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Goodness of Fit Tests of Laplace Distribution Using Selective Order Statistics

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Abstract

In this paper, a comparison for the power and efficiency of a set empirical distribution function(EDF) goodness of fit tests for the Laplace distribution is studied. Moreover, a new method to improve the power of these tests under the ranked set sampling is introduced. Using simulation, under ranked set sampling (RSS), the test are more powerful than their counterparts in simple random sampling (SRS) is shown. Also, the percentage points of these tests under the null hypotheses is obtained.

Keywords: Goodness of fit test; Empirical distribution function; Laplace distribution; Ranked set sampling; Simple random sampling.

INTRODUCTION

McIntyre (1952) introduced a new sampling scheme called the Ranked Set Sampling (RSS). The RSS gives a sample which is more informative than a simple random sample (SRS) about the population of interest. This technique can be described as follows. Select m random samples from the population of interest each of size m . From the i^{th} sample detect, using a visual inspection, the i^{th} order statistic and choose it for actual quantification, say, Y_j, $j=1,...,m$. The RSS is the set of the order statistics $Y_1,...,Y_m$. The technique could be repeated r times to get more observations. Takahasi and Wakimoto (1968) gave the theoretical setups for the RSS. They showed that the mean of an RSS is minimum variance unbiased estimator for the population mean. In fact, two factors affect the efficiency of an RSS, the set size and ranking errors. The larger the set size, the larger the efficiency of the RSS, while the larger the set size the more the difficulty in visual ranking and then the larger the ranking error (Al-Saleh and Al-Omari, 2002). For this, several authors modified the RSS to reduce the error in ranking and to make visual ranking tractable by the experiment. Muttlak (1997) introduced the Median Ranked Set Sampling (MRSS) which consists of quantifying only the median in each set. Samawi et al. (1996) investigated the Extreme Ranked Set Sample (ERSS), i.e. they quantify the small and the largest order statistics. Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS, which is called later by moving extremes ranked set sampling (MERSS). They investigated this modification for the location-scale family and found that the procedure produces more efficient estimators for location and scale parameters. Dell and Clutter (1972) showed that the sample mean of an RSS mean remains unbiased and more efficient than the sample mean of an even if ranking is imperfect. Bhoj (1997) proposed a modification to the RSS and called it new ranked set sampling (NRSS). He used this method to estimate the location and scale parameters of the rectangular and logistic distributions. A comprehensive survey about developments in RSS can be found in Patil et al. (1994a), Sinha et al. (1996), Chen (2000), Patil (2002) and Samawi and Al-Sagheer (2001).

Stephens (1979) gave goodness of fit tests for logistic distribution based on a SRS. A comprehensive survey for goodness of fit tests based on SRS can be found in the book of Stephens (1986). Stocks and Sager (1988) studied the characterization of RSS. Also, they gave an unbiased estimator for the population distribution function based on the empirical distribution function of RSS. Then, they proposed a Kolmogorov-Smirnov goodness of fit test based on the empirical distribution function. Also they derived the null distribution of their proposed test. Ibrahim et al. (2011) introduced a new method to improve the power of the chi-square test for goodness of fit based on RSS. They used the Kullback-Leibler information measure to compare the data collected by both SRS and RSS. Also, they conducted a simulation study for the power of chi-square test of the new method. Al-Subh (2014), studied the Goodness of Fit Test for Gumbel Distribution Based on Kullback-Leibler Information using Several Different Estimators.

For many years the Laplace distribution was popular topic in probability theory due to the simplicity of its characteristic function and density, the curious phenomenon that a random variable with only slightly different characteristic function loses the simplicity of the density function and other numerous attractive probabilistic features enjoyed this distribution. It is symmetric around the location parameter and looks like the normal for the heavy tails. In this paper, we propose a method to improve the power of the empirical distribution function goodness of fit tests for Laplace distribution under the RSS.

This paper is organized as follows. In Section 2, we introduce an empirical distribution function (EDF) goodness of fit tests in SRS and RSS to improve the power of these test statistics under ERSS. We apply these test statistics for the Laplace distribution in Section 3. In Section 4, we are defined two algorithms to calculate the percentage points, the power function and the efficiency at an alternative distribution. In Section 5, a simulation study is conducted to study the efficiency of these test statistics under ERSS with its SRS counterpart. In Section 6, we state our conclusions.

EDF GOODNESS OF FIT TESTS

Stephens (1974 and 1977) and Chandra et al. (1981) gave a practical guide to goodness of fit tests using statistics based on the empirical distribution function (EDF). He has compared the following EDF tests:

a) The Kolmogorov statistics: $D^+,\ D^-,\ D$

$$D^+ = \max_{1\le i\le n} \left[(i/n) - z_i\right].$$

$$D^- = \max_{1\le i\le n} \left[z_i - (i-1)/n\right].$$

$$D = \max \left[D^+, D^-\right].$$

b) The Cramer-von Mises statistics: W^2

$$W^2 = \sum_{i=1}^{n}\left[z_i - (2i-1)/2n\right]^2 + (1/12n).$$

c) The Kuiper statistic: V

$$V = D^+ + D^-.$$

d) The Watson statistics: U^2

$$U^2 = W^2 - n\left(\bar{z} - \frac{1}{2}\right)^2$$

where $\bar{z} = \sum_{i=1}^{n} z_i / n.$

e) The Anderson-Darling statistic: A^2

$$A^2 = -\left\{\sum_{i=1}^{n}(2i-1)\left[\ln z_i + \ln(1 - z_{n+1-i})\right]\right\}/n - n. \tag{2.1}$$

The cdf and pdf of the random variable X which has a Laplace distribution with parameters θ and σ are given, respectively, by

$$F(x;\theta,\sigma) = \begin{cases} \frac{1}{2}e^{-(\theta-x)/\sigma}, x < \theta \\ 1 - \frac{1}{2}e^{-(x-\theta)/\sigma}, x \ge \theta, \end{cases} \tag{2.2}$$

$$f(x;\theta,\sigma) = \frac{1}{2\sigma}e^{-\left(\frac{|x-\theta|}{\sigma}\right)} \tag{2.3}$$

where θ is the location parameter and σ is the scale parameter, $\sigma > 0$, x and $\theta \in (-\infty, \infty)$, denoted as $X : l(\theta, \sigma)$.

Let $X_1, X_2, ..., X_n$, $n = 2m-1$, $m = 1, 2, ...$ be a random sample from the distribution function $F(x)$. Assume that our objective is to test the statistical hypotheses $H_o : F(x) = F_o(x)$ $\forall x$, vs. $H_1 : F(x) \ne F_o(x)$ for some x, where $F_o(x)$ is a known distribution function.

It can be noted that testing the hypotheses $H_o : F(x) = F_o(x)$, $\forall x$ is equivalent to testing the hypotheses $H_o^* : G_i(y) = G_{io}(y)$, $\forall y$ vs. $H_1^* : G_i(y) \ne G_{io}(y)$ for some y, where $G_i(y)$, $G_{io}(y)$ are the cdf's of the i^{th} order statistics of random samples of size $n = 2m-1$ chosen from $F(x)$ and $F_o(x)$, respectively. According to Arnlod et al. (1992) $G_i(y)$ and $G_{io}(y)$ have the following representations:

$$G_i(y) = \sum_{j=i}^{2m-1}\binom{2m-1}{j}[F(y)]^j[1-F(y)]^{(2m-1)-j}$$

and

$$G_{io}(y) = \sum_{j=i}^{2m-1}\binom{2m-1}{j}[F_o(y)]^j[1-F_o(y)]^{(2m-1)-j},$$

respectively. For example, in case of $m = 3,\ i = 1,\ 2$ and 3, and the cdf's $G_i(y)'s$ and $G_{io}(y)'s$ are given by

$$G_1(y) = 1 - [1-F(y)]^3,$$

$$G_{1o}(y) = 1 - [1-F_o(y)]^3.$$

$$G_2(y) = 3F^2(y)(1-F(y)] + F^3(y),$$

$$= 3F^2(y) - 2F^3(y),$$

$$G_{2o}(y) = 3F_o^2(y) - 2F_o^3(y).$$

and

$$G_o(y) = F^3(y),$$

$$G_{3o}(y) = F_o^3(y).$$

It is easy to show that the equation $G_i(y) = G_{i0}(y)$ has the unique solution $F(x) = F_o(x)$.

If we employ the RSS to collect the data using the i^{th} order statistic, then we may use the resulting data to build empirical distribution function goodness of fit tests for the hypotheses H_o^* vs. H_1^*. Let $Y_1,...,Y_r$ be a random sample of size r selected via the i^{th} order statistic. Let T denote a test in (1.1) and T^* denotes its counterpart in the RSS when testing H_o^* vs. H_1^* using the data $Y_1,...,Y_r$.

In this paper, we restrict our attention for the case when $F_o(x)$, i.e., for the Laplace distribution. Moreover, we conduct a simulation study to show that the test T^* is more powerful than the test T when compared based on samples with the same size. The power of the T^* test can be calculated according to the equation

$$\text{Power of} \quad T^*(H) = P_H(T^* > d_\alpha), \tag{2.4}$$

where H is a cdf under the alternative hypothesis H_1^*. Here d_α is the 100α percentage point of the distribution of T^* and H_o. Since the behavior of RSS test statistics relative to SRS test statistics, we will calculate the efficiency of the test statistics as a ratio of powers.

$$eff(T^*, T) = \frac{\text{power of } T^*}{\text{power of } T}, \tag{2.5}$$

T^* is more powerful than T if $eff(T^*, T) > 1$.

TESTING FOR LAPLACE DISTRIBUTION

Let $F_o(x)$, $z_i = F((x_i - \theta)/\sigma)$ and $Y_1,...,Y_r$ be as in the introduction. To test the hypothesis $H_o: F(x) = F_o(x), \ \forall x$, it is equivalent to test

$$H_o^*: G(y) = G_{oi}(y) \ \ \forall y,$$

where $F_0(x)$ defined in (2.2) and

$$G_{io}(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F_o(y)]^j [1 - F_o(y)]^{(2m-1)-j}. \tag{3.1}$$

If θ and σ are unknown, then we may estimate them using their maximum likelihood estimator from $l(\theta, \sigma)$, the likelihood function of the data, i.e.,

$$l(\theta, \sigma) = \prod_{i=1}^{r} \frac{(2m-1)!}{(j-1)!(2m-1-j)!} [F_o(y_i;\theta,\sigma)]^{j-1} [1 - F_o(y_i;\theta,\sigma)]^{2m-1-j} f_o(y_i;\theta,\sigma),$$

where $f_o(y;\theta,\sigma)$ is defined in (2.3).

We perform a goodness of fit test for the hypotheses

$$H_o^*: G_i(y) = G_{io}(y) \quad \text{vs.} \quad H_1^*: G_i(y) \neq G_{io}(y).$$

using the tests give in (2.1) and using the data $Y_1,...,Y_r$.

ALGORITHMS TO POWER COMPARISON

To compare the powers of T^* and T, we first design the following algorithm to calculate the critical values:

1. Simulate $Y_1,...,Y_r$ be a RSS obtained based on the i^{th} order statistic from $G_{i0}(x)$, $i = 1, 2, 3.$
2. Without loss of generality we assume $\theta = 0, \ \sigma = 1$.
3. Find the EDF $F_r^*(x)$ as follows:

$$F_r^*(x) = \frac{1}{r}\sum_{j=1}^{r} I(Y_{(i)j} \le x), \ \ I(Y_{(i)j} \le x) = \begin{cases} 1 & , Y_{(i)j} \le x, \\ 0 & , \ \text{o.w,} \end{cases} \tag{4.1}$$

4. Use $F_r^*(x)$ to calculate the value of T^* as in (2.1).
5. Repeat the steps (1-4) 10, 000 times to get $T_1^*,..., T_{10,000}^*$.
6. The critical value d_α of T^* is given by the $(1-\alpha)100\%$ quantile of $T_1^*,..., T_{10,000}^*$

Secondly, to calculate the power of T^* at H, we need to use simulation. So, we design the following algorithm:

1. Simulate $Y_1,...,Y_r$ be a RSS obtained based on the i^{th} order statistic from H, a distribution under H_1^*, $i = 1, 2, 3$.
2. Find the EDF $F_r^*(x)$ as in (3.1).
3. Calculate the value of T^* as in (2.1) but using the data $Y_1,...,Y_r$.
4. Repeat the steps (1) - (3), 10,000 times to get $T_1^*,..., T_{10,000}^*$.
5. Power of $T^*(H) \approx \frac{1}{10,000}\sum_{t=1}^{10,000} I(T_t^* > d_\alpha)$, where $I(.)$ stands for indicator function

SIMULATION RESULTS

We approximated the power of each test based on a Monte Carlo simulation of 10,000 iterations according to the algorithm of Section power comparison. Table 1 shows the percentage points for the 5-percent level for the null hypotheses of Laplace distribution under three sampling schemes: first $(i=1)$, second $(i=2)$ and largest $(i=3)$ order statistics. We compared the powers and efficiencies of the two tests for different samples sizes: $r=10,\ 20,\ 30, 40,$ different set sizes: $m=1,\ 2,\ 3,\ 4$ (m=1 mean SRS case) and different alternative distributions: $Normal=N(0,\ 1),$ $Logistic=Lo(0,\ 1),$ $Lognormal=LN(0,\ 1),$ $Cauchy=C(0,\ 1),$ $StudentT=S(5),$ and $Uniform=U(0,\ 1).$ The Simulation results are shown in the Tables (2)-(4). For Lognormal and Uniform alternative distributions, computations show that the powers and efficiencies of all test statistics are equal one, so these values have not been reported in Tables (2) and (3). The tables of maximum are deleted for the symmetric distribution.

Table 1. Percentage points for first, median and largest order statistic

	Minimum, $\alpha=0.05.$					Median, $\alpha=0.05.$				
	$r=10$									
$m/$Test	$D\sqrt{n}$	$V\sqrt{n}$	W^2	U^2	A^2	$D\sqrt{n}$	$V\sqrt{n}$	W^2	U^2	A^2
1	1.304	1.643	0.463	0.188	2.531	1.308	1.633	.45	.184	2.529
2	1.290	1.621	0.447	0.179	2.500	1.303	1.639	.463	.186	2.546
3	1.293	1.623	0.451	0.181	2.517	1.294	1.629	.453	.184	2.517
4	1.292	1.624	0.446	0.179	2.456	1.275	1.618	.445	.12	2.486
	$r=20$									
1	1.309	1.653	0.451	0.182	2.450	1.315	1.663	.452	.186	2.487
2	1.315	1.659	0.468	0.185	2.563	1.309	1.676	.459	.186	2.530
3	1.305	1.667	0.445	0.185	2.450	1.314	1.660	.460	.184	2.490
4	1.296	1.664	.442	.182	2.412	1.318	1.673	.460	.186	2.463
	$r=30$									
1	1.324	1.672	.464	.184	2.561	1.323	1.672	.455	.184	2.461
2	1.334	1.688	.471	.187	2.545	1.320	1.687	.460	.186	2.514
3	1.317	1.680	.452	.184	2.434	1.328	1.693	.466	.188	2.525
4	1.309	1.679	.447	.184	2.421	1.314	1.688	.449	.187	2.456
	$r=40$									
1	1.336	1.698	.464	.186	2.510	1.325	1.692	.457	.185	2.498
2	1.341	1.688	.468	.185	2.531	1.337	1.706	.461	.190	2.524
3	1.317	1.691	.456	.186	2.510	1.320	1.689	.454	.184	2.476
4	1.317	1.675	.448	.180	2.423	1.331	1.689	.464	.184	2.499

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Table 2. Efficiency values for SRS and RSS (using first order statistics).

H	T	Minimum , $\alpha = 0.05$.											
		$r = 10$, m			$r = 20$, m			$r = 30$, m			$r = 40$, m		
		2	3	4	2	3	4	2	3	4	2	3	4
$N(0,\ 1)$	$D\sqrt{n}$	1.021	2.521	5.688	1.617	6.383	13.26	2.135	9.962	16.52	3.84	14.2	19.3
	$V\sqrt{n}$	2.185	3.481	5.185	2.676	5.324	7.93	2.737	5.789	8.316	3.569	6.922	8.961
	W^2	1.07	2.767	6.349	1.614	6.5	14.43	2.041	10.2	17.88	3.489	14.55	20.66
	U^2	2.314	4.098	5.804	3.266	6.516	9.547	3.469	7.432	10.26	4.878	9.378	11.46
	A^2	1.393	3.357	7.393	1.944	7.194	16.44	2.725	12.8	21.98	4.595	17.6	23.33
$Lo(0,\ 1)$	$D\sqrt{n}$	1.641	2.034	2.368	1.834	2.592	3.018	1.928	2.715	3.095	2.152	2.753	3.103
	$V\sqrt{n}$	1.065	1	1.084	1.02	1	1.084	0.946	1.018	1.075	1.108	1.112	1.232
	W^2	1.833	2.426	2.889	2.259	3.392	3.741	2.371	3.505	3.989	2.576	3.342	3.749
	U^2	1.062	1.006	1.124	0.935	0.919	1.009	0.898	0.949	0.998	1.01	0.984	1.093
	A^2	1.396	1.667	1.931	1.539	2.112	2.435	1.68	2.271	2.54	1.701	2.061	2.316
$C(0,\ 1)$	$D\sqrt{n}$	2.598	5.12	7.217	4.047	7.262	8.72	5.189	8.261	8.901	5.37	7.23	7.407
	$V\sqrt{n}$	1.849	2.984	4.135	2.098	3.275	4.172	2.251	3.333	3.765	2.279	2.934	3.11
	W^2	2.66	5	6.97	4.065	7.231	8.648	4.825	7.625	8.225	5.1	6.943	7.143
	U^2	1.735	2.833	4.023	1.935	3.047	3.926	2.066	3.07	3.484	2.19	2.853	3.04
	A^2	1.232	1.418	1.388	1.201	1.05	0.794	1.088	0.724	0.459	0.91	0.496	0.267
$S(5)$	$D\sqrt{n}$	0.872	0.915	1.277	0.667	0.921	1.857	1.022	1.63	2.348	0.642	1.062	1.741
	$V\sqrt{n}$	1.378	1.489	1.733	1.322	1.627	2.475	2.286	2.976	3.286	1.259	1.901	2.222
	W^2	0.851	0.872	1.213	0.729	0.949	1.932	0.915	1.426	2.128	0.594	1.261	2.014
	U^2	1.362	1.426	1.723	1.411	1.821	2.786	2.686	3.829	4.429	1.39	2.13	2.545
	A^2	0.974	0.921	1.289	0.851	1.021	2.234	0.911	1.333	1.933	0.71	1.355	2.113

Table 3. Efficiency values for SRS and RSS (using second order statistics).

H	T	Median, $\alpha = 0.05$.											
		$r = 10$, m			$r = 20$, m			$r = 30$, m			$r = 40$, m		
		2	3	4	2	3	4	2	3	4	2	3	4
$N(0,\ 1)$	$D\sqrt{n}$	1	1.038	1.226	1.127	1.127	1.091	1.26	1.22	1.34	1.231	1.25	1.192
	$V\sqrt{n}$	1	1.04	1.08	0.802	0.79	0.679	0.683	0.614	0.644	0.676	0.63	0.676
	W^2	1	1.167	1.229	1.135	1.135	1.173	1.061	1.102	1.245	1.12	1.12	1.18
	U^2	1	0.961	1.039	0.884	0.87	0.812	0.702	0.702	0.833	0.7	0.711	0.767
	A^2	1	1.15	1.3	1.289	1.421	1.526	1.1	1.3	1.45	1.14	1.116	1.302
$Lo(0,\ 1)$	$D\sqrt{n}$	1	1.119	1.178	1.263	1.417	1.48	1.327	1.52	1.677	1.406	1.621	1.793
	$V\sqrt{n}$	1.222	1.485	1.695	1.351	1.642	1.712	1.341	1.51	1.615	1.317	1.48	1.544
	W^2	1.048	1.12	1.12	1.293	1.527	1.647	1.441	1.761	1.989	1.591	1.907	2.16
	U^2	1.229	1.492	1.631	1.368	1.598	1.684	1.333	1.494	1.577	1.326	1.444	1.488
	A^2	1.23	1.568	1.694	1.465	1.886	2.155	1.578	1.946	2.182	1.618	1.932	2.111
$C(0,\ 1)$	$D\sqrt{n}$	1.135	1.326	1.292	1.191	1.291	1.318	1.252	1.276	1.354	1.446	1.454	1.585
	$V\sqrt{n}$	0.902	1.06	1.068	1.219	1.26	1.291	1.124	1.138	1.247	1.251	1.371	1.381
	W^2	1.03	1.162	1.121	1.14	1.202	1.14	1.189	1.152	1.227	1.433	1.433	1.583
	U^2	0.901	1.071	1.021	1.189	1.25	1.245	1.205	1.205	1.329	1.343	1.444	1.447
	A^2	0.797	0.721	0.663	0.96	0.838	0.766	1.102	0.962	0.945	1.369	1.234	1.206
$S(5)$	$D\sqrt{n}$	1.016	1.115	1.148	1.083	1.167	1.117	1.046	1.169	0.831	1.109	1.266	1.328
	$V\sqrt{n}$	0.931	1.138	1.086	1.033	1.082	1.18	1.109	1.203	0.984	1.085	1.324	1.465
	W^2	1.053	1.105	1.228	1.107	1.214	1.125	1.113	1.321	0.943	1.127	1.345	1.291
	U^2	1.034	1.153	1.051	1.145	1.309	1.418	1.115	1.328	0.967	1.141	1.594	1.734
	A^2	1.196	1.37	1.652	1.163	1.327	1.367	1.104	1.458	1.021	1.16	1.46	1.58

From the above tables, we make the following remarks:

1. In general, the efficiency is increasing as the sample size r increases, except for the A^2 test in the Cauchy case.
2. The efficiency is increasing as the set size m increases, except for the A^2 test in the Cauchy case.
3. The EDF tests based on the i^{th} order statistic (i=1, 2, 3) are more efficient than the EDF tests based on the SRS case (m=1) of the same size except for the A^2 test in the Cauchy case.
4. It can be noted from Table 3 (median case) that the efficiencies of the modified tests are equal their counterpart in SRS case.

CONCLUSION

In this paper, we have proposed a method to improve the EDF tests for goodness of fit when the data is collected via the selective order statistics. We have considered the RSS schemes that quantify only one order statistic namely minimum, median and maximum. Since it is easier for the experimenter to detect the extreme order statistics by a visual inspection, then this makes the method easy to apply and robust against ranking error. The theory developed could be extended easily to other distributions.

REFERENCES

[1] Al-Odat M. T. and Al-Saleh M. F. (2001). A variation of ranked set sampling. *Journal of Applied Statistical Science*. **10**, 137–146.

[2] Al-Saleh M. F and Alomari A. I. (2002). Multistage ranked set sampling. *Journal of Statistical planning and Inference*, **102**, 273–286.

[3] Al-Subh, S. A. (2014). Goodness of Fit Test for Gumbel Distribution Based on Kullback-Leibler Information using Several Different Estimators. *Applied Mathematical Sciences*, 8(95), 4703 – 4712.

[4] Arnold, B. C., Balakrishnan, N. and Nagaraja, H. N. (1992). *A First Course in Order Statistics.* John Wiley and Sons, New York.

[5] Bhoj, D. S. (1997). Estimation of parameters of the extreme value distribution using ranked set sampling. *Commun. Statist-Theory Meth*., **26(3),** 653-667.

[6] Chandra, M., Singpurwalla, N. D. and Stephens, M. A. (1981). Kolmogorov statistics for tests of fit for the extreme value and weibull distributions. *Journal of the American Statistical Association*. **76(375),** 729-731.

[7] Chen, Z. (2000). On ranked-set sample quantiles and their applications. *J. Statist. Plann. Inference*. **83**, 125-135.

[8] D'Agostine, R., Stephens. M. (1986). *Goodness of fit Techniques*. Marcel Dekker Inc., New York

[9] Dell, D. R. and Clutter, J. L. (1972). Ranked set sampling theory with order statistics background. *Biometrics*. **28**, 545-555.

[10] Ibrahim, K. Alodat, M.T. Jemain, A.A. & Al-Subh, S.A. (2011). Chi-square test for goodness of fit for logistic distribution using ranked set sampling and simple random sampling. *Statistica & Applicazioni*, IX(2), 111-128.

[11] McIntyre, G. A. (1952). A method of unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research*. **3**, 385-390.

[12] Muttlak, H. A. (1997). Median ranked set sampling. *Journal of applied statistics science*. **6**, 245-255.

[13] Patil, G. P., Sinha, A. K. and Taillie, C. (1994a). Ranked set sampling. In: G.P. Patil and C.R. Rao (eds.), Handbook of Statistics, 12: *Environmental Statistics*. North Holland Elsevier, New York, 167–200.

[14] Patil, G. P. (2002). Ranked set sampling. In Encyclopedia of Environmetrics, El-Shaarawi, A. H. & Piergorsch, W.W. (eds.), **3**, 1684-1690. Wiley: Chichester.

[15] Samawi, H. M., Mohmmad, S. and Abu-Dayyeh, W. (1996). Estimation the population mean using extreme ranked set sampling. *Biometrical Journal*. **38**, 577-586.

[16] Samawi, H. M. and Al-Sagheer, O. A. (2001). On the estimation of the distribution function using extreme and median ranked set sampling. *Biometrical Journal*. **43(3)**, 357-373.

[17] Sinha, B. K., Sinha, B. K. and Purkayastha, S. (1996). On some aspects of ranked set sampling for estimation of normal and exponential parameters. *Statistics and Decisions.* **14**, 223-240.

[18] Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association.* **69**, 730-737.

[19] Stephens, M. A. (1977). Goodness of fit for the extreme value distribution. *Journal of Biometrika.* **64**, 583-588.

[20] Stephens, M. A. (1979). Tests of fit for the logistic distribution based on the empirical distribution function. *Journal of Biometrika.* **66(3)**, 591-595.

[21] Stocks S. L. and Sager T. W. (1988). Characterization of a Ranked -Set Sample with Application to Estimating Distribution Functions. *Journal of the American Statistical Association*. **83(402),** 374-381.

[22] Takahasi, K. and Wakitmoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*. **20**, 1-31.