

Performance of Combined Bulk and Per-Tone Transmit Antenna Selection Algorithm with Low-Complexity in Spatial Multiplexing MIMO-OFDM Systems

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Abstract

In this paper, a transmit antenna selection algorithm based on the null space of the multiple input multiple output (MIMO) channel is employed for per-tone selection at each subcarrier in the combined bulk/per-tone transmit antenna selection scheme for spatial multiplexing MIMO-Orthogonal frequency division multiplexing(OFDM) systems. A selection approach is to successively find and eliminate transmit antennas having the maximum contributions to the null space of the MIMO channel chosen by bulk selection. It is shown that its error performance is close to the optimal antenna selection algorithm and its computational complexity is substantially lower than that of optimal algorithm except for the extreme small number of selected antennas.

Keywords: Multiple input multiple output (MIMO), Orthogonal frequency division multiplexing (OFDM), Spatial multiplexing, Antenna selection, Bulk and per-tone selection, Eigenspace

INTRODUCTION

A multiple-input multiple-output (MIMO) technology can achieve high spectral efficiency by employing multiple antennas at both transmitter and receiver. However, the major constraint in MIMO systems is to increase the cost of the hardware. Antenna selection techniques have been proposed to reduce the hardware complexity while maintaining the benefit of multi-antenna diversity [1],[2]. Although earlier works on antenna selection have focused on the frequency flat fading channels, considerable interests have been given to frequency selective fading channels [1-6]. Orthogonal frequency division multiplexing (OFDM) can mitigate inter-symbol interference in adverse frequency selective fading channels. In the MIMO system combined with OFDM, a data stream at each transmit antenna can be spread across many narrowband orthogonal subcarriers [7]. Unlike in the single-carrier MIMO system, antenna selection in MIMO-OFDM systems can be applied on all subcarriers or each subcarrier individually. The former approach is the bulk selection, where the selected antenna subset is used for transmission on all frequencies [8-10]. The latter one is per-tone selection, where an independent antenna subset selection is performed for each subcarrier [11]. Bulk selection can employ fewer radio frequency (RF) chains than available antennas. Meanwhile, per-tone selection can offer an additional degree of freedom and thus achieve a significantly improved bit-error rate (BER) performance. However, it does

not provide a benefit of saving the number of RF chains. Recently, a hybrid antenna selection method has been proposed in [12]. It first selects a subset of the available antennas (bulk selection) and then at each subcarrier reselects a subset consisting of smaller number of antennas from the chosen antennas (per-tone selection). The combined bulk and per-tone antenna selection scheme has been shown to achieve the performance identical to the per-tone selection system at high signal to noise ratio (SNR).

In [12], an optimal searching method in both bulk and per-tone selection has been considered. It requires searching all possible antenna subsets and thus needs high computational complexity especially for large antenna dimension. To reduce computational complexity in the combined bulk/per-tone antenna selection scheme, suboptimum selection algorithms with low-complexity can be performed. However, little attention has been given to the development of efficient selection algorithms in the hybrid bulk/per-tone selection scheme. In this work, a per-tone selection algorithm based on the eigenspace of the MIMO channel is employed for the combined bulk/per-tone transmit antenna selection in spatial multiplexing MIMO-OFDM systems. It is shown that it can achieve the similar BER performance compared to when the optimal selection algorithm is used in the process of the second per-tone selection. Finally, its computational complexity is compared with that of optimal one.

SYSTEM MODEL

We consider an $N_R \times N_T$ MIMO-OFDM system with Q subcarriers, where N_R and N_T are the number of receive and transmit antennas, respectively. In each transmit-receive antenna link, the frequency selective fading channel is represented by L -ray model. We let $\mathbf{H}_l \in C^{N_R \times N_T}$ denote the channel response matrix for the l th tap, where $l = \{0, 1, \dots, L-1\}$, and L is the number of channel taps. We assume that \mathbf{H}_l is uncorrelated and h_{mn}^l , $m = 1, 2, \dots, N_R$, $n = 1, 2, \dots, N_T$, which is the channel coefficient from the n th transmit antenna to the m th receive antenna element for the l th tap, follows the zero-mean complex Gaussian distribution with variances σ_l^2 , where $\sum_{l=0}^{L-1} \sigma_l^2 = 1$. The impulse response for this frequency-selective channel can be written as

$$\mathbf{H}_\tau = \sum_{l=0}^{L-1} \mathbf{H}_l \delta(\tau-l) \quad (1)$$

where $\delta(\cdot)$ is the Kronecker delta function. If the cyclic prefix is designed to be of at least length $L-1$, the channel frequency response matrix of the q th subcarrier for a Q-tone MIMO-OFDM system can be described using another $N_R \times N_T$ matrix \mathbf{H}_q

$$\mathbf{H}_q = \sum_{l=0}^{L-1} \mathbf{H}_l e^{-j2\pi ql/Q} \quad (2)$$

In this system, the bulk/per-tone transmit antenna selection approach [4] is employed. First the bulk selection is performed to choose N_S out of N_T available transmit antennas for transmission. The data signals corresponding to a given subcarrier are conveyed from N_F antennas of the N_S chosen antennas (i.e., per-tone selection). We define the indicator function of the selected transmit antenna subset in the bulk selection as $\omega_s, s=1,2,\dots,S$,

$$\omega_s = \{I_i\}_{i=1}^{N_T}, \{I_i\} \in \{0,1\} \quad (3)$$

where i is the index of the columns of \mathbf{H}_q , and the indicator function I_i indicates whether the i th column of \mathbf{H}_q (the i th transmit antenna) is selected. If the i th column of \mathbf{H}_q is selected, I_i will be set 1. Otherwise, $I_i=0$. In (3), S is the number of all possible selected transmit-antenna subsets, and thus $S=B_{N_S}^{N_T}$. Here $B_{N_S}^{N_T}$ denotes the number of N_S combinations from N_T elements. Then,

$$\mathbf{H}_{q,s} = [\mathbf{H}_q]_{\omega_s} \in C^{N_R \times N_S} \quad (4)$$

Further, the indicator function of the selected transmit antenna subset at each subcarrier in the per-tone selection is defined as $\omega_{q,f}^*, q=1,2,\dots,Q, f=1,2,\dots,F$,

$$\omega_{q,f}^* = \{I_{q,j}\}_{j=1}^{N_S}, \{I_{q,j}\} \in \{0,1\} \quad (5)$$

where j is the index of the columns of $\mathbf{H}_{q,s}$, and the indicator function $I_{q,j}$ represents whether the j th column of $\mathbf{H}_{q,s}$ is chosen. If the j th column of $\mathbf{H}_{q,s}$ is selected, $I_{q,j}$ will be set 1. In (5), F is the number of all possible selected transmit-antenna subsets, and thus $F=B_{N_F}^{N_S}$, where $B_{N_S}^{N_T}$ denotes the number of N_F combinations from N_S elements. Then, the effective channel matrix for the q th subcarrier after per-tone antenna selection is denoted by

$$\mathbf{H}_{q,f} = [\mathbf{H}_{q,s}]_{\omega_{q,f}^*} \in C^{N_R \times N_F} \quad (6)$$

We assume that perfect channel state information is available at the receiver. Thus the transmit antenna indices selected at the receiver side are fed back to the transmitter. After per-tone selection, the received signal for the q th subcarrier at the receiver can be expressed as:

$$\mathbf{r}_q = \sqrt{\frac{E_s}{N_F}} \mathbf{H}_{q,f} \mathbf{x}_q + \mathbf{n}_q \quad (7)$$

where E_s is the total signal energy and $\mathbf{x}_q = [d_{1,q} \ d_{2,q} \ \dots \ d_{N_F,q}]^T$ is the transmitted data for the q th subcarrier. We assume that \mathbf{x}_q is modulated from the same unit-energy constellation and is uncorrelated. $\mathbf{n}_q \in C^{N_R \times 1} \sim CN(0, N_0 \mathbf{I}_{N_R})$ is the complex additive white Gaussian noise, and \mathbf{I}_{N_R} is an $N_R \times N_R$ identity matrix. The total available power is assumed to be uniformly allocated across all space-frequency subchannels. To recover the transmitted information symbols at the receiver, a ZF equalizer, which is given by $\mathbf{F}_{ZF,q} = \sqrt{N_F/E_s} \mathbf{H}_{q,f}^\dagger$, is employed at the q th subcarrier. Here \dagger denotes the pseudo-inverse. Then the output of the ZF receiver at the q th subcarrier is given by

$$\mathbf{F}_{ZF,q} \mathbf{r}_q = \bar{\mathbf{x}}_q + \sqrt{\frac{N_F}{E_s}} \mathbf{H}_{q,f}^\dagger \mathbf{n}_q \quad (8)$$

ANTENNA SELECTION METHOD

This work considers the combined bulk/per-tone transmit antenna selection [12] in spatial multiplexing MIMO-OFDM systems. In the bulk selection, an optimum antenna selection algorithm based on maximum minimum post-processing SNR (called ZF-SNR-Opt) is performed to choose N_S transmit antennas among N_T transmit antennas. In multi-carrier case, optimal global searching for the antenna subset with the largest minimum post-processing SNR requires the computation of all the smallest post-processing SNRs. It is done for the set of all possible $Q \cdot S (= Q \times B_{N_S}^{N_T})$ subsets of transmit antennas. In other words, it can be formulated as

$$\omega_s = \max \min SNR_{q,s}^{(ZF-opt)}(\omega_{q,s}) \quad (9)$$

where $\omega_{q,s} = \{I_{q,i}\}_{i=1}^{N_T}, \{I_{q,i}\} \in \{0,1\}$. Here $\omega_{q,s}$ is the indicator function of the possible transmit antenna subset used in the bulk selection and the SNR expression of the j th layer over the q th subcarrier, $j=1,2,\dots,N_S$, is given by [5]

$$SNR_{q,s,j}^{(ZF-opt)}(\omega_{q,s}) = \frac{E_s}{N_0 [\mathbf{H}_{q,s}^H \mathbf{H}_{q,s}]_{jj}^{-1}} \quad (10)$$

After bulk selection of N_s transmit antennas, the per-tone selection is performed to determine N_F transmit antennas of the N_s chosen antennas at each subcarrier. If an optimum selection algorithm is considered for per-tone selection [12], it selects the transmit antenna subset with the largest minimum SNR. That is, the per-tone selection based on optimal searching computes the following at the q th subcarrier.

$$\omega_{q,f}^* = \max \min SNR_{q,f}^{(ZF-opt)}(\omega_{q,f}) \quad (11)$$

Here $\omega_{q,f}$ is the indicator function of the possible transmit antenna subset given in the per-tone selection and the SNR value of the k th layer over the q th subcarrier, $k = 1, 2, \dots, N_F$, is also given as

$$SNR_{q,f,k}^{(ZF-opt)}(\omega_{q,f}) = \frac{E_s}{N_0 [\mathbf{H}_{q,f}^H \mathbf{H}_{q,f}]_{kk}^{-1}} \quad (12)$$

Now, we want to reduce overall computational complexity in the combined bulk/per-tone transmit antenna selection scheme. If a suboptimum selection algorithm with reduced complexity is used in bulk selection process, its error performance relative to optimal one could be significantly degraded. Thus we are interested in performing low-complexity selection algorithms only in per-tone selection while an optimal searching method is still used in bulk selection. Therefore, we focus on the reduction of computational complexity in per-tone selection when $N_F = 1$.

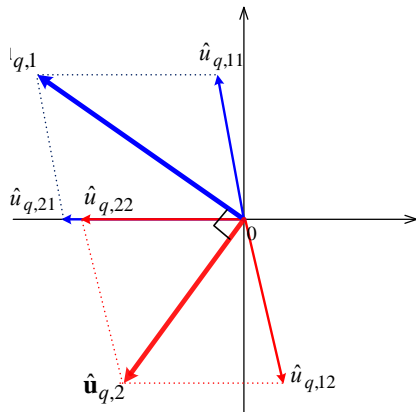


Figure 1. Eigenspace of the combined full channel matrix for $N_T = 3$, $N_s = 2$, $N_R = 2$, $N_F = 1$, and $L = 5$.

This work considers a suboptimum transmit antenna selection algorithm based on null space of the MIMO channel [13]. It is adapted for per-tone selection at each subcarrier. Here the nature of eigenspace of the spatially combined channel matrix is utilized. To pictorially illustrate it, the combined full channel matrix for $N_T = 3$, $N_R = 2$, $N_s = 2$, $N_F = 1$, and $L = 5$ is considered. Then, the combined 2×2 matrix $\tilde{\mathbf{H}}_q$ is given by

$$\tilde{\mathbf{H}}_q = \begin{bmatrix} |\underline{h}_{(q,11)}|^2 + |\underline{h}_{(q,21)}|^2 & \underline{h}_{(q,11)}^* \underline{h}_{(q,12)} + \underline{h}_{(q,21)}^* \underline{h}_{(q,22)} \\ \underline{h}_{(q,12)}^* \underline{h}_{(q,11)} + \underline{h}_{(q,22)}^* \underline{h}_{(q,21)} & |\underline{h}_{(q,12)}|^2 + |\underline{h}_{(q,22)}|^2 \end{bmatrix} \quad (13)$$

where $\underline{h}_{(q,m'n')}$, $m' = 1, 2$, $n' = 1, 2$, is the (m', n') th element of the matrix $\mathbf{H}_{q,s}$. Then as one example seen in Fig. 1, its unitary matrix and diagonal matrix of eigenvalues by a Matlab command *eig* are obtained as, respectively, the followings.

$$\mathbf{U}_q = \begin{bmatrix} \hat{\mathbf{u}}_{q,1}^T \\ \hat{\mathbf{u}}_{q,2}^T \end{bmatrix}^T = \begin{bmatrix} -0.1335 + 0.6423i & 0.1536 - 0.7389i \\ -0.7547 & -0.6561 \end{bmatrix} \quad (14)$$

$$\mathbf{S}_q = \begin{bmatrix} 0.3382 & 0 \\ 0 & 3.3980 \end{bmatrix} \quad (15)$$

Here the left eigenvector, $\hat{\mathbf{u}}_{q,1} = [\hat{u}_{q,11} \ \hat{u}_{q,21}]^T$, associated with the smallest eigenvalue of the combined matrix $\tilde{\mathbf{H}}_q$, spans the null space of $\tilde{\mathbf{H}}_q$. The right eigenvector, $\hat{\mathbf{u}}_{q,2} = [\hat{u}_{q,12} \ \hat{u}_{q,22}]^T$, corresponding to the largest eigenvalue, spans the range of $\tilde{\mathbf{H}}_q$. Then the combined matrix $\tilde{\mathbf{H}}_q$ can be described as

$$\tilde{\mathbf{H}}_q = \mathbf{U}_q \mathbf{S}_q \mathbf{U}_q^H = \begin{bmatrix} s_{q,1} |\hat{u}_{q,11}|^2 + s_{q,2} |\hat{u}_{q,12}|^2 & s_{q,1} \hat{u}_{q,11} \hat{u}_{q,21}^* + s_{q,2} \hat{u}_{q,12} \hat{u}_{q,22}^* \\ s_{q,1} \hat{u}_{q,21} \hat{u}_{q,11}^* + s_{q,2} \hat{u}_{q,22} \hat{u}_{q,12}^* & s_{q,1} |\hat{u}_{q,21}|^2 + s_{q,2} |\hat{u}_{q,22}|^2 \end{bmatrix} \quad (16)$$

By comparing the diagonal terms in the expressions of (13) and (16), the following relationships can be obtained.

$$s_{q,1} |\hat{u}_{q,11}|^2 = |\underline{h}_{(q,11)}|^2 + |\underline{h}_{(q,21)}|^2 - s_{q,2} |\hat{u}_{q,12}|^2 \quad (17)$$

$$s_{q,1} |\hat{u}_{q,21}|^2 = |\underline{h}_{(q,12)}|^2 + |\underline{h}_{(q,22)}|^2 - s_{q,2} |\hat{u}_{q,22}|^2 \quad (18)$$

In this particular example, to remove one transmit antenna, we exploit the eigenvector, $\hat{\mathbf{u}}_{q,1}$, representing the null space and consisting of two elements of $\hat{u}_{q,11}$ and $\hat{u}_{q,21}$. Each component in the null space can be considered to express the influence of each transmit antenna's channel component on the null space. Through the length of each component vector, we can find the maximum effect on the null space and then delete the corresponding antenna. That is, the antenna index to be removed can be given by follows.

$$\hat{v}_{q,remove} = \arg \max (|\hat{u}_{q,11}|, |\hat{u}_{q,21}|) \quad (19)$$

This algorithm (called Null-Space) modified for OFDM systems can be described as in TABLE 1. It exploits null

space of the channel matrix, $\mathbf{H}_{q,s}$, determined in bulk selection. To obtain the null space, the channel matrix of

$$\underline{\mathbf{H}}_{q,s} = \left[\mathbf{h}_{(q,1)}, \mathbf{h}_{(q,2)}, \dots, \mathbf{h}_{(q,N_F+1)} \right] \quad (20)$$

where $\mathbf{h}_{(q,v)}$ is denoted by the v th column vector of the matrix $\mathbf{H}_{q,s}$, is combined at each subcarrier as $\tilde{\mathbf{H}}_q = \underline{\mathbf{H}}_{q,s}^H \underline{\mathbf{H}}_{q,s}$, which is decomposed into its eigenvalues and eigenvectors. Then find the eigenvector $\mathbf{u}_{q,1}$, associated with the smallest eigenvalue of $\tilde{\mathbf{H}}_q$. It describes the null space of the combined matrix $\tilde{\mathbf{H}}_q$. Since each element of $\mathbf{u}_{q,1}$ represents the contribution of the corresponding transmit antenna's channel component to the null space, the channel column with the largest contribution can be removed. Here $\hat{r}_{q,1}^{(1)}$ indicates the index of maximum length of the eigenvector $\hat{\mathbf{u}}_{q,1}$. After finding the first transmit antenna, which will be eliminated, the matrix $\underline{\mathbf{H}}_{q,s}$ can be updated as

$$\underline{\mathbf{H}}_{q,s} = \left[\mathbf{h}_{(q,1)}, \dots, \mathbf{h}_{(q,\hat{r}_{q,1}^{(1)}-1)}, \mathbf{h}_{(q,\hat{r}_{q,1}^{(1)}+1)}, \dots, \mathbf{h}_{(q,N_F+1)} \right] \quad (21)$$

by removing one antenna before combining. Then go to the next step to remove the second antenna.

TABLE 1. Transmit antenna selection algorithm based on null-space

1. for $q=1,2,\dots,Q$
2. set $\underline{\mathbf{H}}_{q,s} = \left[\mathbf{h}_{(q,1)}, \mathbf{h}_{(q,2)}, \dots, \mathbf{h}_{(q,N_F+1)} \right]$
3. compute $\tilde{\mathbf{H}}_q = \underline{\mathbf{H}}_{q,s}^H \underline{\mathbf{H}}_{q,s}$
4. for $k=1,2,\dots,(N_S - N_F)$
5. $\left[\mathbf{U}_q, \mathbf{T}_q \right] = \text{eig} \left(\tilde{\mathbf{H}}_q \right)$
6. find the eigenvector, $\mathbf{u}_{q,1}$, corresponding to the smallest eigenvalue of $\tilde{\mathbf{H}}_q$
7. $\left[\hat{\mathbf{u}}_{q,1}, \hat{\mathbf{r}}_{q,1} \right] = \text{sort} \left| \mathbf{u}_{q,1} \right|$
8. $\hat{v}_q = \hat{r}_{q,1}^{(1)}$
9. update $\underline{\mathbf{H}}_{q,s} = \left[\mathbf{h}_{(q,1)}, \dots, \mathbf{h}_{(q,\hat{v}_q-1)}, \mathbf{h}_{(q,\hat{v}_q+1)}, \dots, \mathbf{h}_{(q,N_F+1)} \right]$
10. obtain $\tilde{\mathbf{H}}_q = \underline{\mathbf{H}}_{q,s}^H \underline{\mathbf{H}}_{q,s}$
11. end
12. $\mathbf{H}_{q,s} = \underline{\mathbf{H}}_{q,s}$
13. end

The complexity of the Null-Space antenna selection algorithm can be given by

$$\begin{aligned} & O \left(0.5N_S(N_S+1)N_RQ - 0.5N_F(N_F+1)N_RQ \right. \\ & \left. + (N_S - N_F)Q \left((N_F+1)^3 + (N_F+1) \right) \right) \end{aligned} \quad (22)$$

Here the combined matrix, $\mathbf{H}_q = \mathbf{H}_{q,s}^H \mathbf{H}_{q,s} \in C^{N_S \times N_S}$, can be computed only in the first step and thus the computation of the matrix $\tilde{\mathbf{H}}_q$, with smaller size, is unnecessary in the following steps. Further, an additional computation for ZF filtering in the receiver using the matrix $\mathbf{H}_{q,f}$ can be saved. The Null-Space selection algorithm can offer a significantly reduced computational complexity compared with the optimum selection algorithm, whose complexity is given by

$$O \left(\frac{(N_F^3 + N_F + 0.5N_F(N_F+1)N_F)N_S!}{(N_F!(N_S - N_F)!)} Q \right) \quad (23)$$

4. Simulation Results

The simulated spatial multiplexing MIMO-OFDM system assumes 64 FFT points, unless specified otherwise, under 16 quadrature amplitude modulation (QAM) and quadrature phase shift keying (QPSK) signalings. The total number of available transmit antennas is given as N_T and the number of RF chains in transmitter is N_S . After per-tone selection, N_F transmit antennas are used for sending the signals. Meanwhile, the number of receive antennas is assumed to be N_R . The propagation channel in every transmit and receive antenna pair is modeled as the independently and identically distributed (i.i.d.) complex Gaussian distribution $CN(0, 1)$ with $L=5$ taps.

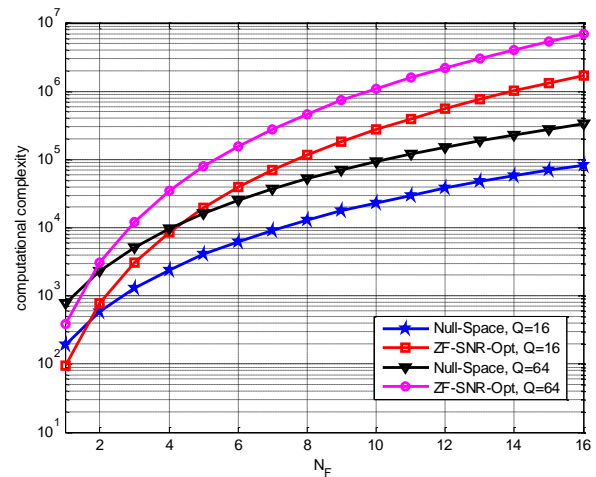


Figure 2. Computational complexity comparison for Null-Space algorithm and ZF-SNR-Opt algorithm for per-tone selection with $N_F = N_R$ and $N_F = N_S - 1$.

In Figs. 2, 3, and 4, computational complexity comparisons between optimal searching algorithm and Null-Space

algorithm are presented with $N_F = N_S - 1$, $N_F = N_S - 2$, and $N_F = N_S - 3$, respectively. It is assumed that $N_R = N_F$. It is shown that the complexity of Null-Space selection algorithm is significantly lower than the optimal one except for $N_F = 1$. As the number of available RF chains increases, the complexity difference between Null-Space algorithm and optimal one gets bigger. It is also observed that as the gap between N_S and N_F increases, the Null-Space algorithm has a better advantage in terms of complexity over optimal one. Hence the optimal algorithm requires higher computational complexity except for the case of $N_F = 1$.

$E_b/N_0 + 10\log_{10}(N_{bps})$ where E_b is the information bit energy and N_{bps} is the signal modulation order. In Fig. 6, the BER performance of the OFDM antenna diversity system with $N_T = 6$, $N_S = 4$, and $N_F = N_R = 2$ is seen. It is found that the Null-Space algorithm achieves the error rate performance approaching to that of ZF-SNR-Opt one with reduced complexity. Independent of signal modulation order, its performance is close to that of optimal one. By comparing Fig. 5 with Fig. 6, the antenna diversity system with $N_T = 6$, $N_S = 4$, and $N_F = N_R = 2$ can provide more diversity than that with $N_T = 5$, $N_S = 4$, and $N_F = N_R = 3$.

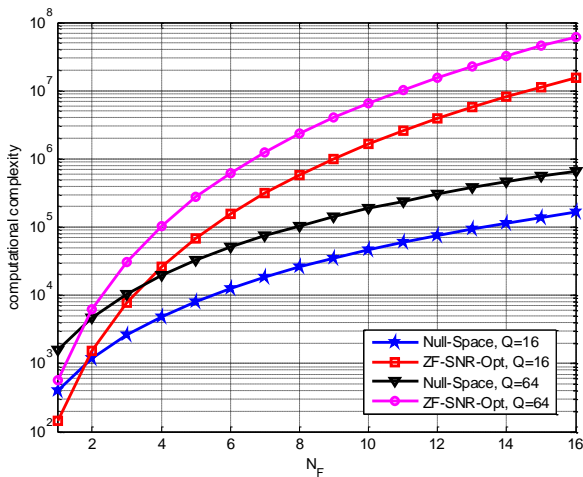


Figure 3. Computational complexity comparison for Null-Space algorithm and ZF-SNR-Opt algorithm for per-tone selection with $N_F = N_R$ and $N_F = N_S - 2$.

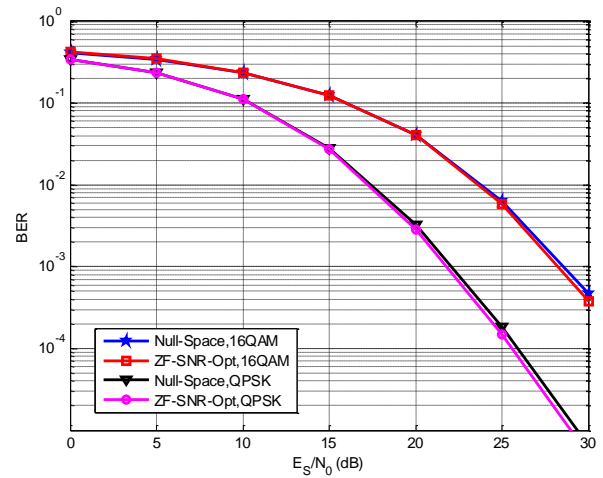


Figure 5. BER comparison of Null-Space selection algorithm and ZF-SNR-Opt algorithm with $N_T = 5$, $N_S = 4$, $N_F = 3$, and $N_R = 3$.

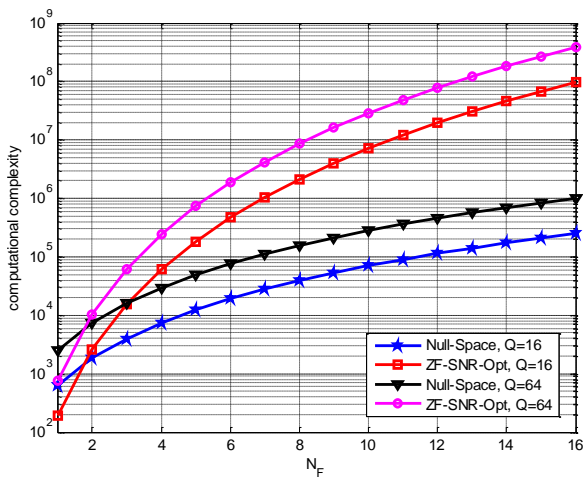


Figure 4. Computational complexity comparison for Null-Space algorithm and ZF-SNR-Opt algorithm for per-tone selection with $N_F = N_R$ and $N_F = N_S - 3$.

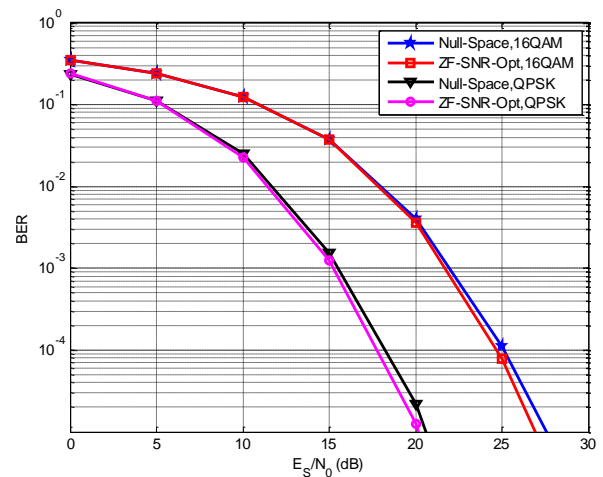


Figure 6. BER comparison of Null-Space selection algorithm and ZF-SNR-Opt algorithm with $N_T = 6$, $N_S = 4$, $N_F = 2$, and $N_R = 2$.

Fig. 5 shows the BER result versus E_s/N_0 in decibels for the OFDM antenna diversity system with system parameters such as $N_T = 5$, $N_S = 4$, and $N_F = N_R = 3$. Here $E_s/N_0 =$

CONCLUSION

This work has studied an efficient per-tone selection algorithm for hybrid bulk and per-tone transmit antenna selection in spatial multiplexing MIMO-OFDM systems with ZF receiver. It is based on the eigenspace of the MIMO channel spatially combined, which is obtained by successively removing one transmit antenna with the largest contribution to its null space. The Null-Space algorithm has achieved near-optimal BER performance while holding low-complexity except for the case of $N_f = 1$. An advantage of its complexity gets prominent as the number of RF chains increases from $N_f = 2$. Thus the Null-Space algorithm is a promising one for per-tone selection for the practical antenna selection MIMO-OFDM systems.

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