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A New 3-D Rational Fraction Chaotic Map with Symmetry

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Abstract

In this letter we have proposed a 3-D rational fraction chaotic map with six terms. Our proposed map can display new types of chaotic attractors. The chaotic attractors results from a reverse border-collision period- doubling bifurcation scenario route to chaos.

Keywords: 3-D rational fraction map, symmetric chaotic attractor, border-collision period-doubling bifurcation route to chaos.

INTRODUCTION

Chaotic behavior or sensitivity to initial conditions, as a most fascinating phenomenon in nonlinear polynomial system, has been intensively studied. While, the chaotic behavior in rational fraction discrete maps have not been as widely studied as 2-D and 3-D polynomial mappings [1,2,3,4,5,6,7,8,9,10], though in recent years has been a considerable increase in their study [11,12,13].

In this paper, we extend the 2-D rational fraction discrete chaotic map [11] to 3-D rational fraction discrete chaotic map (2). Our proposed map can display several symmetric chaotic attractors obtained by a reverse border-collision period-doubling bifurcation scenario route to chaos.

The new 3-D rational fraction map and its dynamical analysis

Let us consider the following new 3-D rational map defined by:

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{ax_n}{1+y_n^2} \\ x_n + by_n \\ y_n + cz_n \end{pmatrix} \quad (1)$$

Where a, b and c are bifurcation parameters, and $(x_n, y_n, z_n) \in \mathbb{R}^3$ is the state variable, $(x_0, y_0, z_0) \in \mathbb{R}^3$ is the initial state, and $n = 0,1,2, \ldots$ is the discrete time. From (1), also we know that the system has the property of symmetry $(x, y, z) \to (-x, -y, -z)$ consequently one needs to study its dynamics only in a half-space for all values parameters of the system.

The fixed point of the map (1) which satisfies the following conditions:

$$\frac{ax}{1+y^2} = x,\ x + by = y,\ y + cz = z \quad (2)$$

From (2), we can get that the map (1) has only one fixed point $S = (0,0,0)$ when $a \geq -1, b \neq 1$ and $c \neq 1.$ at the fixed point, the Jacobi matrix of the system (1) is given by:

$$J_S = \begin{pmatrix} a & 0 & 0 \\ 1 & b & 0 \\ 0 & 1 & c \end{pmatrix} \quad (3)$$

and its characteristic polynomial is $(a - \lambda)(b - \lambda)(c - \lambda) = 0$. Hence, from (3), we can study the local stability of the system when $a \geq -1, b \neq 1$ and $c \neq 1$. The eigenvalues of the Jacobi matrix corresponding to S are $\lambda_1 = a, \lambda_2 = b$ and $\lambda_3 = c$.

Than one have the following results:

(a) if $|a| < 1, |b| < 1$ and $|c| < 1$, thus, S is stable.

(b) if $|a| > 1$ or $|b| > 1$ or $|c| > 1$, thus, S is an unstable fixed point.

(c) if ($|a| > 1, |b| < 1$ and $|c| < 1$) or ($|a| < 1, |b| > 1$ and $|c| < 1$) or ($|a| < 1, |b| < 1$ and $|c| > 1$), thus S is a saddle.

(d) if $|a|$ =1, or $|b| = 1$, or $|c| = 1$, thus S is a non-hyperbolic fixed point.

Bifurcation analysis of the map

In this section, we will illustrate some observed of chaotic attractors, the dynamical behaviors of the map (1) are investigated numerically. Figures 1 and 2 shows respectively the bifurcation diagrams and the diagrams of the variation of the largest Lyapunov exponent of the map (1) from the initial point $x_0 = y_0 = z_0 = 0.01$ and from the initial point $x_0 = y_0 = z_0 = -0.01$ that are obtained at different values of parameter, $b \in [-1,1]$. However, we deduce from the bifurcation diagrams in Fig.1(a), Fig.1(b), Fig.1(c) and Fig.1(d) that the proposed map (1) exhibits a reverse border-collision period-doubling scenario bifurcation route to chaos for the selected values of the bifurcation parameter b. The attractors evolution of the system is shown in figures 5 and 6. Figures.5 represents the attractors are formed from the initial point $x_0 = y_0 = z_0 = 0.01$, and Figures.6 represents the attractors are formed from the initial point $x_0 = y_0 = z_0 = -0.01$. We can observe the attractors generated from the two symmetrical initial conditions $x_0 = y_0 = z_0 = 0.01$ and $x_0 = y_0 = z_0 = -0.01$ in the $x - y$ plane are both symmetric.

We fix the two symmetrical initial conditions $x_0 = y_0 = z_0 = 0.01$ and $x_0 = y_0 = z_0 = -0.01$ and we chose $a = 4, c = -0.1$ and let the parameter b vary in the interval $b \in [-1,1]$, the map (1) exhibits the following dynamical behaviors as shown in Fig.1 (a) and Fig.2 (a): For the range $-1 < b \leq -0.30$ the map (1) is generally chaotic which is verified by the corresponding largest Lyapunov exponent is positive as shown in Fig.2 (a). Fig.5 (a), Fig.5 (b), Fig.5 (c), Fig.5 (d), Fig.5 (e) and Fig.5 (f), shows the chaotic attractors of the map (1). For the range $-0.30 < b \leq -0.1$ the map (1) is in a reverse series of period-doubling bifurcation and periodic orbits-6 as shown in Fig.1 (a), Fig.1 (b), Fig.1(c) and Fig.1 (d). Finally Fig.5 (a), Fig.5 (b), Fig.5 (c), Fig.5 (d), Fig.5 (e) and Fig.5 (f) represents the attractors are obtained from the initial point $x_0 = y_0 = z_0 = 0.01$, and Fig.56 (a), Fig.6 (b), Fig.6 (c), Fig.6 (d), Fig.6 (e) and Fig.6 (f) represents the attractors are obtained from the initial point $x_0 = y_0 = z_0 = -0.01$.

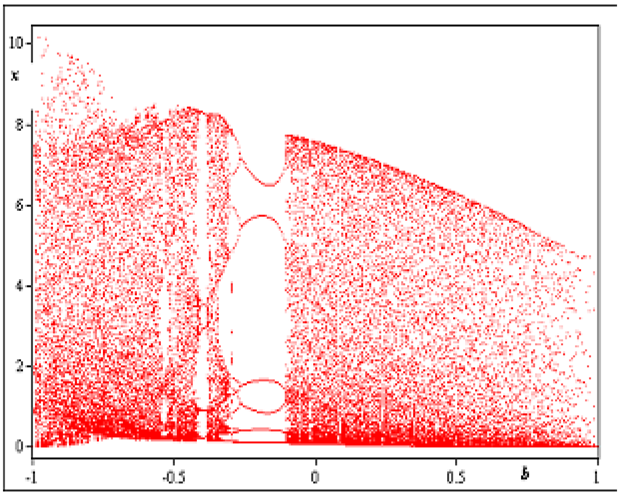

Fig.1(a): Bifurcation diagram, for the map (1) obtained for $a = 4, c = -0.1$ and $-1 \leq b \leq 1$ with $(x_0, y_0, z_0) = (0.01,0.01,0.01)$.

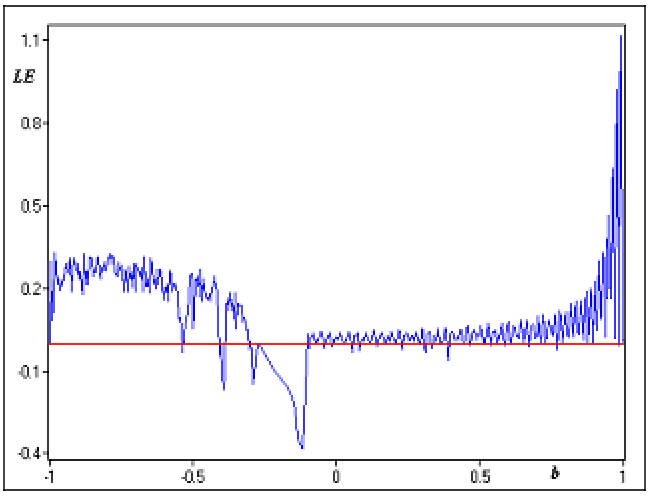

Fig. 2(a): Variation of Lyapunov exponent obtained for $a = 4, c = -0.1$ and $-1 \leq b \leq 1$ with $(x_0, y_0, z_0) = (0.01,0.01,0.01)$.

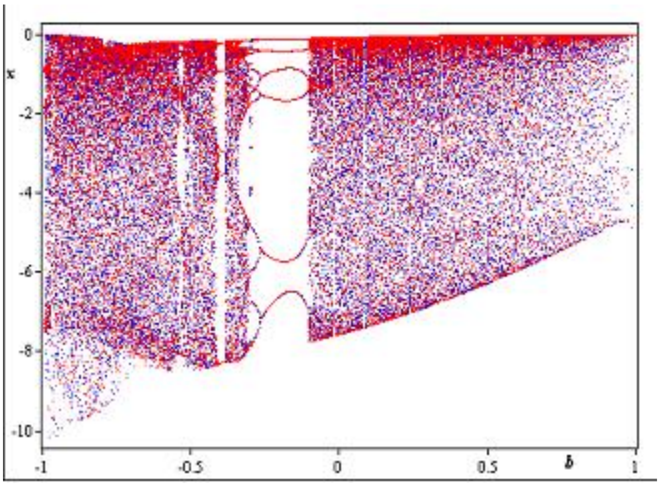

Fig.1(b): Bifurcation diagram, for the map (1) obtained for $a = 4, c = -0.1$ and $-1 \leq b \leq 1$ with $(x_0, y_0, z_0) = -(0.01,0.01,0.01)$.

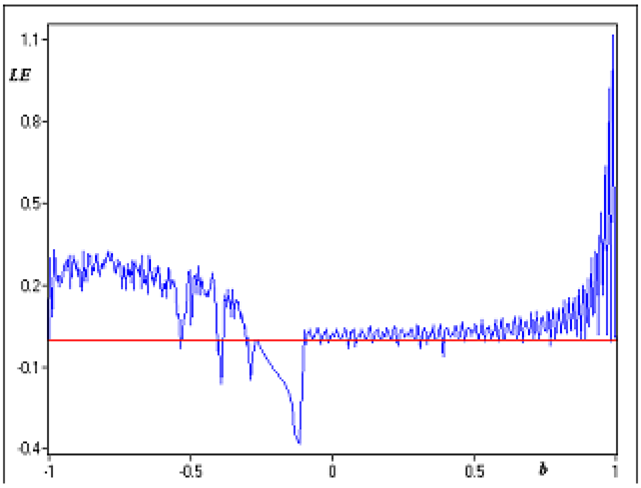

Fig.2(b): Variation of Lyapunov exponent obtained for $a = 4, c = -0.1$ and $-1 \leq b \leq 1$ with $(x_0, y_0, z_0) = -(0.01,0.01,0.01)$.

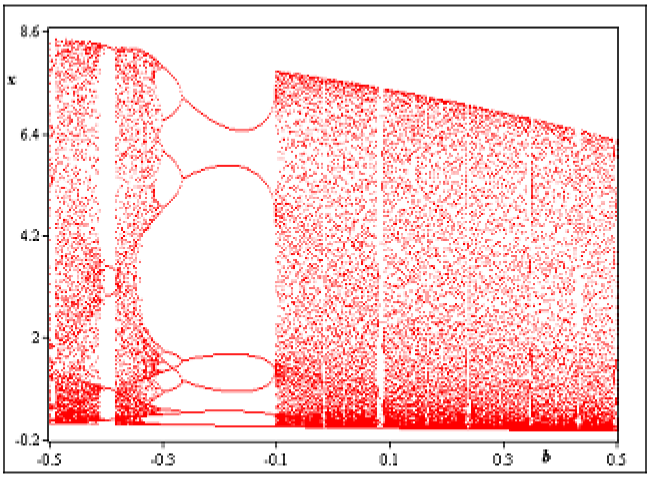

Fig.1(c): Bifurcation diagram, for the map (1) obtained for $a = 4, c = -0.1$ and $-0.5 \leq b \leq 0.5$ with $(x_0, y_0, z_0) = (0.01,0.01,0.01)$.

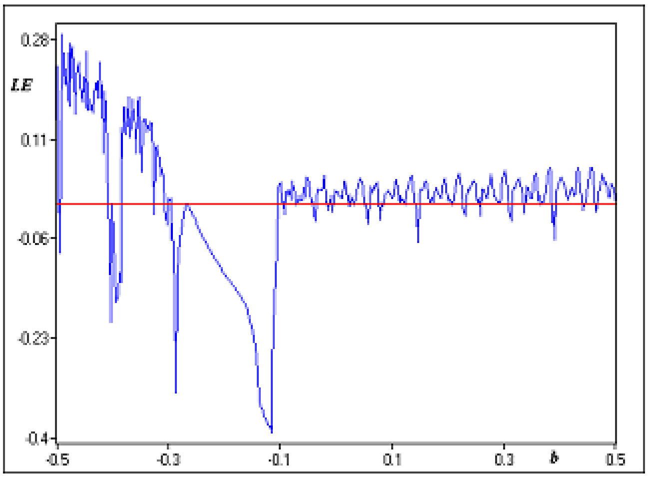

Fig. 2(c): Variation of Lyapunov exponent obtained for $a = 4, c = -0.1$ and $-0.5 \leq b \leq 0.5$ with $(x_0, y_0, z_0) = (0.01,0.01,0.01)$.

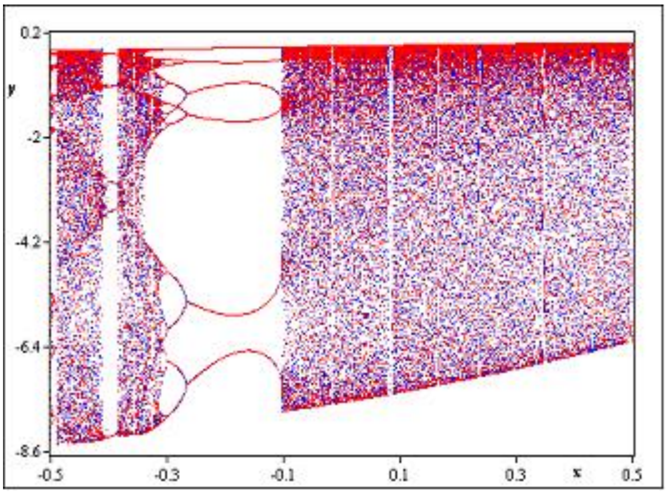

Fig.1(d): Bifurcation diagram, for the map (1) obtained for $a = 4, c = -0.1$ and $-0.5 \leq b \leq 0.5$ with $(x_0, y_0, z_0) = -(0.01,0.01,0.01)$.

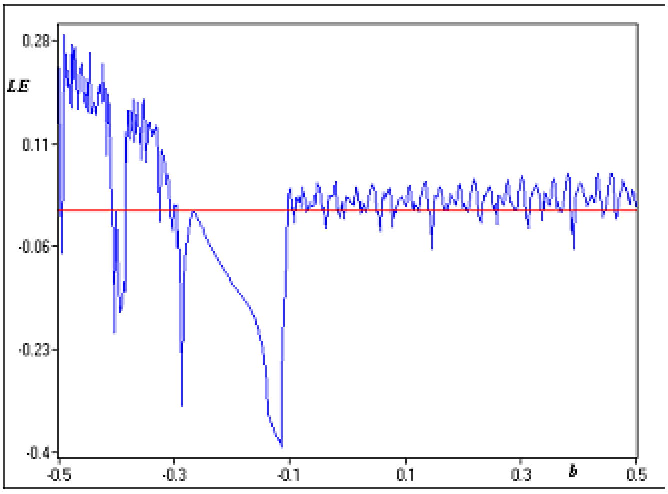

Fig. 2(d): Variation of Lyapunov exponent obtained for $a = 4, c = -0.1$ and $-0.5 \leq b \leq 0.5$ with $(x_0, y_0, z_0) = -(0.01,0.01,0.01)$.

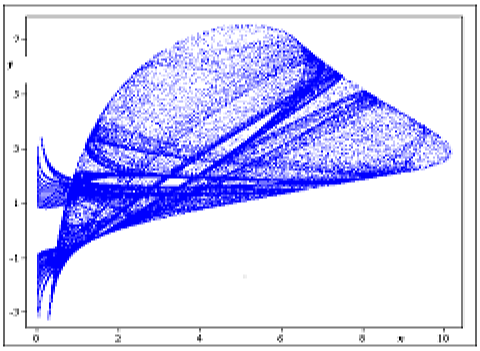

Fig 5(a): $b = -0.96$.

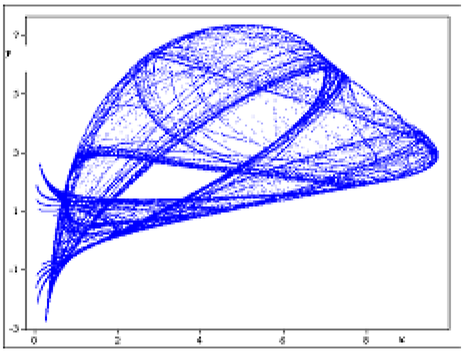

Fig 5(b): $b = -0.86$.

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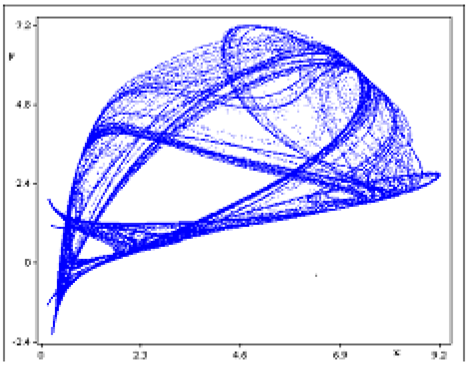
Fig 5(c): $b = -0.76$.

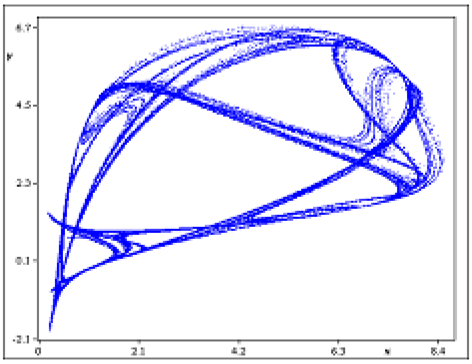
Fig 5(d): $b = -0.66$.

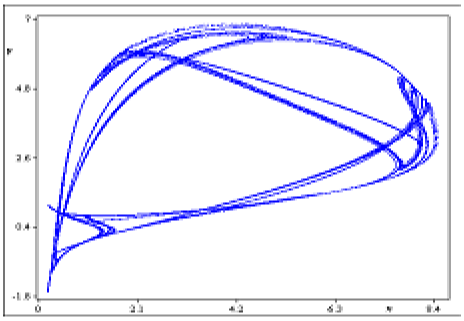
Fig 5(e): $b = -0.56$.

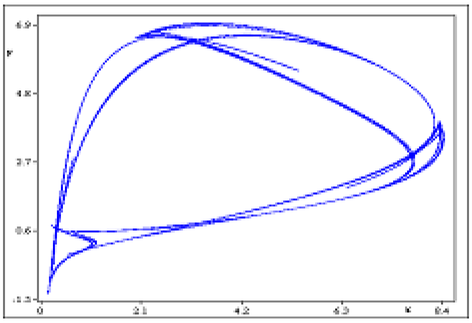
Fig 5(f): $b = -0.46$.

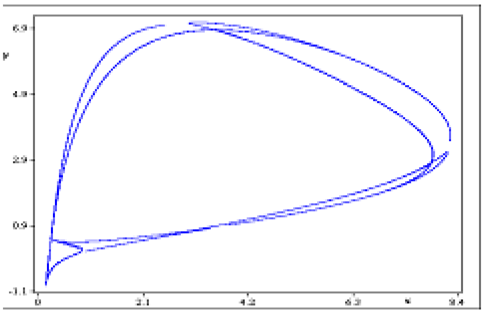
Fig 5(g): $b = -0.36$.

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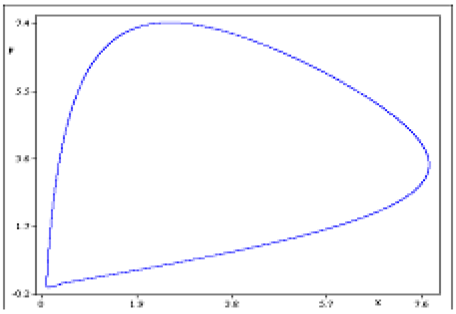
Fig 5(h): $b = -0.1$.

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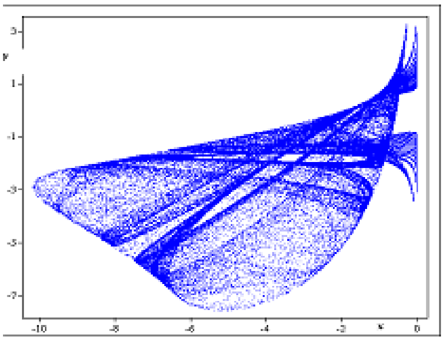

Fig 6(a): $b = -0.96$.

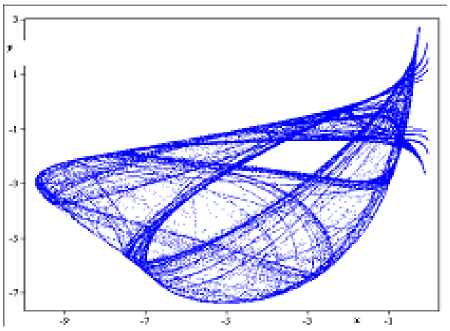

Fig 6(b): $b = -0.86$.

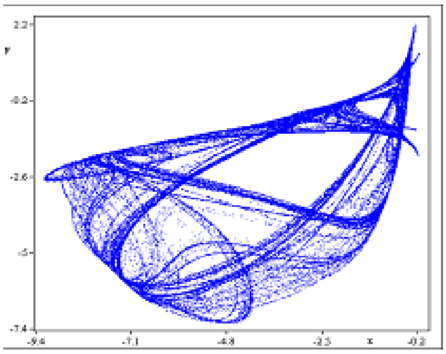

Fig 6(c): $b = -0.76$.

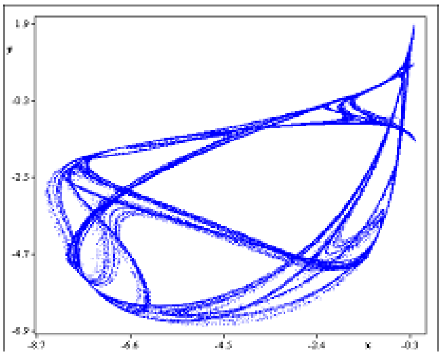

Fig 6(d): $b = -0.66$.

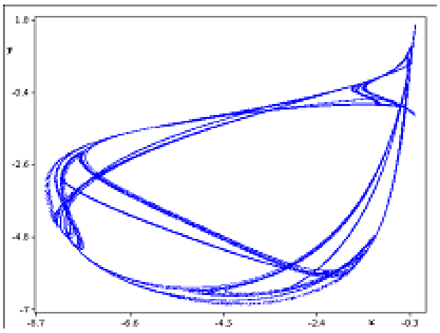

Fig 6(e): $b = -0.56$.

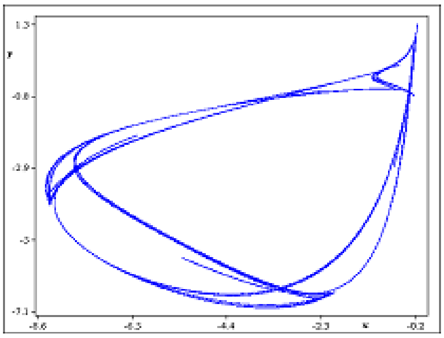

Fig 6(f): $b = -0.46$.

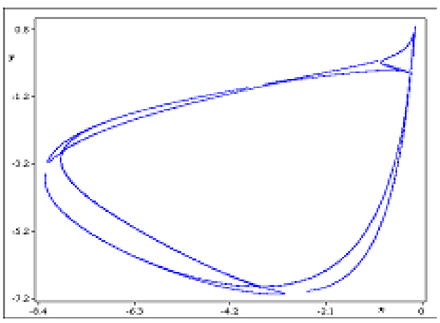

Fig 6(g): $b = -0.36$.

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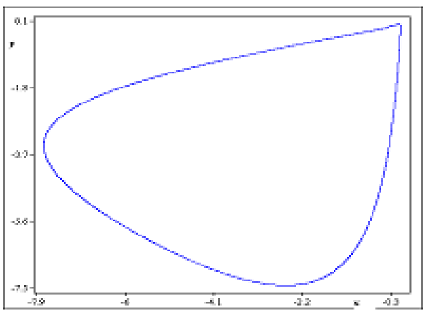

Fig 6(h): $b = -0.1$.

CONCLUSION

In this paper we investigate the results of an analytical and numerical study of the 3-D rational fraction discrete chaotic map capable of generating symmetric chaotic attractors via a reverse border-collision period- doubling bifurcation scenario route to chaos.

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