho_r\left(e_{r0} - E_{q0}\right) \tag{31}$$

$$\frac{d\psi_{rd0}}{dt} = -\omega_s . \rho_{rd} E_{rq0} \tag{32}$$

$$\frac{d\psi_{rq0}}{dt} = \omega_s . \rho_{rq} E_{rd0} \tag{33}$$

As a result, a system of algebraic and differential equations will represent the synchronous machine when a first order harmonic currents flow in stator winding. These equations are:

$$X\dot{I}_1 + 2j\left(\dot{E}_{q0} + \dot{E}_{rq0}\right) - 2\dot{E}_{rd0} = j\left(\dot{V}_1 + r_a . \dot{I}_1\right) \tag{34}$$

$$X\dot{I}_1 + 2j\left(\dot{E}_{q0} + \dot{E}_{rq0}\right) - 2\dot{E}_{rd0} = j\left(\dot{V}_2 + r_a . \dot{I}_2\right) \tag{35}$$

$$\psi_{r0} = \frac{X_{ad}}{2X_r}\left(I'_{\beta 1} + I''_{\alpha 1}\right) + E_{q0} + \frac{X_{ad}}{X_r}.E_{rq0} \tag{36}$$

$$\psi_{rd0} = \frac{X_{ad}^2}{2X_{rd}}\left(I'_{\beta 1} + I''_{\alpha 1}\right) + \frac{X_{ad}}{X_{rd}}.E_{q0} + E_{rq0} \tag{37}$$

$$\psi_{rq0} = \frac{X_{aq}^2}{2X_{rq}}\left(I'_{\alpha 1} - I''_{\beta 1}\right) - E_{rd0} \tag{38}$$

$$\frac{d\psi_{ro}}{dt} = \omega_s.\rho_r\left(e_{r0} - E_{q0}\right) \tag{39}$$

$$\frac{d\psi_{rdo}}{dt} = -\omega_s.\rho_{rd}E_{rq0} \tag{40}$$

$$\frac{d\psi_{rqo}}{dt} = \omega_s.\rho_{rq}E_{rd0} \tag{41}$$

CONCLUSIONS

The method and discussions in this paper presents a new form of equations of a synchronous machine when complicated faults occur in external network. These equations will be implemented in obtaining a new equation for negative sequence impedance of the machine, which will be presented in Part II.

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