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Comparative Analysis of Two Stochastic Models on Hardware/Software System Considering Various Kinds of Failures, Errors and Recovery Coverages

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Abstract

The present paper deals with two stochastic Models developed for a single-unit hardware-software system considering occurrence of various kinds of hardware/software failures, hardware/software interactions failures, human errors and common cause failures. The aspects of fault location and fault detection coverages in the system have also been incorporated along with other recovery mechanisms. The fault detection coverage has been considered in the first Model whereas fault location and fault detection coverages have been considered in the second Model. Various measures of system performance are computed and the Models are compared with respect to their reliability and cost considerations. Various conclusions are drawn on the basis of graphical analyses.

Keywords: Hardware-software system, Hardware/Software interaction failures, human errors, common cause failures, fault location/detection coverage, Markov Process, regenerative point technique.

INTRODUCTION

The advancement of technology in modern society is not overstated as the technology has profoundly influenced the living style of an individual. To achieve the maximum benefit of advancement in technology, improved, trustworthy and reliable hardware-software systems are presently in great demand. In the past researchers investigated reliability of the hardware-software system dealing with hardware reliability and software reliability. The combined reliability Models, i.e. including both hardware and software subsystems were also discussed by a few researchers including Friedman and Tran [3], Hecht and Hecht [4], Welke et al.[18] etc. assuming in general that these components are independent of each other and the aspects of interactions between the hardware and software components were not taken up. However, some researchers including Iyer and Velardi [6], Martin and Mathur [11], Kanoun and Ortalo-Borrel [9], Haung et al.[5] have justified that there exist remarkable interactions between hardware and software components and have significant effect on reliability of the system. Taking this into consideration, Teng et al. [16] discussed the reliability of the combined system by considering different Models for hardware, software and hardware based software failure, however not in integrated way. Recently, Kumar and Kumar [10] extended the above work by considering combined reliability Model for the systems having hardware, software and hardware based software failures with different types of recovery methods.

Environmental factors such as human error and common cause failures may affect the system performance. Human errors are important while predicting the reliability and safety measures of any computer system. In a real life situation, many faults are caused directly or indirectly due to human errors such as wrong action, poor communication, wrong interpretation, poor handling, poor maintenance and operation procedure, etc. According to the work by Meister [15] about 30% of failures are directly or indirectly due to human errors. A few researchers anlaysed some systems considering failures in the systems due to human errors. Kumar and Kumar [12] analysed two unit cold standby system considering hardware, human error failures and preventative maintenance. Mahmoud and Moshref [14] investigated a two-unit cold standby system considering hardware, human error failures and preventative maintenance. Further, common cause failure in the system is another key factor that should be incorporated to predict the system performance. The common cause failure may generally occur due to equipment design deficiency, power supply, humidity, temperature, etc. The availability analysis of embedded computer system with two types of failure and common cause failure was presented by Jain et al.[8]. Chae et al. [7] discussed system reliability in the presence of common-cause failures.

The present paper deals with two stochastic Models developed for a single-unit hardware-software system considering occurrence of various kinds of hardware/software failures, hardware/software interactions failures, human errors and common cause failures. The aspects of fault location and fault detection coverages in the system have also been incorporated along with other recovery mechanisms. The fault detection coverage has been considered in the first Model whereas fault location and fault detection coverages have been considered in the second Model. Various measures of system performance are computed and the Models are compared with respect to their reliability and cost considerations. Various conclusions are drawn on the basis of graphical analyses.

STATES OF THE SYSTEM

HS	Normal operative mode
$\hat{\mathrm{H}}\mathrm{S}$	Partial (degraded) hardware mode
$\mathrm{H}\hat{\mathrm{S}}$	Partial (degraded) software mode
$\mathrm{H}\tilde{\mathrm{S}}$	Undetected software mode
$\tilde{\mathrm{H}}\mathrm{S}$	Undetected hardware mode
$\bar{\mathrm{H}}\mathrm{S}$	Complete hardware failure mode
$\mathrm{H}\bar{\mathrm{S}}$	Complete software failure mode
$\bar{\mathrm{H}}\bar{\mathrm{S}}$	Complete Hardware-software failure mode

NOTATIONS

λ_{h1} : Hardware failure rate from normal mode to hardware degraded mode.

λ_{h2} : Hardware failure rate from hardware degraded mode to complete hardware failed mode.

λ_{h3} : Hardware failure rate from normal mode to complete hardware failed mode.

λ_{s1} : Software failure rate from normal mode to software degraded mode.

λ_{s2} : Software failure rate from hardware degraded mode to complete hardware failed mode.

λ_{s3} : Software failure rate from normal mode to complete software failed mode.

λ_{s4} : Hardware based software interaction failure rate.

λ_{s5} : Hardware based software interaction failure rate when hardware fault is undetected.

λ_{he} : Hardware failure rate due to human error.

λ_{se} : Software failure rate due to human error.

λ_{hse} : Hardware-Software failure rate due to human error.

λ_{cc} : Common cause failure rate.

C_{dh} : Hardware fault detection coverage.

C_{ls} : Software fault location coverage.

C_{lh} : Hardware fault location coverage.

C_{ds} : Software fault detection coverage.

$g_{h1}(t)$: Automatic hardware repair rate by software methods.

$g_{h2}(t)$: Hardware repair rate by external engineer.

$g_{s1}(t)$: Automatic software repair by software methods.

$g_{s2}(t)$: Software repair rate from complete hardware failed mode to normal mode.

$g_{cc}(t)$: Complete system repair by external engineer.

MODEL -1

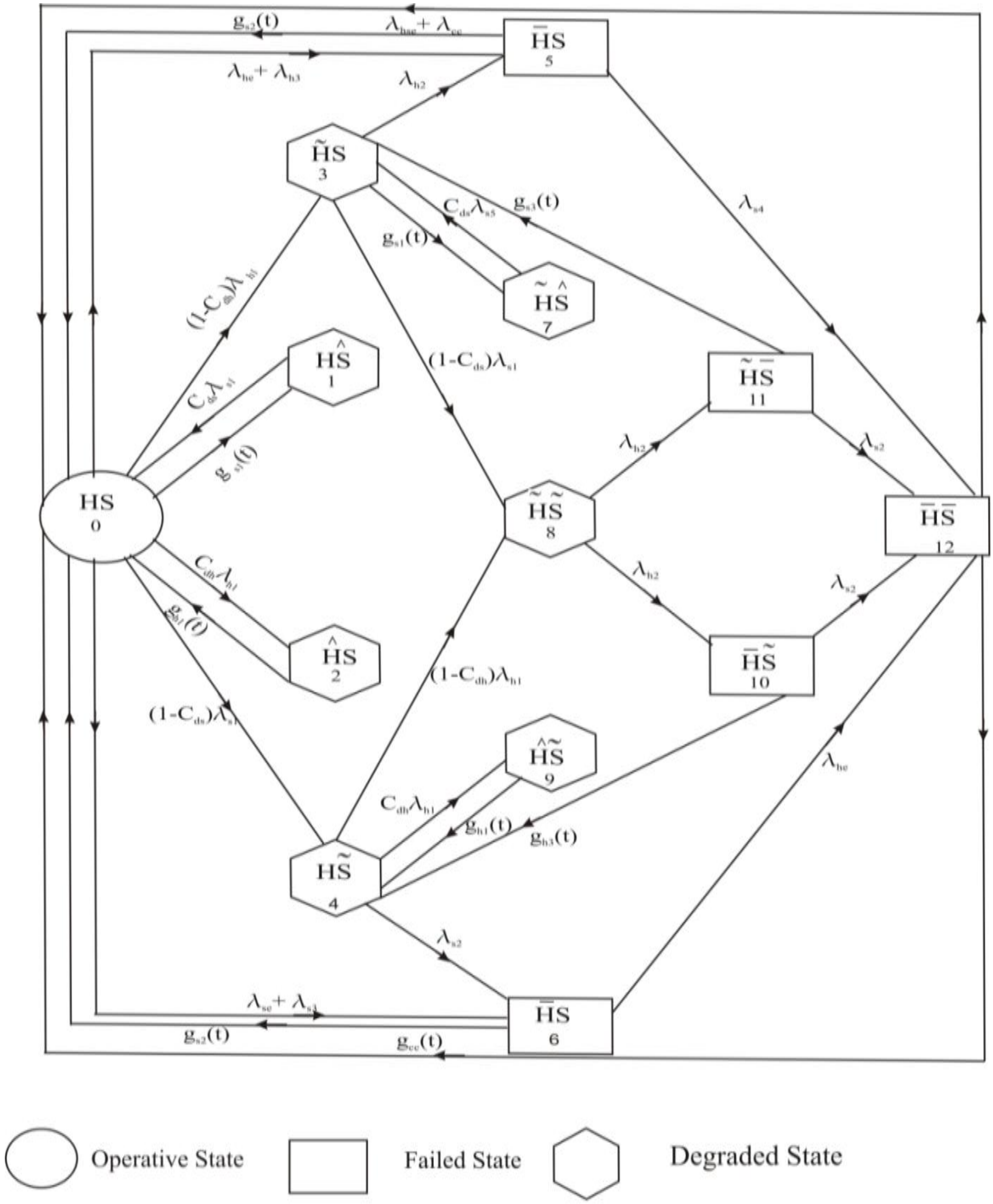

Figure 1.

MODEL - 2

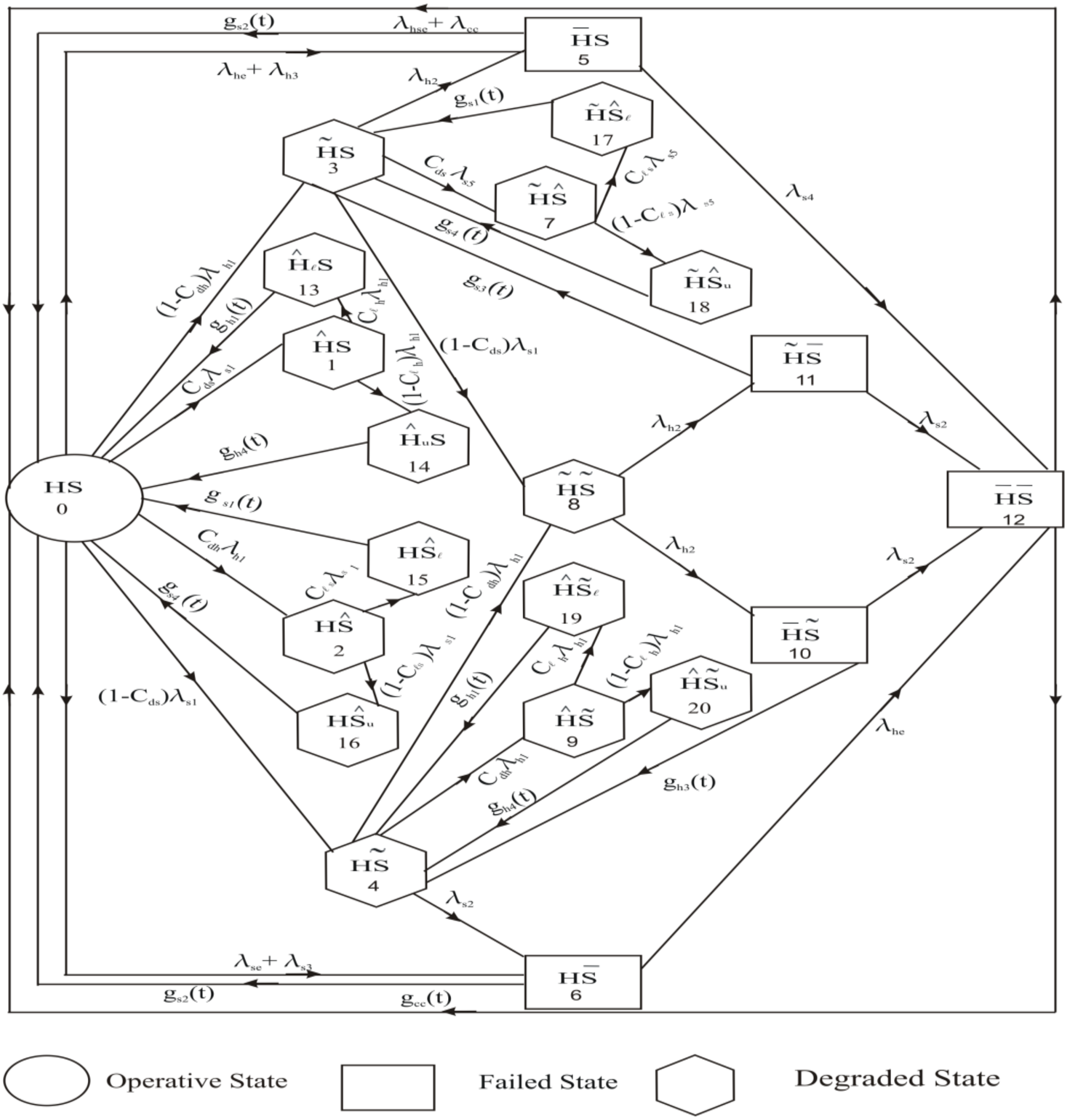

Figure 2.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIME

For Model-1:

The transition probabilities p_{ij} are given by

$$p_{01} = \frac{C_{dh}\lambda_{h1}}{A},\ p_{02} = \frac{C_{ds}\lambda_{s1}}{A},\ p_{03} = \frac{(1-C_{dh})\lambda_{h1}}{A},\ p_{04} = \frac{(1-C_{ds})\lambda_{s1}}{A},\ p_{05} = \frac{(\lambda_{he} + \lambda_{h3})}{A},$$

$$p_{06}=\frac{(\lambda_{se}+\lambda_{s3})}{A},\ p_{0,12}=\frac{(\lambda_{hse}+\lambda_{cc})}{A},\ p_{10}=p_{20}=1,\ p_{35}=\frac{\lambda_{h2}}{\lambda_{h2}+\lambda_{s5}},\ p_{37}=\frac{C_{ds}\lambda_{s5}}{\lambda_{h2}+\lambda_{s5}},$$

$$p_{38}=\frac{(1-C_{ds})\lambda_{s5}}{\lambda_{h2}+\lambda_{s5}},\ p_{46}=\frac{\lambda_{s2}}{\lambda_{h1}+\lambda_{s2}},\ p_{48}=\frac{(1-C_{dh})\lambda_{h1}}{\lambda_{h1}+\lambda_{s2}},\ p_{48}=\frac{C_{dh}\lambda_{h1}}{\lambda_{h1}+\lambda_{s2}},\ p_{50}=g^{*}_{h2}(\lambda_{s4}),$$

$$p_{5,12}=1-g^{*}_{h2}(\lambda_{s4}),\ p_{60}=g^{*}_{s2}(\lambda_{he}),\ p_{6,12}=1-g^{*}_{s2}(\lambda_{he}),\ p_{8,10}=\frac{\lambda_{h2}}{\lambda_{h2}+\lambda_{s2}}\ p_{8,11}=\frac{\lambda_{s2}}{\lambda_{h2}+\lambda_{s2}}$$

$$p_{10,4}=g^{*}_{h3}(\lambda_{s2}),\ p_{10,12}=1-g^{*}_{h3}(\lambda_{s2}),\ p_{11,3}=g^{*}_{s3}(\lambda_{h2}),\ p_{11,12}=1-g^{*}_{s3}(\lambda_{h2}),$$

$$p_{73}=p_{94}=p_{12,0}=1,\ A=\lambda_{s1}+\lambda_{h1}+\lambda_{s3}+\lambda_{h3}+\lambda_{he}+\lambda_{se}+\lambda_{hse}+\lambda_{cc}$$

The mean sojourn times μ_{ij} are obtained as under

$$\mu_0=\frac{1}{A},\ \mu_1=-g_{h1}^{*'}(0),\ \mu_2=-g_{s1}^{*'}(0),\ \mu_3=\frac{1}{\lambda_{s5}+\lambda_{h2}},\ \mu_4=\frac{1}{\lambda_{s2}+\lambda_{h1}},\ \mu_5=\frac{1-g^{*}_{h2}(\lambda_{s4})}{\lambda_{s4}},\ \mu_6=\frac{1-g^{*}_{s2}(\lambda_{he})}{\lambda_{he}},$$

$$\mu_7=-g_{s1}^{*'}(0),\ \mu_8=\frac{1}{\lambda_{s2}+\lambda_{h2}},\ \mu_9=-g_{h1}^{*'}(0),\ \mu_{10}=\frac{1-g^{*}_{h3}(\lambda_{s2})}{\lambda_{s2}},\ \mu_{11}=\frac{1-g^{*}_{s3}(\lambda_{h2})}{\lambda_{h2}},\ \mu_{12}=-g_{cc}^{*'}(0)$$

For Model-2:

The non-zero element p_{ij} are given by

$$p_{01}=\frac{C_{dh}\lambda_{h1}}{A},\ p_{02}=\frac{C_{ds}\lambda_{s1}}{A},\ p_{03}=\frac{(1-C_{dh})\lambda_{h1}}{A},\ p_{04}=\frac{(1-C_{ds})\lambda_{s1}}{A},\ p_{05}=\frac{(\lambda_{he}+\lambda_{h3})}{A},$$

$$p_{06}=\frac{(\lambda_{se}+\lambda_{s3})}{A},\ p_{0,12}=\frac{(\lambda_{hse}+\lambda_{cc})}{A},\ p_{1,13}=C_{lh},\ p_{1,14}=(1-C_{lh}),\ p_{2,15}=C_{ls},\ p_{2,16}=(1-C_{ls}),$$

$$p_{35}=\frac{\lambda_{h2}}{\lambda_{h2}+\lambda_{s5}},\ p_{37}=\frac{C_{ds}\lambda_{s5}}{\lambda_{h2}+\lambda_{s5}},\ p_{38}=\frac{(1-C_{ds})\lambda_{s5}}{\lambda_{h2}+\lambda_{s5}},\ p_{46}=\frac{\lambda_{s2}}{\lambda_{h1}+\lambda_{s2}},\ p_{48}=\frac{(1-C_{dh})\lambda_{h1}}{\lambda_{h1}+\lambda_{s2}},$$

$$p_{49}=\frac{C_{dh}\lambda_{h1}}{\lambda_{h1}+\lambda_{s2}},\ p_{50}=g^{*}_{h2}(\lambda_{s4})\ p_{5,12}=1-g^{*}_{h2}(\lambda_{s4}),\ p_{60}=g^{*}_{s2}(\lambda_{he})$$

$$p_{6,12}=1-g^{*}_{s2}(\lambda_{he}),\ p_{7,17}=C_{ls},\ p_{7,18}=(1-C_{ls}),\ p_{8,10}=\frac{\lambda_{h2}}{\lambda_{h2}+\lambda_{s2}},\ p_{8,11}=\frac{\lambda_{s2}}{\lambda_{h2}+\lambda_{s2}},\ p_{9,19}=C_{lh},$$

$$p_{9,20}=(1-C_{lh}),\ p_{10,4}=g^{*}_{h3}(\lambda_{s2}),\ p_{10,12}=1-g^{*}_{h3}(\lambda_{s2}),\ p_{11,3}=g^{*}_{s3}(\lambda_{h2}),\ p_{11,12}=1-g^{*}_{s3}(\lambda_{h2}),$$

$$A=\lambda_{s1}+\lambda_{h1}+\lambda_{s3}+\lambda_{h3}+\lambda_{he}+\lambda_{se}+\lambda_{hse}+\lambda_{cc}.$$

The mean sojourn times μ_{ij} are obtained as under

$$\mu_0=\frac{1}{A},\ \ \mu_1=\frac{1}{\lambda_{h1}},\ \mu_2=\frac{1}{\lambda_{s1}},\ \mu_3=\frac{1}{\lambda_{s5}+\lambda_{h2}},\ \mu_4=\frac{1}{\lambda_{s2}+\lambda_{h1}},\ \mu_5=\frac{1-g^{*}_{h2}(\lambda_{s4})}{\lambda_{s4}},\ \mu_6=\frac{1-g^{*}_{s2}(\lambda_{he})}{\lambda_{he}}$$

$$\mu_7 = \frac{1}{\lambda_{s5}},\ \mu_8 = \frac{1}{\lambda_{s2}+\lambda_{h2}},\ \mu_9 = \frac{1}{\lambda_{h1}},\ \mu_{10} = \frac{1-g_{h3}^{*}(\lambda_{s2})}{\lambda_{s2}},\ \mu_{11} = \frac{1-g_{s3}^{*}(\lambda_{h2})}{\lambda_{h2}},\ \mu_{12} = -g_{cc}^{*\prime}(0),\ \mu_{13} = -g_{h1}^{*\prime}(0),$$

$$\mu_{14} = -g_{h4}^{*\prime}(0),\ \mu_{15} = -g_{s1}^{*\prime}(0),\ \mu_{16} = -g_{s4}^{*\prime}(0),\ \mu_{17} = -g_{s1}^{*\prime}(0),$$

$$\mu_{18} = -g_{s4}^{*\prime}(0),\ \mu_{19} = -g_{h1}^{*\prime}(0),\ \mu_{20} = -g_{h4}^{*\prime}(0).$$

OTHER MEASURES OF SYSTEM PERFORMANCE

By probabilistic arguments for the regenerative process, the recursive relations for various measures of the system performance are obtained for the both Models. On solving the recursive relations using Laplace and Laplace-Stieltjes transforms, we get the following measures in steady state:

For Model-1:

Mean time to system failure	$T_0 = N / D$
Mean up time	$A_0 = N_1 / D_1$
Mean Degradation time	$D_0 = N_2 / D_1$
Expected Number of Hardware Repairs by External Engineer	$HR_0 = N_3 / D_1$
Expected Number of Hardware Repairs by Software methods	$HM_0 = N_4 / D_1$
Expected Number of Software Repairs by External Engineer	$SR_0 = N_5 / D_1$
Expected Number of Software Repairs by Software methods	$SM_0 = N_6 / D_1$
Expected Number of Visits by External Engineer	$VR_0 = N_7 / D_1$

For Model-2:

Mean time to system failure	$T_1 = N_8 / D_2$
Mean up time	$A_1 = N_9 / D_3$
Mean Degradation time	$D_1 = N_{10} / D_3$
Expected Number of Hardware Repairs by External Engineer	$HR_1 = N_{11} / D_3$
Expected Number of Hardware Repairs by Software methods	$HM_1 = N_{12} / D_3$
Expected Number of Software Repairs by External Engineer	$SR_1 = N_{13} / D_3$
Expected Number of Software Repairs by Software methods	$SM_1 = N_{14} / D_3$
Expected Number of Visits by External Engineer	$VR_1 = N_{15} / D_3$

where,

$$N = (\mu_0 + \mu_1 p_{01} + \mu_2 p_{02})(1 - p_{37})(1\text{-}p_{49}) + p_{03}(1\text{-}p_{49})(\mu_3 + \mu_7 p_{37} + \mu_8 p_{38}) + p_{04}(1\text{-}p_{37})(\mu_4 + \mu_9 p_{49} + \mu_8 p_{48})$$

$$D = (1 - p_{01} - p_{02})(1 - p_{37})(1\text{-}p_{49})$$

$$N_1 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][\mu_0]$$

$$D_1 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{05}\mu_5 + p_{06}\mu_6 + p_{0,12}\mu_{12}] + [1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{03}\mu_3 + p_{03} p_{35}\mu_5 + p_{03} p_{35} p_{5,12}\mu_{12} + p_{03} p_{37}\mu_7 + p_{03} p_{38}(\mu_8 + p_{8,10} p_{10,12}\mu_{12} + p_{8,11} p_{11,12}\mu_{12} + p_{8,10}\mu_{10} + p_{8,11}\mu_{11} + p_{8,10} p_{10,4}\mu_4 + p_{8,11} p_{11,3}\mu_3)] + [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][p_{04}\mu_4 + p_{04} p_{46}\mu_6 + p_{04} p_{46} p_{6,12}\mu_{12} + p_{04} p_{49}\mu_9 + p_{04} p_{48}(\mu_8 + p_{8,10} p_{10,12}\mu_{12} + p_{8,11} p_{11,12}\mu_{12} + p_{8,10}\mu_{10} + p_{8,10} p_{10,4}\mu_4 + p_{8,11}\mu_{11} + p_{8,11} p_{11,3}\mu_3)]$$

$$N_2 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{01}\mu_1 + p_{02}\mu_2] + [1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{03}\mu_3 + p_{03} p_{37}\mu_7 + p_{03} p_{38}\mu_8] + [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][p_{04}\mu_4 + p_{04} p_{48}\mu_8 + p_{04} p_{49}\mu_9]$$

$$N_3 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{05} p_{50} + p_{0,12}] + p_{03} p_{38} p_{8,10} p_{10,4}[1 - p_{49} - p_{48} p_{8,10} p_{10,4}] + p_{04} p_{48} p_{8,10} p_{10,4}[1 - p_{37} - p_{38} p_{8,11} p_{11,3}]$$

$$N_4 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}] p_{01} p_{10} + p_{04}[1 - p_{37} - p_{38} p_{8,11} p_{11,3}] p_{49} p_{94}$$

$$N_5 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{06} p_{60} + p_{0,12}] + p_{03} p_{38} p_{8,11} p_{11,3}[1 - p_{49} - p_{48} p_{8,10} p_{10,4}] + p_{04} p_{48} p_{8,11} p_{11,3}[1 - p_{37} - p_{38} p_{8,11} p_{11,3}]$$

$$N_6 = p_{02}[1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}] + p_{03} p_{37}[1 - p_{49} - p_{48} p_{8,10} p_{10,4}]$$

$$N_7 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{05} + p_{06} + p_{012}] + [p_{03} p_{35} + p_{03} p_{38} p_{8,10} + p_{03} p_{38} p_{8,11}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}] + [p_{04} p_{46} + p_{04} p_{48} p_{8,10} + p_{04} p_{48} p_{8,11}][1 - p_{37} - p_{38} p_{8,11} p_{11,3}]$$

$$N_8 = (\mu_0 + \mu_1 p_{01} + \mu_2 p_{02})(1 - p_{37})(1 - p_{49}) + p_{03}(1 - p_{49})(\mu_3 + \mu_7 p_{37} + p_{37} p_{7,17}\mu_{17} + \mu_8 p_{38} + p_{37} p_{7,18}\mu_{18}) + p_{04}(1 - p_{37})(\mu_4 + \mu_9 p_{49} + p_{49} p_{9,19}\mu_{19} + \mu_8 p_{48} + p_{49} p_{9,20}\mu_{20})$$

$$D_2 = (1 - p_{01} - p_{02})(1 - p_{37})(1 - p_{49})$$

$$N_9 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][\mu_0]$$

$$D_3 = [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][1 - p_{49} - p_{48} p_{8,10} p_{10,4}][\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{05}\mu_5 + p_{06}\mu_6 + p_{0,12}\mu_{12}] + [1 - p_{49} - p_{48} p_{8,10} p_{10,4}][p_{03}\mu_3 + p_{03} p_{35}\mu_5 + p_{03} p_{35} p_{5,12}\mu_{12} + p_{03} p_{37}\mu_7 + p_{03} p_{38}(\mu_8 + p_{8,10} p_{10,12}\mu_{12} + p_{8,11} p_{11,12}\mu_{12} + p_{8,10}\mu_{10} + p_{8,11}\mu_{11} + p_{8,10} p_{10,4}\mu_4 + p_{8,11} p_{11,3}\mu_3)] + [1 - p_{37} - p_{38} p_{8,11} p_{11,3}][p_{04}\mu_4 + p_{04} p_{46}\mu_6 + p_{04} p_{46} p_{6,12}\mu_{12} + p_{04} p_{49}\mu_9 + p_{04} p_{48}(\mu_8 + p_{8,10} p_{10,12}\mu_{12} + p_{8,11} p_{11,12}\mu_{12} + p_{8,10}\mu_{10} + p_{8,10} p_{10,4}\mu_4 + p_{8,11}\mu_{11} + p_{8,11} p_{11,3}\mu_3)]$$

$N_{10} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{01}\mu_1 + p_{02}\mu_2] + [1 - p_{49} - p_{48}$
$p_{8,10}p_{10,4}][p_{03}\mu_3 + p_{03}p_{37}\mu_7 + p_{03}p_{38}\mu_8 + p_{03}p_{37}p_{7,17}\mu_{17} + p_{03}p_{37}p_{7,18}\mu_{18}] + [1 - p_{37}$
$- p_{38}p_{8,11}p_{11,3}][p_{04}\mu_4 + p_{04}p_{48}\mu_8 + p_{04}p_{49}\mu_9 + p_{04}p_{49}p_{9,19}\mu_{19} + p_{04}p_{49}p_{9,20}\mu_{20}]$

$N_{11} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{05}p_{50} + p_{0,12} + p_{01}p_{1,14}] + p_{03}p_{38}p_{8,10}$
$p_{10,4}[1 - p_{49} - p_{48}p_{8,10}p_{10,4}] + p_{04}[p_{48}p_{8,10}p_{10,4} + p_{49}p_{9,20}p_{20,4}][1 - p_{37} - p_{38}p_{8,11}p_{11,3}]$

$N_{12} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{01}p_{10} + p_{01}p_{1,13}p_{13,0}]$
$+ p_{04}[1 - p_{37} - p_{38}p_{8,11}p_{11,3}][p_{49}p_{94} + p_{49}p_{9,19}p_{19,4}]$

$N_{13} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{02}p_{2,16}p_{16,0} + p_{06}p_{60} + p_{0,12}p_{12,0}] + p_{03}$
$[p_{37}p_{7,18}p_{18,3} + p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}] + p_{04}p_{48}p_{8,11}p_{11,3}[1 - p_{37} - p_{38}p_{8,11}p_{11,3}]$

$N_{14} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{02}p_{20} + p_{02}p_{2,15}\ p_{15,0}] + p_{03}[1 - p_{49}$
$- p_{48}p_{8,10}p_{10,4}][p_{37} + p_{37}p_{7,17}p_{17,3}]$

$N_{15} = [1 - p_{37} - p_{38}p_{8,11}p_{11,3}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}][p_{05} + p_{06} + p_{012}] + [p_{03}p_{35} + p_{03}p_{38}p_{8,10} + p_{03}$
$p_{38}p_{8,11}][1 - p_{49} - p_{48}p_{8,10}p_{10,4}] + [p_{04}p_{46} + p_{04}p_{48}p_{8,10} + p_{04}p_{48}p_{8,11}][1 - p_{37} - p_{38}p_{8,11}p_{11,3}]$

Profit analysis

The expected total profit incurred to the system in steady state is given by

For Model-1:

$$\mathrm{P}_0 = C_0A_0 + C_1D_0 - C_2HM_0 - C_3HR_0 - C_4SM_0 - C_5SR_0 - C_6VR_0 - C_I$$

For Model-2:

$$\mathrm{P}_1 = C_0A_1 + C_1D_1 - C_2HM_1 - C_3HR_1 - C_4SM_1 - C_5SR_1 - C_6VR_1 - C_I$$

where

C_0 = revenue per unit mean up time of the system.

C_1 =revenue per unit degradation time of the system.

C_2 = cost per unit hardware repair by software methods.

C_3 = cost per unit hardware repair by external engineer.

C_4 = cost per unit software repair by software methods.

C_5 = cost per unit software repair by external engineer.

C_6 = cost per visit of the external engineer.

C_I = software installation cost.

COMPARATIVE ANALYSIS

The Model-1and Model-2 are compared on the basis of their reliability, availability and profit to judge which and when one Model is better than the other. For the comparative analyses purpose, the following particular case is considered:

$g_{s1}(t) = \alpha_1 e^{-\alpha_1 t}$ $g_{s2}(t) = \alpha_2 e^{-\alpha_2 t}$ $g_{s3}(t) = \alpha_3 e^{-\alpha_3 t}$ $g_{h1}(t) = \beta_1 e^{-\beta_1 t}$ $g_{h2}(t) = \beta_2 e^{-\beta_2 t}$ $g_{h3}(t) = \beta_3 e^{-\beta_3 t}$
$g_{cc}(t) = \gamma e^{-\gamma t}$.

The values of the various failures rates and probability of hardware degradation detection, as given in Teng et al. [16] and Trivedi et al. [17], are taken, i.e.

$$\lambda_{h1} = .02; \lambda_{h2} = .0248; \lambda_{h3} = .0284; \lambda_{s1} = .00614; \lambda_{s2} = .00674; \lambda_{s3} = .00692;$$
$$\lambda_{s4} = .05; \lambda_{s5} = .00246 \alpha_1 = .1; \alpha_2 = .16; \alpha_3 = .24; \beta_1 = .12, \beta_2 = .18, \beta_3 = .2$$

Various costs, failure and repair rates are also assumed as under:

$$\gamma = 2; \lambda_{he} = .001; \lambda_{se} = .006; \lambda_{hse} = .0002; \lambda_{cc} = .0005; c_{dh} = .9; c_{ds} = .6, C_0 = 10000,$$
$$C_1 = 100, C_2 = 500, C_3 = 50, C_4 = 400, C_5 = 200, C_I = 2000$$

Several graphs are plotted for difference of mean time to system failures, mean up time, mean degradation time and profits of the two Models with respect to various failure rates and costs.

Fig. 3 and 4 depicts the behavior of the difference of mean time to system failure $(T_0 - T_1)$ corresponding to Model-1 and Model-2 with respect to various failure rates viz. λ_{h1} and λ_{s2} for different values of and hardware/software fault detection coverage.

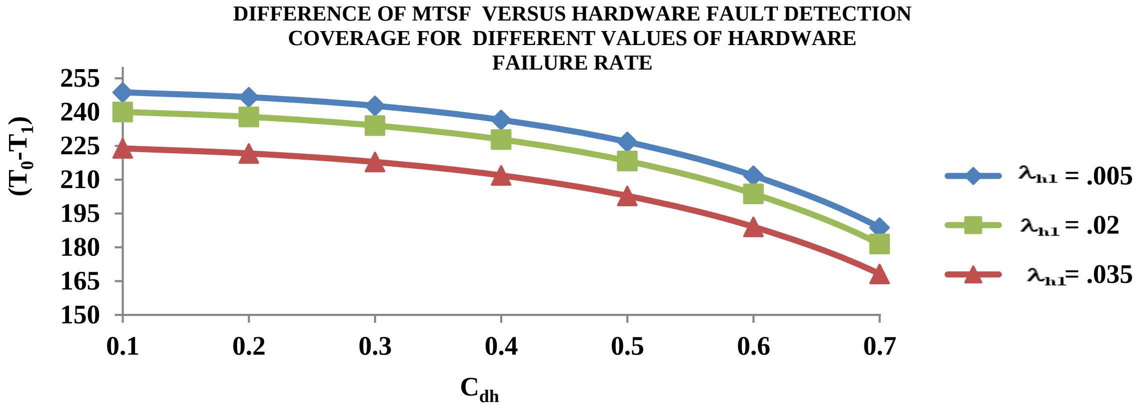

Figure 3.

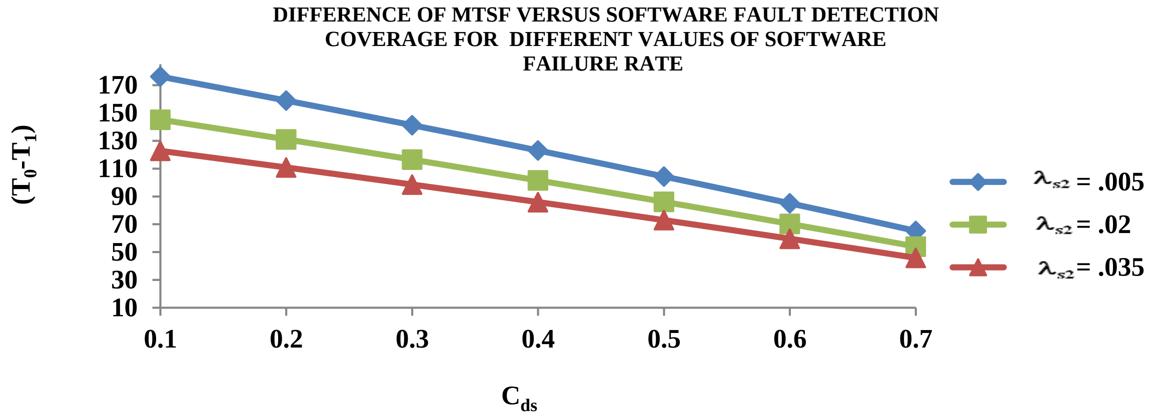

Figure 4.

It can be seen that the difference of mean time to system failure $(T_0 - T_1)$ decreases with the increase in the values of hardware/software fault detection coverage and failure rates viz. λ_{h1} and λ_{s2}. We observed that mean time to system failure of Model-1 is greater than the mean time to system failure of Model-2.

The curve in the fig. 5, 6 and 7 depicts the behavior of difference of the expected uptimes of the system corresponding to Model-1 and Model-2, i.e. $(A_0 - A_1)$ with respect to various failure rates and hardware/software fault detection coverage.

It can be seen that the difference of up time of system $(A_0 - A_1)$ increases with the increase in the values of hardware/software fault detection coverage and failure rates whereas decreases with hardware repair rate. We observed that expected up time of system of Model-1 is greater than that of Model-2.

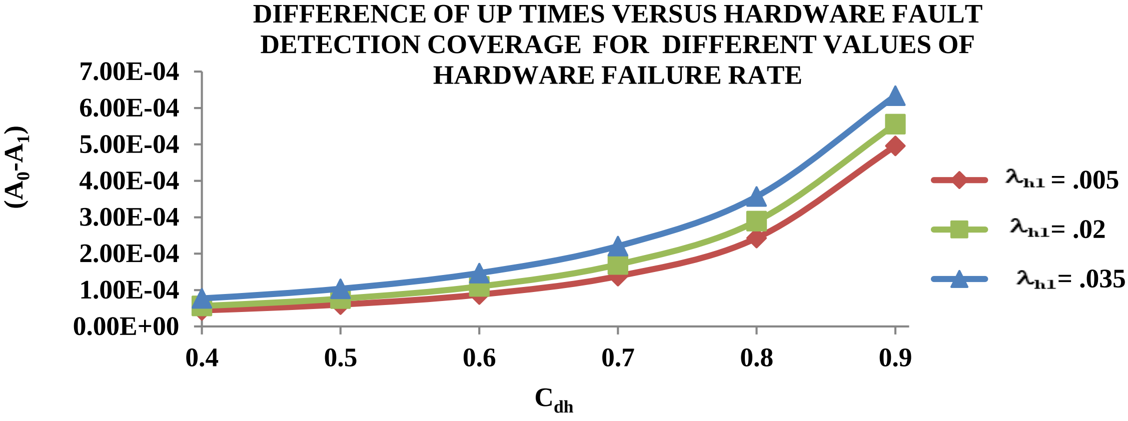

Figure 5.

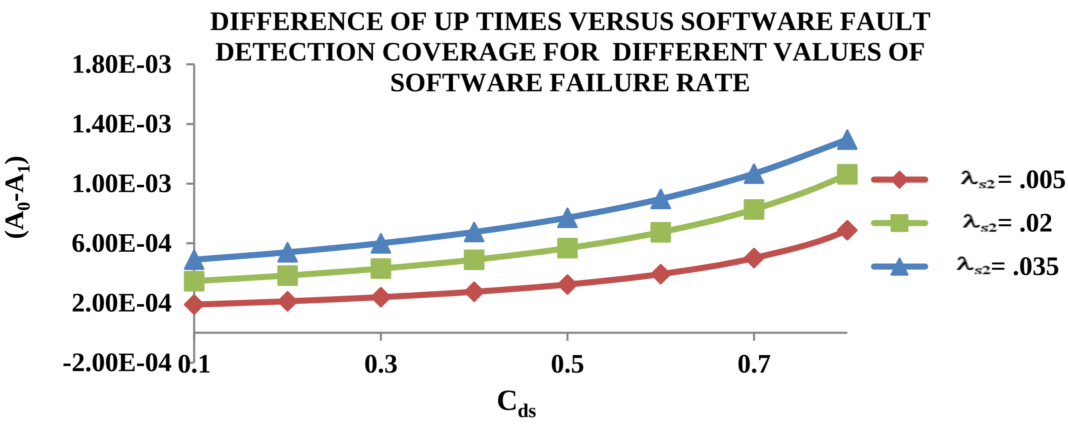

Figure 6.

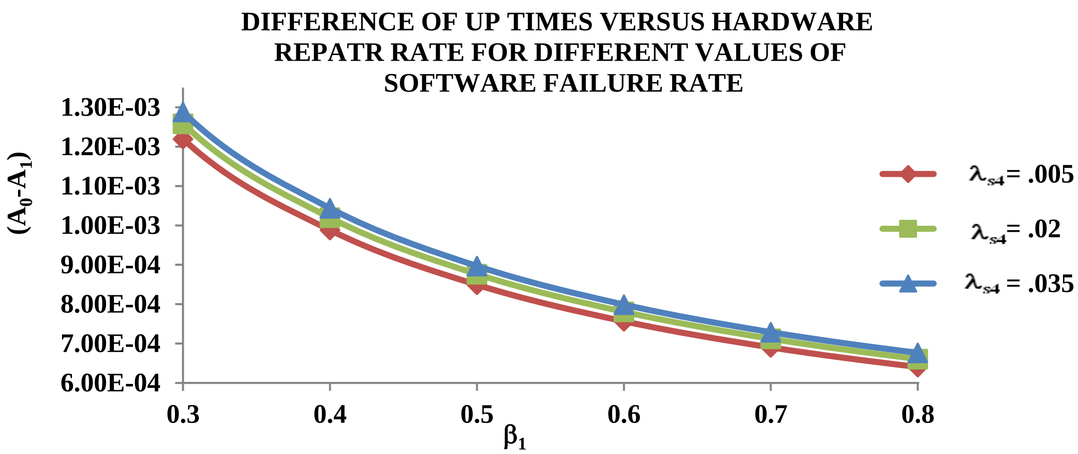

Figure 7.

Fig. 8 shows the behavior of difference of mean degradation time to the system $(D_0 - D_1)$ with respect to hardware fault detection coverage C_{dh} and hardware failure rate λ_{h1}.

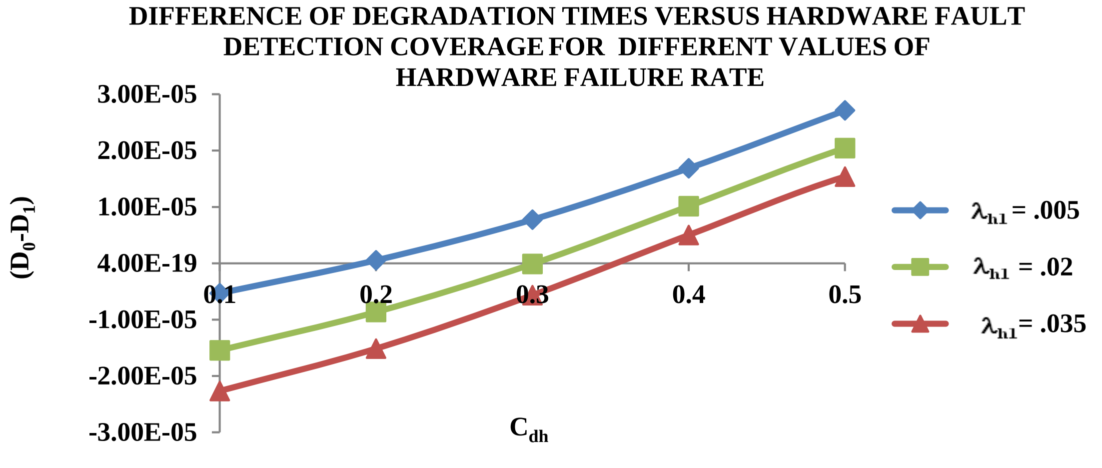

Figure 8.

From the **fig. 8**, it is evident that difference of mean degradation time to the system $(D_0 - D_1)$ of the system increases with hardware fault detection coverage C_{dh} and has lower values for the higher values of hardware failure rate λ_{h1}.

It can also be observed from the graph that for λ_{h1} = .005, $(D_0 - D_1)$ is positive or negative according as C_{dh} > or < 0.192. Therefore, the mean degradation time to the system of Model-1 is greater than mean degradation time to the system of Model-2 whenever C_{dh} > 0.192. Similarly for λ_{h1} = .02 and λ_{h1} = .035 the mean degradation time to the system of Model-1 is greater than mean degradation time to the system of Model-2 whenever C_{dh} > 0.298 and C_{dh} > 0.354 respectively.

Fig. 9 shows the behavior of difference of the profits of the system corresponding to the Model-1 and Model-2, i.e. $(P_0 - P_1)$ with respect to revenue per unit up time C_0 and hardware fault detection coverage C_{dh}.

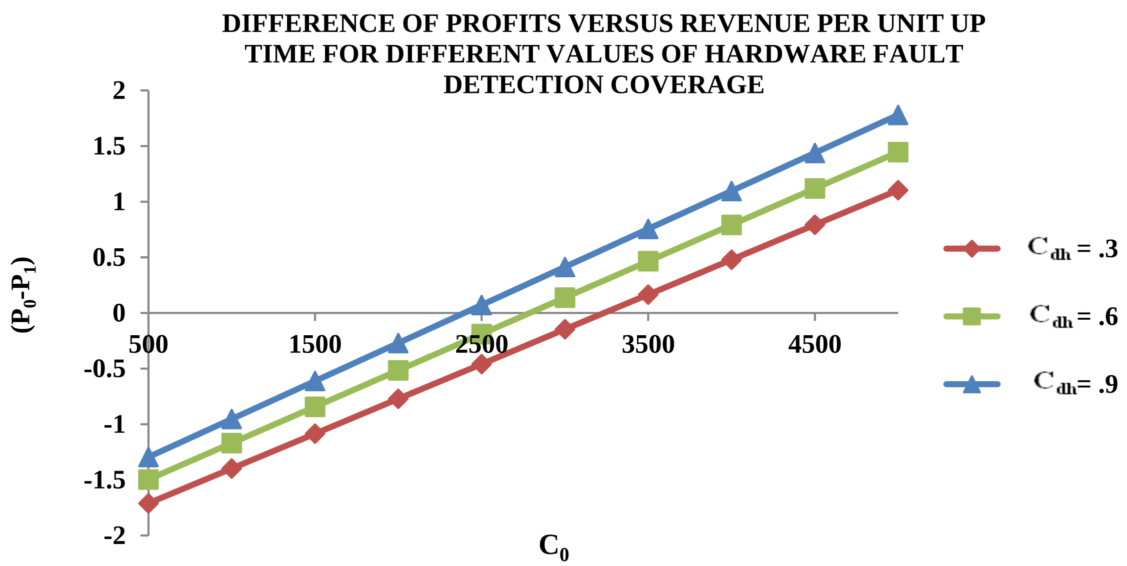

Figure 9.

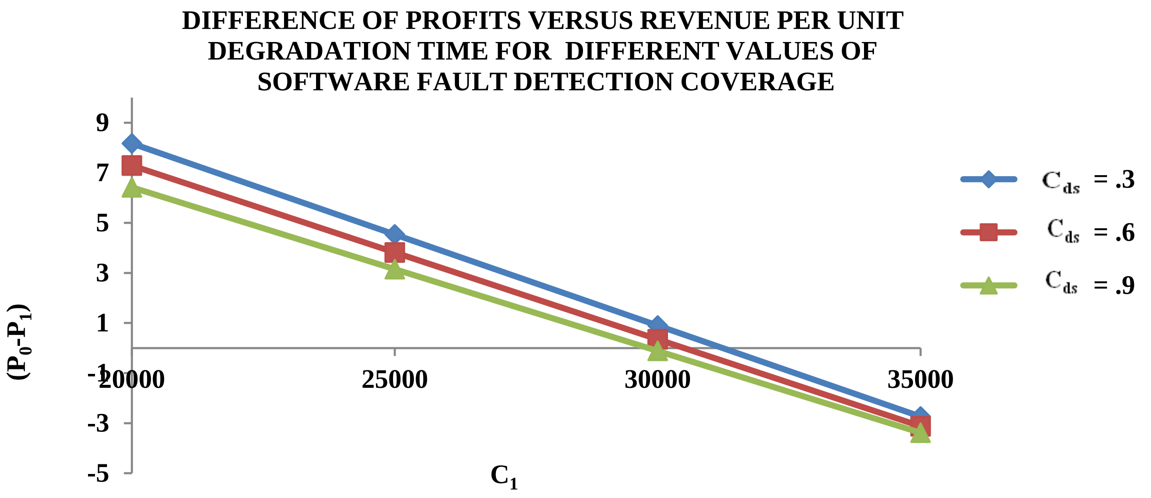

Figure 10

From the fig. 9, it is evident that difference of profits $(P_0 - P_1)$ of the system increases with increase in the values of revenue cost per unit up time C_0 and has higher values for the higher values of hardware fault detection coverage C_{dh} .

It can also be observed from the graph that for $C_{dh} = .3$, $(P_0 - P_1)$ is positive or negative according as C_0 > or < 2360.5. Therefore, Model-2 is better (more profitable) than Model-1whenever C_0 >2360.5. Similarly for C_{dh} = .6 and C_{dh} = .9, Therefore, Model 2 is better (more profitable) than Model 1 whenever C_0 > 2780.5 and C_0 > 3230.5 respectively.

Fig. 10 shows the behavior of difference of the profits of the system corresponding to the Model-1 and Model-2, i.e. $(P_0 - P_1)$ with respect to revenue per unit degradation time for different values of software fault detection coverage C_{ds} .

From the fig.10, it is evident that difference of profits $(P_0 - P_1)$ of the system decreases with increase in the values of revenue cost per unit degradation time C_1 and has lower values for the higher values of software fault detection coverage C_{ds} .

From the fig.10, it is evident that difference of profits $(P_0 - P_1)$ of the system decreases with increase in the values of revenue cost per unit degradation time C_1 and has lower values for the higher values of hardware fault detection coverage C_{ds} .It can also be observed from the graph that for C_{ds} = .3, $(P_0 - P_1)$ is positive or negative according as C_1 < or >29995. Therefore, Model-2 is better (more profitable) than Model-1whenever C_1 < 29995. Similarly for C_{ds} = .6 and C_{ds} = .9, Therefore, Model-2 is better (more profitable) than Model-1 whenever C_1 < 30550 and C_1 <31450 respectively.

CONCLUSION

From the comparative analyses of the Models we concluded that Model-1is better than the Model-2 in terms of their mean time to system failure and mean up time for fixed value of the hardware/software failure rate. However, in terms of profits, either of the Models may be better than the other Model depending on cut- off values of various failures rates, hardware/software failure rates, fault detection coverage, and revenue per unit degradation time of system and revenue per unit up time of system. These results may be very significant and useful for the system developers and engineers.

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