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Apply Markov Decision Process with Variable Discrete Random Demand Problem

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Abstract

We describe a formulation of the dynamic lot–sizing single product problem when demand is discrete random and time variant. Assuming that the probability of each demand alternative is known for each period. The problem is difficult to solve if there is size increasing because of the large amount of alternative demands. It is required to take longer computational time to obtain optimal solution. The solutions are usually available for problems with a variable discrete random demand in a short period. This study was investigated the solution by applying the Markov decision process (MDP) and develop a linear model for the MDP to solve the problem. A case study is used for uncertain demand to determine fixed price. According to testing, this solution was able to solve the problem consisting short period and used with the problem of a variable discrete random demand over a long period. The MDP is not an integer programming model with short computational time. The results have been the conducted to solve a large problem.

Keywords: dynamic lot sizing system; variable discrete random demand; Markov decision process

INTRODUCTION

The concept of finding the right order quantity starts with finding the most economic order quantity (EOQ) which was presented by Harris (1913). The assumption of the EOQ model consists of infinite periods ($T=+\infty$), continuous and constant demand rate (r), no constraint in capacity, no shortage, and zero lead time. Thus, finding EOQ $= \sqrt{\frac{2c_3 r}{c_1}}$ by c_3 is ordering cost and c_1 is holding cost per period. Next, application concept of the EOQ based finding the order appropriate in other condition was researched. Hax and Candea (1984) collected the models cover various assumptions such as allowing for shortage, opportunity cost, quantity discount etc.

If the demand for a product changes with time but has constant demand rate in a period, Wagner and Whitin (WW) (1958) proposed a dynamic schedule to solve the problem. The model assumption is that demand for goods is changing over time, no capacity constraints, and zero inventory at the end of planning. Veinott (1963) shown that if the demand for goods and the cost on the inventory system is concave function, the problem can be solved $O(T^2)$ by dynamic programming. Zangwill (1966, 1969), Gupta and Brennan (1992) applied WW model under considering shortage constraint. Martel and Gascon (1998) studied WW model in cases where the cost of goods is calculated as a percentage of the cost of goods. Additionally, the WW model had used to other researches such as, Ghare and Schrader (1963), Shah (1977), Tadikamalla (1978), Nahmias and Wang (1979), and Jain and Silver (1994). Because the dynamic programming may be difficult to calculate if period increases, the heuristic approach was developed to solve the problem in short time. The powerful heuristic method such as, Part Period Balancing (De Matteis and Mendoze, 1968), Least Unit Cost (Gorham, 1968), EOQ based period order quantity (Berry, 1972), Silver and Meal (Silver and Meal, 1973) etc.

Under the demand time-varying with uncertain in each period but can know, Hadley and Whitin (1963) proposed dynamic programming to solve the problem which no capacity and no shortage constraints are the assumption of model. Their results shown that the problem solving under the demand time-varying with uncertain in each period case is harder than the problem under the demand time-varying with certain in each period case. In other words, the number of the periods and a lot of choices in decision under uncertain demand in each period make it harder to solve with large problem size.

The remainder of the paper is organized as follows. Section 2 provides a literature review on the relevant subject. Section 3 describes the problem, states the assumptions, and gives the parameters, variables and the model formulation. A numerical example is provided in Section 4. Finally, in Section 5, the conclusion of the paper is presented.

LITERATURE REVIEW

In this section, we review the classical dynamic lot sizing problem with time-varying discrete random demand. A typical dynamic lot-sizing problem is to determine the amount of replenishment of items, with known but time-varying and total cost is minimized. The so-called classical dynamic lot sizing problem which was first analyzed by Wagner and Whitin (1958). The classical problem assumes uncapacitated production and no shortages. They provided a dynamic programming algorithm for this problem. These authors proposed an $O(T^2)$ algorithm based on the zero inventory order (ZIO) policies to reduce the state space. Many authors (Aggarwal and Park, 1993; Federgruenand and Tzur, 1991; Wagelmans et al. 1992) improved the time complexity for obtaining an optimal solution from $O(T^2)$ to $O(T \log T)$, where T represents the length of the planning horizon. Later, (Veinott, 1964) approached the model as a minimum cost flow problem considering convex costs. Zangwill (1966) presented polynomial algorithms for the extension of Wagner and Whitin model in which backlogging is allowed and the cost functions are concave.

Even though the Wagner–Whitin algorithm provides an optimal solution has been considered difficult to understand and requiring high computational resources. However, several authors developed heuristic solution procedures to solve the dynamic lot-sizing problem. Silver and Meal (1973) have contributed faster heuristics to solve the problem. Recent work on dynamic lot sizing have added in the area of stochastic dynamic lot sizing problem consider mainly demand as the uncertain parameter. Bookbinder and Tan (1988) convert the stochastic problem to an equivalent deterministic problem that has the same form as the deterministic dynamic lot sizing problem. Sox (1997) developed an optimal algorithm for the single item dynamic lot sizing problem with random demand and non-stationary. Sox et al. (1999) survey the most current research literature on the stochastic lot scheduling problem, which deals with scheduling production of multiple products with stochastic demand. Guan and Miller (2008) studied the stochastic version of the deterministic lot-size problem and proposed a polynomial time algorithm to obtain the optimal solution. Piperagkas et al. (2012) have investigated the dynamic lot-size problem under stochastic and non-stationary demand over the planning horizon. The problem is solving by three popular meta-heuristic methods from the fields of evolutionary computation and swarm intelligence, namely particle swarm optimization, differential evolution and harmony search.

In addition, Yin et al. (2002) proposed a formulation and solution procedure for inventory planning with the Markov decision process (MDP) models. Ahiska et al. (2013) formulated of problem the MDP in order to determine the optimal policy which is a list of the optimal action to follow the optimal order quantities from both suppliers in each possible state of the system. The literature review confirms that the MDP approach is very efficient in handling stochastic decision problems (Bai et al. 2016; Li and Jiang, 2013; Lin et al. 2013).

In this paper, we formulated a Markov decision process (MDP) model of the dynamic lot-sizing single product problem. The demand is assumed to be discrete random variable. The model considered lead time and shortages are allowed.

MODEL FORMULATION

Model assumptions

- Order can be placed at the beginning of period and product is received in next L, next period.
- The beginning of first period has enough inventory to meet demand lead time. There is inventory at the end of period.
- The demand is at the beginning of period. It is divided into several alternatives and varied in each period as found its demand characteristics in Table 1.
- There are some cases of shortage when demand exceeds available quantity.
- Lead time is allowed.

Table 1. Demand characteristics

Period (t)	Demand (v)			
	1 Prob. = P_1	2 Prob. = P_2	…….	S Prob.= P_S
1	$d_{1,1}$	$d_{1,2}$	…….	$d_{1,K}$
2	$d_{2,1}$	$d_{2,2}$	…….	$d_{2,K}$
…….	…….	…….	…….	…….
…….	…….	…….	…….	…….
N-1	$d_{N-1,1}$	$d_{N-1,2}$	…….	$d_{N-1,S}$
N	$d_{N,1}$	$d_{N,2}$	…….	$d_{N,K}$

Notation

The mathematical notation is given as follows.

Decision Variables

$x(i,s,n)$ Steady state probability of beginning available inventory (i) in period n and decision under policy for inventory level (s)

Parameter

$p(i,j,s,n)$ Transition probability of the beginning inventory quantity (i) in period n to be the beginning inventory quantity (j) in period n+1 under policy for inventory level (s)

$p(i,j,s,n) = p_{n,k}$ when $i \leq s$ by $s \in S_{k(n)}$

and $j = (s - D_{n,k})$ and $p_{n,k}$ the probability of occurred

$D_{n,k} = p_{n,k}$ when $i > s$ by $s \in S_{k(n)}$

and $j = (i - D_{n,k})$

and $p_{n,k}$ the probability of occurred $D_{n,k}$

$$S_{k(n)} = D_{n,k}, \sum_{t=n}^{n+1} D_{t,k_t}, \sum_{t=n}^{n+2} D_{t,k_t}, .., \sum_{t=n}^{N} D_{t,k_t}$$

k_t, k=1,2,…, K_t, = L+1,L+2,…,N

by K_t Number alternative of order level total in period t

$$= k\sum_{n=t}^{N} k^{N-t}$$

I_n State of inventory quantity at possible in the beginning in period n

$$I_0 = 0 \qquad \text{When L=0}$$

$$= \sum_{n=1}^{L} D_{n,1} \qquad \text{When L>0}$$

J_{n+1} State of available quantity of inventory in beginning in period n+1 or state of available quantity of inventory at the end of period n

$Q_{n,k}$ Quantity is purchased under extra unit discount at beginning of period n alternative k member of policy s_n when $s_n \in S_n$

$d(i,s,n)$ Expected cost for decision under inventory level s policy at the beginning of inventory i of period n

$$d(i,s,n) \begin{cases} p_{n,k}(O_n + c_n(s-i) + c_{1n}(s-D_{n,k}))...\text{if } s >= D_{n,k} \\ p_{n,k}(O_n + c_n(s-i) + c_{2n}(D_{n,k}-s))......\text{if } s < D_{n,k} \end{cases}$$

When $i \leq s$ by $s \in S_{k(n)}$ and $p_{n,k}$ the probability of occurred $D_{n,k}$ $= \begin{cases} p_{n,k}(c_{1n}(i-D_{n,k}))...\text{if } i >= D_{n,k} \\ p_{n,k}(c_{2n}(D_{n,k}-i))......\text{if } i < D_{n,k} \end{cases}$

When $i > s$ by $s \in S_{k(n)}$ and $p_{n,k}$ the probability of occurred $D_{n,k}$

Linear model for Markov decision process

Objective function

$$\text{Min} = \sum_{n=1}^{N} \sum_{s \in S_{k(n)}} \sum_{i \in I_n} x(i,s,n)d(i,s,n) \qquad (1)$$

Subject to

$$\sum_{j \in J_{n+1}, s \geq j} x(j,s,n+1) = \sum_{i \in I_n} \sum_{s \in S_{k(n)}} x(i,s,n)p(i,j,s,n)$$

$$n = 1,2,3,..N-1,\ j \in J_{n+1} \qquad (2)$$

$$\sum_{i \in I_n} \sum_{s \in S_{k(n)}} x(i,s,n) = 1 \quad n = 1,2,3,....N \qquad (3)$$

$$x(j,s,n+1)=0 \qquad n=L+1,L+2,...,N-1,$$
$$j \in J_{n+1} < D_{n,1} \qquad (4)$$

$$x(i,s,n) \geq 0\ n = 1,2,3,...,N,\ i \in I_n\ s \in S_{k(n)},\ s \geq i \quad (5)$$

The objective function as shown in equation (1) is to minimize the total expected cost at steady state for the beginning inventory quantity i in period n under policy of inventory order up to level s. Equation (2) determine the total probability at steady state of the beginning inventory level j under in inventory level s in next period (n+1) equal to total of multiply of probability at steady state to the beginning inventory quantity i at other period n with probability in transition probability with the beginning inventory i there will be inventory end of period j under the policy to have inventory levels s in period n. Equation (3) the sum of probabilities at steady state of the beginning inventory quantity i in period n under policy for to inventory level s equal 1. Equation (4) determine the total probability at steady state of the beginning inventory volume j of next period (n+1) under policy of inventory level s equal zero. Equation (5) is used to determine the total probability at steady state of the beginning inventory quantity i period n and have decision under policy to inventory level s is non-negativity.

NUMERICAL EXAMPLE

Example data

The data used in these examples consists of the demand in each period variable discrete random demand as shown in Table 2. In case and problem size to test is shown in table 3.

Table 2. Data of demand for problem

Period (week)	Demand					
	Optimistic		Most likely		Pessimistic	
	Prob	Q'ty	Prob	Q'ty	Prob	Q'ty
1	0.2	25	0.6	40	0.2	50
2	0.2	10	0.6	15	0.2	30
3	0.2	33	0.6	67	0.2	100
4	0.2	50	0.6	55	0.2	60
5	0.2	30	0.6	35	0.2	50
6	0.2	18	0.6	25	0.2	32
7	0.2	25	0.6	40	0.2	50

Table 3 Case and size problem in tests

Problems	Period (week)	Lead time (week)
1	4	0
2	4	1
3	4	2
4	5	1
5	5	2
6	6	2
7	6	3
8	7	3
9	8	3

Holding cost per unit = $0.4 / PCs /week

Back order or lost sales cost per unit = $0.6 / PCs

Order cost = $100 / Order

Numerical results

In this section, we solved a numerical example of the proposed model. The model is coded in optimization package like LINGO 12 and was run on computer with 1.80 GHz Intel processor Core i3, and 4 GB RAM. The results of the numerical example by the Markov decision process is shown in Table 4.

Table 4. Results from the Markov decision process

Problems	Order	Total cost
1	Period 1 : order placed 60 units Period 3 : order placed 117 units Period 3 : order placed 38 for case inventory from period 3 on hand 17	$257
2	Period 1 : order placed Period 2 : order receipt 180	$228.7
3	Period 1 : order placed Period 2 : order receipt 150	$162.56
4	Period 1, 3 : order placed Period 2 : order placed 187 units Period 4 : order receipt 90 units	$265.86
5	Period 1 : order placed Period 4 : order receipt 90 units	$206.76
6	Period 1, 3 : order placed Period 3 : order receipt 195 units Period 5 : order receipt 68 units	$241.68
7	Period 1 : order placed Period 4 : order receipt 128 units	$198.67
8	Period 1, 3 : order placed Period 4 : order receipt 162 units Period 6 : order receipt 57 units	$247.52
9	Period 1, 4 : order placed Period 4 : order receipt 190 units Period 7 : order receipt 60 units	$301.62

In period 1, there is no initial inventory will be replenished to purchase 60 units, and in period 1, demand 25 units will need 35 units inventory, and if demand 40 will need 20 units inventory, if demand 50 units will need 10 inventory units. In period 2, if the beginning inventory is 35 units, it will use no order inventory policy. If the beginning inventory is 20 units, it will follow ordering policy. If there is 10 units inventory, it will use no ordering policy, and demand in period 2 are 10, 15, 30 units with the ending inventory -20, -10, -5, 0, 5, 10, 20, 25 respectively. Period 3 will use ordering policy with 117 unit inventory level. The same policy does not matter with the quantity of beginning inventory and demand of period 3 is 33, 67, 100 units will required the ending inventory of 17,50,84, while period 4 will follow ordering policy as if the beginning inventory is 17, it will use 55 units inventory level. If the beginning inventory is 50 units, it will require 50 units policy. If the beginning inventory is 84 units, it will need no order policy while other policy will lead to $257 expected cost.

CONCLUSIONS

We describe a formulation of the dynamic lot–sizing single product problem when demand is discrete random and time variant. Assuming that the probability of each demand alternative is known for each period. The problem is difficult to solve if there is size increasing because of the large amount of alternative demands. This makes the computation time to find the right choice is a long period and solutions are usually available for problems with a variable discrete random demand in a short period which of the applications. Markov decision process (MDP) and develop a linear model for the MDP to help solve the problem. The results demonstrate that solution was able to solve the problem in a short computational time and used the problem of variable discrete random demand for a long period.

In addition, the linear model for the MDP, it is not an integer programming model and short computational time. The results have been the conducted to solve a large problem. However, if problem is demand variation for multiple products, it will result in a very high inventory level. This will also affect computational time and the model used to solve problem which is a typical linear model. It is not an integer programming model, which there is one disadvantage. The answer is no upper and lower bounds so it is impossible to find the basic answer to be used in urgent cases.

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