

Modelling a Spiral Porous Fin with Temperature Dependent and Independent Thermal Conductivity

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Abstract

In this study, temperature distribution equation for a spiral porous fin is presented. This new configuration is conceived regarding the interest of the porous fin. Numerical method is applied. First, we did the study with a thermal conductivity independent of temperature, after the thermal conductivity is taking linearly dependent of temperature. It is assumed that heat transfer through the porous media is simulated by using the passage velocity from the Darcy's model. Effects of porosity, Rayleigh number modified, fin parameter, and the pitch of the spiral fin on fin efficiency are examined. It is observed that the fin temperature is increased by increasing the thermal conductivity parameter and the fin efficiency will be improved.

Keywords: spiral porous fin, thermal conductivity, Darcy' model, efficiency.

INTRODUCTION

Extended surfaces that are well known as fins are commonly used to enhance heat transfer in many industries. Fins are widely used to increase the rate of heat transfer from a primary wall surface; and that's why applications for finned surfaces are widely seen in air-conditioning, refrigeration, cryogenics and electronic devices as well as in heat exchangers and power generators working with high temperature.

Beside the traditional applications, we can found fins in nuclear technology where heat transfer is dominated by low thermal convection, high thermal radiation and by internal heat generation. Because of the high temperature range involved, the thermo physical properties of the fins and the volumetric heat source become temperature dependent. Therefore it is essential that the variation in thermal conductivity of the fin must be taken into account in the thermal analysis; thus, a renewed attention has been received by many researchers during the last decade, termed as highly non linear heat transfer fin.

Many researches are applied on straight solid fins such Sharqawy and Zubair [1]. Sabaghi et al [2] investigated on the semi spherical solid fins. Aziz and Bouaziz [3] studied the longitudinal fin with temperature dependent internal heat generation and thermal conductivity. Fin efficiency and optimization in the case of temperature-dependent of the thermal conductivity is studied by Bouaziz and Hanini [4]. Also, temperature distribution for annular fins with temperature-dependent thermal conductivity was studied by Ganji et al [5].

On the other hand heat transfer takes place in porous materials of various types. A primary aim is commonly to maximize either the thermal resistance or the rate of thermal equilibration between the material and a fluid passing through it - i.e. to facilitate heat exchange, and on the other hand when a porous surface is used, less material is consumed, knowing that to reduce the size and price of fins are the first fins industry goals. The need is often justified by the high cost of high thermal conductivity metals that are employed in the manufacture of surfaces of fins.

The application of porous materials are relevant to a wide number of fields and industries, including aerospace, energy, transportation, construction, electronics, biomedical and others.

The concept of using porous fins by introducing the Darcy model [6, 7] is firstly proposed by Kiwan and Al-Nimr [8], they studied the heat transfer with radiation losses in porous fins, and mainly they investigated the heat transfer enhancement using porous fins.

Saedodin and Sadeghi [9] studied heat transfer of a cylindrical porous fin through fourth order Runge-Kutta method and they found that the heat transfer rate from porous fin could exceed that of a solid fin. Hatami and Ganji [10] studied the heat transfer in porous fins manufactured on the material Si_3N_4 and Al.

There is a continuous effort engaged to modify the fin shape in order to obtain high heat transfer rate per unit mass of fins. With this consideration, different tapered shapes have been suggested by many researchers. Hatami and Ganji [11] studied and investigate the refrigeration efficiency for fully wet circular porous fins with variable sections by combined heat and mass transfer analysis. Refrigeration efficiency analysis for fully wet semi spherical porous fins is studied by Hatami et al. [12]. Bhanja and Kundu [13] investigated the T shaped porous fins. Kundu and Bhanja [14] presented an analytical study on the performance and most favorable design of porous fin of various profiles operating in convective environment. They observed that heat transfer rate through porous fins is significantly increased for any geometric fin compared to solid fins. Moradi et al. [15] analyzed the performance of a triangular porous fin using differential transform method. Das [16] carried out forward and inverse solutions of conductive, convective and radiative cylindrical porous fins.

The solid spiral fin is investigated by Wen-Jyi and Cha'o-Kuang [17]. They studied the transient response of a spiral fin with its base, subjected to a sinusoidal form in temperature. Kiatpachai et al. [18] worked on air side performance of serrated welded spiral fin and tube heat exchangers. Fajiang et

al [19] studied experimental investigation of heat transfer and flowing resistance for air flow cross over spiral finned tube heat exchanger.

The spiral fins seems to have a greater heat transfer coefficient than the circular form, because of the influence of downstream rotational vortices. Similarly, a spiral heat exchanger is more compact than many other types of heat exchangers. The main advantages of a spiral heat exchangers are its high overall heat transfer coefficient, compact size for a given heat exchange area, relatively low pressure drop and ease of cleaning. Spiral heat exchanger flow may be countercurrent flow, concurrent flow or cross flow.

The porous spiral fins has not been yet considered so it is the first time that a study on modeling of spiral porous fins is investigated in this paper. Both considerations are analyzed from the dependent/independent thermal conductivity of the porous spiral fin.

MATHEMETICAL ANALYSIS

A steady state analysis is carried out on a spiral porous fin subject to convection as shown in Fig. 1, with the first part the thermal conductivity is considered as independent of temperature, after we do on the second part the study with the temperature-dependent thermal conductivity. In this regard, the following assumptions are made to simplify the analysis.

1. Heat transfer coefficient is constant.
2. Heat transfer is considered to be occurred just in one direction r.
3. The interaction between the porous media and the fluid is modeled by Darcy's formulation.
4. The fluid and the solid matrix are in thermal equilibrium.
5. The effect of air pressure drop due to air flow is neglected.

These are essentially classical assumptions that are typically used for the analysis of conducting – convecting finned surfaces.

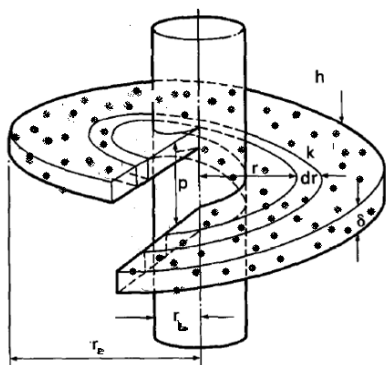


Figure 1: Schematic of spiral porous fin.

According to an element considered in Fig.1, by the assumption that minus heat enters to control volume, energy balance for heat can be written as

$$q_r - q_{r+dr} = \dot{m}C_p(T(r) - T_\infty) + hA(1 - \varepsilon)(T(r) - T_\infty) \quad (1)$$

Where ε is porosity and $(1 - \varepsilon)$ denotes to the fraction of fin area which is solid or subtracted the pores area which is necessary for convection surface. C_p is the specific heat at a constant pressure of the fluid; A is the section area of fin, h the heat transfer convective coefficient, T_∞ the air temperature for convection while q denotes the heat flux.

The mass flow rate of the fluid passing through the porous material is

$$\dot{m} = \rho v_w S \quad (2)$$

Where $S = 2\pi r' dr$ and r' is the radial direction. The Length of a whorl of one circular helix r' linked to the pitch of the spiral fin P is

$$r' = 2\pi \sqrt{\left(\frac{P}{2\pi}\right)^2 + r^2}$$

The passage velocity from the Darcy's model is well known, expressed by the thermal dilatation coefficient β^* , the permeability K of the porous medium and the Kinematic viscosity ν of the air.

$$v_w = \frac{gK\beta}{\nu} (T(r) - T_\infty) \quad (3)$$

From Fourier's law of conduction

$$q = -k_{eff} A \frac{dT}{dr} \quad (4)$$

With A is the cross-sectional of the fin, $A = 2\pi r' \delta r' \delta$ linked to the fin thickness and k_{eff} is the effective thermal conductivity of the porous fin, by part of solid and fluid and which can be obtained from the following equation

$$k_{eff} = \varepsilon k_f + (1 - \varepsilon) k_s \quad (5)$$

By substituting eqs. (2-5) into eq. (1) yields

$$\frac{d}{dr} \left\{ k_{eff} \sqrt{\left(\frac{P}{2\pi}\right)^2 + r^2} \frac{dT}{dr} \right\} = \frac{\rho g K \beta C_p}{\delta \nu} \sqrt{\left(\frac{P}{2\pi}\right)^2 + r^2} (T(r) - T_\infty) + \sqrt{\left(\frac{P}{2\pi}\right)^2 + r^2} \frac{h(1-\varepsilon)}{\delta} (T(r) - T_\infty) \quad (6)$$

Referring to Fig.1 and considering the following non-dimensional parameters (temperature, radius and the pitch)

$$\theta = \frac{T(r) - T_\infty}{T_b - T_\infty} \quad (7)$$

$$R = \frac{r}{r_b} \quad (8)$$

$$P_a = \frac{P}{2\pi r_b} \quad (9)$$

r_b referred to the base of the fin. The above equation (6) can be written in the convenient forms.

Case1.

Temperature-independent thermal conductivity case

The non dimensional form is:

$$\frac{d}{dR} \left(\sqrt{P_a^2 + R^2} \frac{d\theta}{dR} \right) = \frac{\rho g K \beta c_p r_b^2}{k_{eff} v \delta} (T_b - T_\infty) \sqrt{P_a^2 + R^2} \theta^2 + \frac{h(1-\epsilon)r_b^2}{\delta k_{eff}} \sqrt{P_a^2 + R^2} \theta \quad (10)$$

The following parameters are defined as

$$Ra = \frac{g \beta r_b^3 (T_b - T_\infty) \rho c_p}{v k_f} \text{ Rayleigh number}$$

$$N^2 = \frac{h r_b^2}{\delta k_{eff}} \text{ fin Parameter}$$

$$Ra^* = \frac{Ra Da}{\psi k_r} \text{ Modified Rayleigh number}$$

Where the Darcy (Da) number appears and where the permeability is included. k_r (k_{eff}/k_f) is the conductivity ratio. Finally, equation (10) can be written

$$\frac{d^2 \theta}{dR^2} = Ra^* \theta^2 + (1 - \epsilon) N^2 \theta - \frac{R}{P_a^2 + R^2} \theta' \quad (11)$$

The boundary conditions will be as follows: the fin tip is assumed to be insulated, so no heat transfer will occur at the insulated tip and the fin base has a temperature, by these definitions, we write:

$$\theta(1) = 1$$

$$\frac{d\theta}{dR} \Big|_{R=Re} = 0 \quad (12)$$

The subscript e is for the tip of the fin.

Case 2.

Temperature-dependent thermal conductivity case

Here, we put a linear form without loss of generality

$$k_{eff} = k^* (1 + \beta^* \theta)$$

The corresponding governing equation can be obtained with the same precedent path

$$\frac{d^2 \theta}{dR^2} = \frac{-\beta^*}{1 + \beta^* \theta} \left(\frac{d\theta}{dR} \right)^2 + \frac{1}{1 + \beta^* \theta} Ra^* \theta^2 + \frac{(1 - \epsilon)}{1 + \beta^* \theta} N^2 \theta - \frac{R}{P_a^2 + R^2} \theta' \quad (13)$$

Non linear terms appeared which corresponding to the temperature –dependent thermal conductivity.

The boundary conditions will be as indicated in eq. (12),

RESULTS AND DISCUSSION

In this study we investigated the convective heat transfer of a porous spiral fin with temperature-independent thermal conductivity corresponding to the first case. To validate our analysis and results, the plot dressed in the fig.2 compares the results for the present spiral fin with those for the circular fin. In the case where the pitch is equal to zero, we obtain similar curves, which demonstrate the correct mathematical way.

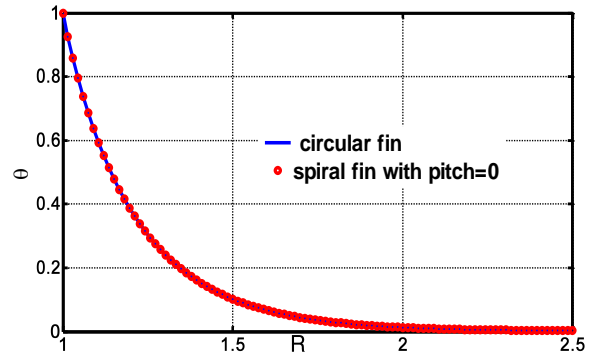


Figure.2: Comparison between a circular fin and a spiral fin for a pitch equal to zero

Case 1: $\beta = 0$

The effect of the pitch of the spiral fin on the non dimensional temperature is shown in Fig.3, it can be seen that by decreasing the pitch, the temperature of the fin decreases slightly and no important effect can be obtained.

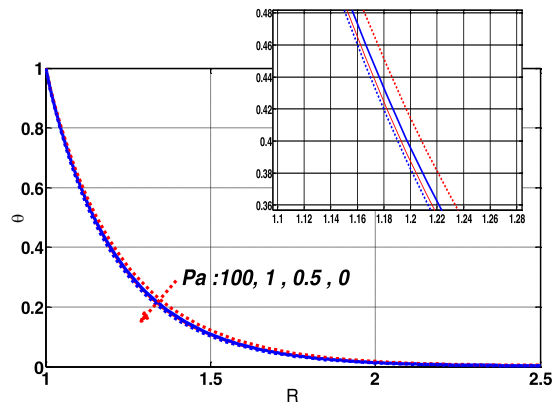


Figure 3: Effect of the pitch of the spiral fin on dimensionless temperature

Fig.4 shows the effect of porosity on temperature profiles. As seen, increasing the porosity makes higher temperature, because the conducting exchange between the fluid and the solid surface of the fin is augmented due to large specific areas. As result, the fin becomes more conductive and more efficient.

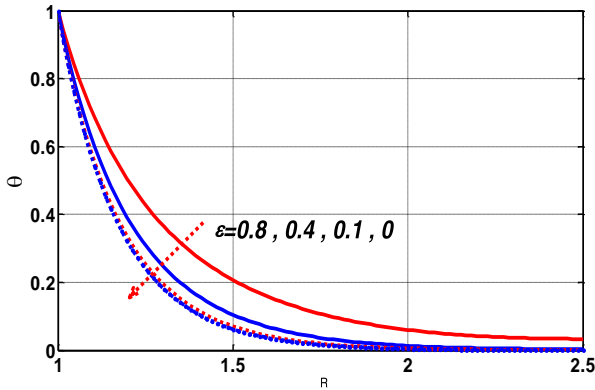


Figure 4: Effect of porosity for a spiral fin on temperature distribution

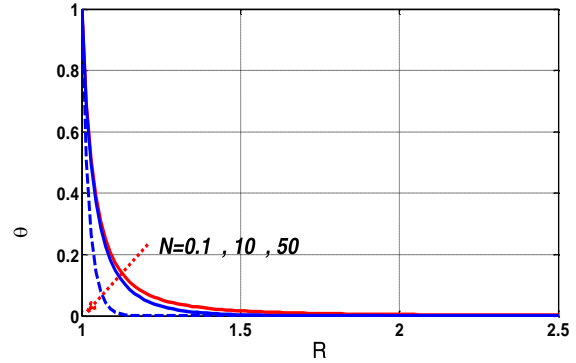


Figure 6: Effect of the fin parameter on temperature distribution.

Fig.5 explains that increasing the Ra^* number decreases the fin efficiency, because the modified Rayleigh number contains Ra number which represents the buoyancy force so the convection mechanism is dominant. This number contains also the Darcy number which is proportional to the permeability, Darcy number increases when the permeability increases so collision among the fluid flow and pores of porous media is less great and we obtain lower fin temperature.

Fig.6 illustrates the effect of the fin parameter on the temperature distribution, while the other parameters are constant. As seen, when this parameter increases the convective transfer is dominant over the conductive transfer within the fin. Consequently, the fin efficiency falls until the fin is not necessary to use. It is evident that a weak fin parameter is desired in these circumstances.

Case 2: $\beta \neq 0$

As before, in fig. 7, the temperature distributions prepared with the mathematical model without β and with β taking a value different of zero is presented. The two curves are identical.

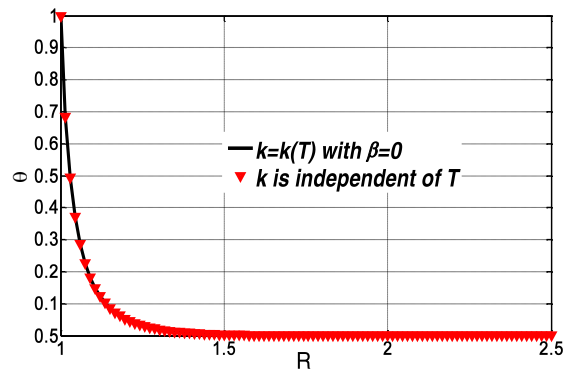


Figure 7: Comparison between situations $k=k(T)$ for $\beta=0$

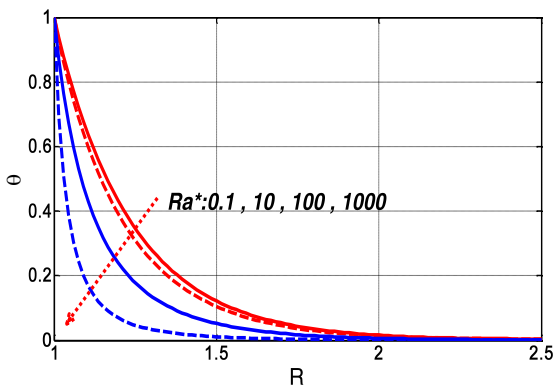


Figure 5: Effect of modified Rayleigh number on temperature distribution.

As drawn in fig. 8, the temperature distribution is more and more affected by the values of β taking the large positive values or negative values. The fin efficiency is better for the fin manufactured by the material with positive β than with negative β . This is physically suspected because the fin material has a more and more great thermal conductivity.

Like the precedent result, in fig. 9, the great value of the pitch is also slightly advantageous in this case. As previously, the same trends are obtained for both the effect of the modified Rayleigh number and the fin parameter, and large values of Ra and N lead to a great fin efficiency, fig. 10 and fig. 11 respectively.

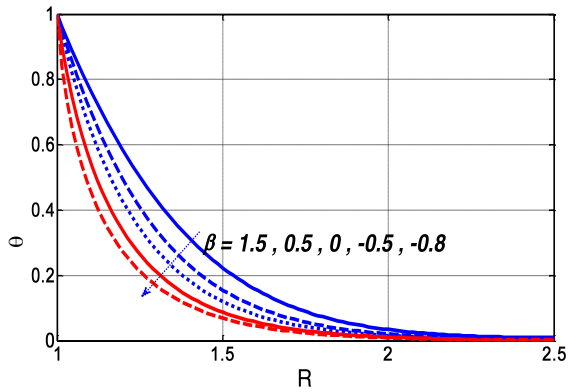


Figure 8: Effect of conductivity parameter on temperature distribution. ($\beta \neq 0$)

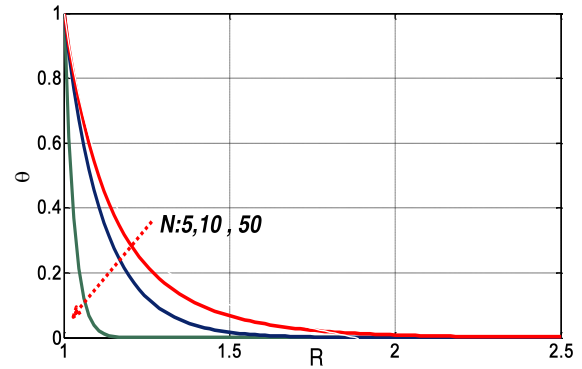


Figure 11: Effect of fin parameter on temperature distribution $\beta \neq 0$

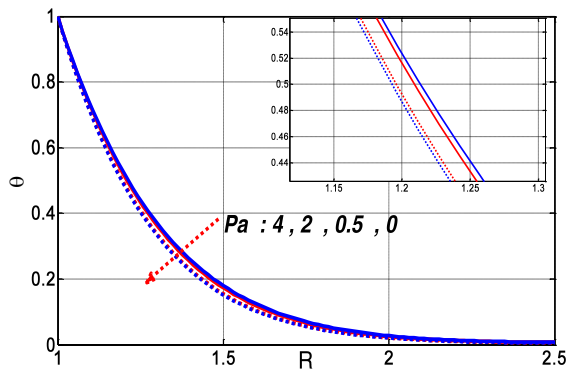


Figure 9: Effect of the spiral fin pitch on temperature distribution. ($\beta \neq 0$)

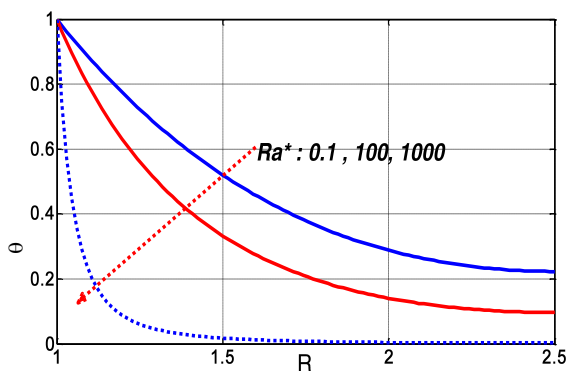


Figure 10: Effect of modified Rayleigh number on temperature distribution. ($\beta \neq 0$)

CONCLUSION

In this paper heat transfer equations for a spiral porous fin are presented and a numerical method has been applied to access to the temperature distribution. The following main points can be drawn from the present study.

- Large pitches from the spiral fin make a slightly better heat transfer when k is dependent or independent of temperature.
- Increasing the porosity makes higher temperature to fin.
- Increasing the modified Rayleigh number decreases fin efficiency; this is verified for a thermal conductivity dependent or independent of temperature.
- When the fin parameter N decrease the fin transfer is better in the two cases.
- Fin temperatures and efficiencies are increased by increasing in the thermal conductivity parameter β .

In several cases, it is interesting to add fins manufactured with porous material and in a spiral form. However, the opportunity of this last geometric form can be more favorable if it is corroborated by a conjugate problem with the flow around the fin.

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