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Estimation for Unknown Parameters of the Extended Burr Type-XII Distribution Based on Type-I Hybrid Progressive Censoring Scheme

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Abstract: In this paper, Bayesian and non-Bayesian estimations are used to make point and interval estimations for extended Burr Type-XII distribution in case of Type-I hybrid progressive censoring scheme. Markov Chain Monte Carlo (MCMC) approximation is needed to solve the complicated integrations in the case of Bayesian techniques. Numerical results using generation data sets are presented.

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INTRODUCTION

The Burr-XII distribution was first introduced by Burr (1942), and has gained great attention due to the importance of using it in many practical situations. Inferences on the Burr-XII distribution have been studied by many authors. Wingo (1993), Moore and Papadopoulos (2000), Mousa and Jaheen (2002), Wu and Yu (2005), Soliman (2005), Xiuchun, et al. (2007), Jaheen and Okasha (2011) and Jong-Wuu Wu et al. (2014). Gharib et al. (2010) have derived a new distribution called extended Burr-XII distribution (EBXIID) by using the Marshall-Olkin technique. They have discussed some properties for the proposed distribution and shown that the extended Burr-XII distribution may be obtained as a compound distribution with mixing exponential distribution.

Censored sampling have great interest due to the cost and time. In many cases, the experimenter may not always obtain complete information on failure times for all experimental units. Lifetime distributions under censored sampling have been attracting great interest in the recent years due to their wide applications. The most common censoring schemes are Type-I and Type-II censoring, in which the experiment will be terminated at a predetermined time T in the case of Type-I censoring or m^{th} failure in the case of Type-II censoring. A new model of censoring scheme called Type-I hybrid progressive censoring scheme. Its a mixture of Type-2 progressive and hybrid censoring scheme (For more details, see Balakrishnan and Aggarwala (2000) and Childs et al. (2003)). Its considered n independent and identical items which placed on a life-test. In the beginning, the T and value of m are fixed beforehand. The experiment is terminated at a random

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time $T_* = min(X_{(m:m:n)}, T)$. Suppose, the failure times are denoted by $X_i, i = 1, 2, \ldots, m, m < n$. Let $\{R_1, \ldots, R_m\}$ are prescribed censoring schemes number of units removed from the experiment satisfying $\sum_{i=1}^{m} R_i + m = n$. When the first failure occurs $X_{(1:m:n)}$, the R_1 of the remaining units are randomly removed from the experiment. Similarly, when the second failure occurs $X_{(2:m:n)}$, the rest of units R_2 are randomly moved from the test and so on. There are two cases of the end of the experiment, If the m^{th} failure happen before a prescribed time T, the experiment is finished at the time point $X_{(m:m:n)}$. Furthermore, the test quit at time T satisfying $X_{(j:m:n)} < T < X_{(j+1:m:n)}$ and all the remaining live test units $R_j^* = n - R_1 - \cdots - R_j - j$ are removed. The observed sample might be one of the following two cases:

$$\begin{aligned} &Case \quad I : \{X_{(1:m:n)}, X_{(2:m:n)}, \cdots, X_{(m:m:n)}\}, \\ &\quad \text{if} \quad X_{(m:m:n)} < T, \\ &Case \quad II : \{X_{(1:m:n)}, X_{(2:m:n)}, \cdots, X_{(j:m:n)}\}, \\ &\quad \text{if} \quad X_{(j:m:n)} < T < X_{(j+1:m:n)}. \end{aligned}$$

Now, If the failure times of the n items originally on the test are from an extended Burr Type-XII population with cdf $F(x)$ and pdf $f(x)$ as follow:

$$f(x) = \frac{\alpha c \beta x^{c-1}(1+x^c)^{\beta-1}}{((1+x^c)^{\beta} - \bar{\alpha})^2}, \quad \alpha, \beta, c, x > 0, 0 \le \alpha \le 1, \quad \bar{\alpha} = 1 - \alpha \tag{1}$$

and

$$F(x) = 1 - \frac{\alpha}{(1+x^c)^{\beta} - \bar{\alpha}}. \tag{2}$$

Also, the hazard rate function and reliability function are; respectively:

$$H(x) = \frac{c\beta x^{c-1}(1+x^c)^{\beta-1}}{(1+x^c)^{\beta} - \bar{\alpha}} \tag{3}$$

and

$$S(x) = \frac{\alpha}{(1+x^c)^{\beta} - \bar{\alpha}}. \tag{4}$$

Remarks 1.

1. For $\alpha = 1$, (1) reduce to the case of Burr-XII distribution.
2. For $\alpha = 1$, c = 1, (1) reduce to the case of standard Lomax distribution.
3. For $\beta = 1$, (1) reduce to the case of log-logistic distribution.

MAXIMUM LIKELIHOOD ESTIMATION (MLE)

This section deals with obtaining the point estimation for unknown parameters c and β using maximum likelihood technique. The joint probability density function can written from as follows:

$$f_{1,2,\cdots,m:m:n}(x_1, x_2, \ldots, x_m) \propto \begin{cases} \prod_{i=1}^{m} f(x_{i:m:n})\,[1 - F(x_{i:m:n})]^{R_i}, \\ \quad \text{Case I: } X_m < T; \\ \prod_{i=1}^{j} f(x_{i:m:n})\,[1 - F(x_{i:m:n})]^{R_i}\,[1 - F(T)]^{R_j^*}, \\ \quad \text{Case II: } T < X_m. \end{cases} \tag{5}$$

where

$$R_j^* = n - R_1 - \cdots - R_j - j \text{ and } 0 < x_{i:m:n}, T < \infty.$$

The maximum likelihood for EBXIID can be written from (5), as follow:

$$\begin{aligned} \ell \propto c^d \beta^d \prod_{i=1}^{d} &\Big[\big(x_{i:m:n}^{c-1}(1 + x_{i:m:n}^c)^{\beta-1}\big) \\ &\times \big((1 + x_{i:m:n}^c)^{\beta} - \bar{\alpha}\big)^{-2} \big((1 + x_{i:m:n}^c)^{\beta} - \bar{\alpha}\big)^{-R_i}\Big] \\ &\times \big((1 + T^c)^{\beta} - \bar{\alpha}\big)^{-dR_j^*} \end{aligned} \tag{6}$$

where

$$d = \begin{cases} j, & T < X_m; \\ m, & X_m < T. \end{cases}$$

Taking logarithm of ℓ gives:

$$\begin{aligned} L = \log \ell \propto\ & d \log c + d \log \beta \\ &+ (c-1)\sum_{i=1}^{d} \log x_{i:m:n} + (\beta - 1)\sum_{i=1}^{d} \log(1 + x_{i:m:n}^c) \\ &- 2\sum_{i=1}^{d} \log((1 + x_{i:m:n}^c)^{\beta} - \bar{\alpha}) \\ &- \sum_{i=1}^{d} R_i \log((1 + x_{i:m:n}^c)^{\beta} - \bar{\alpha}) \\ &- R_j^* \log((1 + T^c)^{\beta} - \bar{\alpha}) \end{aligned} \tag{7}$$

The point estimation for c and β can obtained by differentiating (7) to c and β, equating the new equations to zero and solving to obtain $\hat{c}$ and $\hat{\beta}$. The analytical solution may be very difficult to find. So, Matematica 8 are used to solve the two equations. Also, the corresponding MLE of $S(t)$ and $H(t)$ can be written as:

$$\hat{H}(x) = \frac{\hat{c}\hat{\beta}x^{\hat{c}-1}(1+x^{\hat{c}})^{\hat{\beta}-1}}{(1+x^{\hat{c}})^{\hat{\beta}} - \bar{\alpha}} \tag{8}$$

$$\hat{S}(x) = \frac{\alpha}{(1+x^{\hat{c}})^{\hat{\beta}} - \bar{\alpha}} \tag{9}$$

ASYMPTOTIC CONFIDENCE INTERVALS

In this section, the approximate confidence interval for $c, \beta, S(t)$ and $H(t)$ are obtained. c, β was approximately bivariate normal with mean $(\hat{c}, \hat{\beta})$ and variance-covariance matrix I_0^{-1}. see Greene (2000), Agresti (2002). Where

$$I_0(c,\beta) = \begin{bmatrix} -\frac{\partial^2 L}{\partial c^2} & -\frac{\partial^2 L}{\partial c \partial \beta} \\ -\frac{\partial^2 L}{\partial \beta \partial c} & -\frac{\partial^2 L}{\partial \beta^2} \end{bmatrix}. \tag{10}$$

Inverse of the matrix in (10), then the variance-covariance matrix will be:

$$I_0^{-1}(c,\beta) = \begin{bmatrix} var\hat{c} & covar(\hat{c},\hat{\beta}) \\ covar(\hat{c},\hat{\beta}) & var\hat{\beta} \end{bmatrix}. \tag{11}$$

The (1-α)% approximate confidence interval for c, β, $S(t)$ and $H(t)$; respectively:

$$\left(\hat{c} \pm Z_{\frac{\eta}{2}} \sqrt{var\hat{c}}\right),$$

$$\left(\hat{\beta} \pm Z_{\frac{\eta}{2}} \sqrt{var\hat{\beta}}\right),$$

$$\left(\hat{S}(t) \pm Z_{\frac{\eta}{2}} \sqrt{var\hat{S}(t)},\right)$$

and

$$\left(\hat{H}(t) \pm Z_{\frac{\eta}{2}} \sqrt{var\hat{H}(t)}\right),$$

where, $var(c)$ and $var(\beta)$ are the variances of the unknown parameters c and β which obtained from inverse fisher information matrix and $Z_{\frac{\eta}{2}}$ is the percentile of the two tail probability $\frac{\eta}{2}$ calculated from standard normal distribution.

The $\sqrt{var\hat{H}(t)}$ and $\sqrt{varS(t)}$ can be calculated from the next equations:

$$\sqrt{var\hat{H}(t)} \simeq \left[G_1^T I_0^{-1} G_1\right]_{(\hat{c},\hat{\beta})}, \tag{12}$$

and

$$\sqrt{var\hat{S}(t)} \simeq \left[G_2^T I_0^{-1} G_2\right]_{(\hat{c},\hat{\beta})}, \tag{13}$$

where

$$G_1 = \begin{pmatrix} \frac{\partial H(t)}{\partial c} & \frac{\partial H(t)}{\partial \beta} \end{pmatrix}, G_2 = \begin{pmatrix} \frac{\partial S(t)}{\partial c} & \frac{\partial S(t)}{\partial \beta} \end{pmatrix}$$

and I_0^{-1} is the variance-covariance matrix defined in (11).

BAYES ESTIMATION

Bayesian estimation for unknown parameters are studied in this section. First, let the parameters c and β are independent and have gamma distributions as prior functions in the form; respectively:

$$\pi_1(c) = \frac{b^a c^{a-1} e^{-bc}}{\Gamma(a)}, \qquad c > 0, \tag{14}$$

and

$$\pi_2(\beta) = \frac{\xi^\gamma \beta^{\gamma-1} e^{-\xi\beta}}{\Gamma(\gamma)}, \qquad \beta > 0 . \tag{15}$$

The joint prior distribution of c and β can be given as:

$$g(c,\beta) = \pi_1(c)\pi_2(\beta). \tag{16}$$

The joint posterior density function can be written as:

$$\pi(c,\beta|\underline{x}) = \frac{\ell(c,\beta|\underline{x})\pi_1(c)\pi_2(\beta)}{\int_0^\infty \int_0^\infty \ell(\underline{x}|c,\beta)\pi_1(c)\pi_2(\beta)dcd\beta}. \tag{17}$$

The Bayes estimator under squared error loss function of any function g is the posterior mean and has been given by:

$$\hat{g}(c,\beta|\underline{x}) = E(g(c,\beta|\underline{x})) = \frac{\int_0^\infty \int_0^\infty g(c,\beta)\ell((c,\beta|\underline{x}))\pi_1(c)\pi_2(\beta)dcd\beta}{\int_0^\infty \int_0^\infty \ell(c,\beta|\underline{x})\pi_1(c)\pi_2(\beta)dcd\beta}. \tag{18}$$

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In general, eq. (18) can't reduce to closed form and the exact solution may be very hard. In this case, the MCMC method to generate sample of (17) maybe the best procedure to compute the double integrations in (18). Next section shows the MCMC approach including Metropolis-Hastings within Gibbs Sampling technique.

BAYESIAN ESTIMATION USING MCMC APPROACH

Now, considered MCMC approach to generate samples from the posterior. Its clear that, the solution of (18) is very hard to calculate. So, the MCMC maybe suitable approximation to solve (18) and hence estimate $\hat{c}$ and $\hat{\beta}$. The Metropolis-Hastings algorithm is a very general MCMC method developed by Metropolis et al. (1953) and extended by Hastings (1970). It can be used to obtain random samples from any arbitrarily complicated target distribution of any dimension that is known up to a normalizing constant. In fact, Gibbs Sampler is a special case of a Monte Carlo Markov chain algorithm. It generates a sequence of samples from the full conditional probability distributions of two or more random variables. Gibbs sampling requires decomposing the joint posterior distribution into full conditional distributions for each parameter and then sampling from them. In order to use the method of MCMC for estimating the parameters of the EBXIID, let us consider independent priors (14 and 15), respectively, for the parameters c and β. The joint posterior density function can be obtained up to proportionality by multiplying the likelihood with the prior and this can be written as:

$$\pi(c,\beta|\underline{x}) \propto \beta^{d+\gamma-1} c^{d+a-1} e^{-bc-\xi\beta} \times \prod_{i=1}^{d}\left[\left(x_{i:m:n}^{c-1}(1+x_{i:m:n}^{c})^{\beta-1}\right)\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-2}\right] \times \prod_{i=1}^{d}\left[\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-R_i}\right]\left((1+T^{c})^{\beta}-\bar{\alpha}\right)^{-dR_j^*} \tag{19}$$

From (19), the posterior density function of c given β and β given c are, respectively:

$$g_1(c|\beta,\underline{x}) \propto c^{d+a-1} e^{-bc} \times \prod_{i=1}^{d}\left[\left(x_{i:m:n}^{c-1}(1+x_{i:m:n}^{c})^{\beta-1}\right)\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-2}\right] \times \prod_{i=1}^{d}\left[\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-R_i}\right]\left((1+T^{c})^{\beta}-\bar{\alpha}\right)^{-dR_j^*}, \tag{20}$$

and

$$g_2(\beta|c,\underline{x}) \propto \beta^{d+\gamma-1} e^{-\xi\beta} \times \prod_{i=1}^{d}\left[\left((1+x_{i:m:n}^{c})^{\beta-1}\right)\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-2}\right] \times \prod_{i=1}^{d}\left[\left((1+x_{i:m:n}^{c})^{\beta}-\bar{\alpha}\right)^{-R_i}\right]\left((1+T^{c})^{\beta}-\bar{\alpha}\right)^{-dR_j^*} \tag{21}$$

Its clear that the posterior density function of c and β given in (20 and 21) cannot be reduced analytically to well known distributions and therefore it is not possible to sample directly by standard methods, but the plot of it shows that it is similar to normal distribution. So, to generate random numbers from this distribution, the Metropolis-Hastings method with normal proposal distribution will be used. The proposal scheme to generate c and β from the posterior density functions and in turn obtain the Bayes estimates and the corresponding credible intervals can be written as:

1. Start with $c^{(0)}$
2. Set $\nu = 1$
3. Generate c^{ν} from g_1 with the normal $N(c^{\nu-1}, var(c))$, where $var(c)$ is the variance of parameter c is obtained from (11).
4. Also, generate β^{ν} from g_2 with the normal $N(\beta^{\nu-1}, var(\beta)$, where $var(\beta)$ is the variance of parameter β is obtained from (11).
5. Compute c^{ν} and β^{ν}
6. Set $\nu = \nu + 1$
7. Repeat 3-6 steps N times
8. Rearrange the values c^i and β^i, $i = Q+1, Q+2, \cdots, N$.
9. Obtain the Bayes estimates of c and β with respect to the squared error loss function as:

$$E(c|data) = \frac{1}{N-Q}\sum_{i=Q+1}^{N} c_i$$

and

$$E(\beta|data) = \frac{1}{N-Q}\sum_{i=Q+1}^{N} \beta_i,$$

where Q is burn-in.

Table 1. Censoring Schemes

Censoring Schemes	R
$CS1$	$R_1 = \{4, 2, 4, 3, 2, 4, 2, 4, 3, 2\}$
$CS2$	$R_2 = \{1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 2, 1, 1, 2\}$
$CS3$	$R_3 = \{1, 2, 1, 0, 2, 0^4, 2, 1, 2, 1, 0, 2, 1, 2, 1, 0, 2\}$
$CS4$	$R_4 = \{4, 3, 4, 3, 5, 4, 5, 4, 3, 5\}$
$CS5$	$R_5 = \{4, 2, 4, 3, 2, 1, 2, 1, 3, 2, 1, 2, 3, 3, 2\}$
$CS6$	$R_6 = \{4, 2, 0, 3, 2, 0, 2, 0, 3, 2, 0, 2, 0, 3, 2, 0, 0, 0, 3, 2\}$
$CS7$	$R_7 = \{0^8, 2, 0^6, 8, 3, 0^3, 5, 0^8, 2\}$
$CS8$	$R_8 = \{4, 2, 4, 3, 2, 4, 2, 0, 3, 2, 0, 2, 0, 3, 2, 0, 2, 0, 3, 2\}$
$CS9$	$R_9 = \{1, 2, 1, 0, 0, 1, 0, 1, 0, 2, 1, 0, 1, 0, 2, 1, 2, 1, 0, 2, 1, 2, 1, 0, 2, 1, 2, 1, 0, 2\}$

where 0^r means that 0 repeated r times.

Table 2. Mean, mse, LL, UL, IL and cov for **c** and β using maximum likelihood method

CS	n	m	$mean$	mse	LL	UL	IL	Cov
				$\hat{c}$				
$CS1$	40	10	3.1883	0.2799	1.7554	4.6213	2.8659	0.998
$CS2$		15	3.2365	0.2549	2.0473	4.4258	2.3785	1
$CS3$		20	3.2465	0.2496	2.1846	4.3084	2.1238	0.96
$CS4$	50	10	3.178	0.3164	1.6978	4.6582	2.9604	0.988
$CS5$		15	3.1288	0.2422	1.9738	4.2839	2.3101	0.99
$CS6$		20	3.2408	0.2336	2.2214	4.2603	2.0389	0.99
$CS7$		30	3.5002	0.4066	2.5855	4.4149	1.8293	0.908
$CS8$	60	20	3.2121	0.2184	2.2205	4.2037	1.9832	0.982
$CS9$		30	3.3399	0.2555	2.4541	4.2257	1.7716	0.968
				$\hat{\beta}$				
$CS1$	40	10	2.1563	0.5735	−0.5248	4.8373	5.3621	1
$CS2$		15	2.3397	0.774	0.187	4.4925	4.3055	1
$CS3$		20	2.4628	0.7187	0.6246	4.301	3.6763	1
$CS4$	50	10	2.3708	1.7815	−1.1122	5.8539	6.9661	0.998
$CS5$		15	2.1618	0.6219	0.0939	4.2298	4.1359	0.988
$CS6$		20	2.3025	0.5208	0.578	4.0271	3.4492	1
$CS7$		30	3.0717	1.9015	1.1867	4.9566	3.7699	1
$CS8$	60	20	2.241	0.5724	0.5052	3.9768	3.4716	1
$CS9$		30	2.5566	0.83	0.981	4.1323	3.1513	1

10. $(1-\eta)\%$ credible intervals of c and β can be computed as

$$\left(c_{\left(\frac{\eta}{2}(N-Q)\right)}, c_{\left(\frac{1-\eta}{2}(N-Q)\right)} \right)$$

and

$$\left(\beta_{\left(\frac{\eta}{2}(N-Q)\right)}, \beta_{\left(\frac{1-\eta}{2}(N-Q)\right)} \right).$$

NUMERICAL RESULTS

Confidence interval for c, β, S and H using maximum likelihood and Bayes include MCMC technique are calculated numerically. ($ns = 500$) samples generated from Type-I hybrid progressive samples from the EBXIID with different censoring schemes R contains $N = 11000$ values with discard the first $Q = 1000$ values in the case of MCMC as burn-in. The performances of ML and Bayesian (with joint gamma priors) methods are compared via mean squared errors technique. In Tables (2–5) below, Average point estimation (mean), mean squared error (mse), interval estimation lower limit (LL), interval estimation upper limit (UL), interval length (IL) and coverage probability (cov)

Table 3. Mean, mse, LL, UL, IL and cov for **S** and **H** using maximum likelihood method

CS	n	m	$mean$	mse	LL	UL	IL	Cov
					$\hat{S}$			
$CS1$	40	10	0.8076	0.0547	0.6877	0.9274	0.2398	0.956
$CS2$		15	0.801	0.0558	0.6882	0.9139	0.2257	0.96
$CS3$		20	0.7922	0.0529	0.6807	0.9037	0.223	0.94
$CS4$	50	10	0.7999	0.057	0.6839	0.9159	0.232	0.962
$CS5$		15	0.8012	0.05	0.6952	0.9073	0.2121	0.97
$CS6$		20	0.8021	0.0521	0.7004	0.9039	0.2035	0.94
$CS7$		30	0.7915	0.0537	0.6957	0.8874	0.1918	0.928
$CS8$	60	20	0.8045	0.0502	0.7089	0.9001	0.1912	0.926
$CS9$		30	0.7998	0.0479	0.7107	0.8888	0.178	0.936
					$\hat{H}$			
$CS1$	40	10	1.5311	0.35	0.4861	2.5761	2.0901	0.996
$CS2$		15	1.622	0.3908	0.7528	2.4912	1.7384	0.992
$CS3$		20	1.7055	0.3959	0.9138	2.4973	1.5835	1
$CS4$	50	10	1.6098	0.4763	0.4122	2.8073	2.3951	0.988
$CS5$		15	1.5661	0.3435	0.7204	2.4119	1.6915	0.99
$CS6$		20	1.6144	0.3484	0.8678	2.361	1.4931	0.982
$CS7$		30	1.8584	0.503	1.1428	2.5741	1.4314	0.916
$CS8$	60	20	1.5843	0.3489	0.8528	2.3158	1.463	0.986
$CS9$		30	1.6987	0.379	1.0546	2.3429	1.2883	0.966

Table 4. Mean, mse, LL, UL, IL and cov for **c** and β using Bayesian method under mcmc approximation

CS	n	m	$mean$	mse	LL	UL	IL	Cov
					$\hat{c}$			
$CS1$	40	10	2.5773	0.2801	1.5873	3.6847	2.0974	0.932
$CS2$		15	2.7538	0.1601	1.8649	3.7472	1.8824	0.962
$CS3$		20	2.8562	0.1352	2.0226	3.7848	1.7622	0.96
$CS4$	50	10	2.5251	0.3222	1.5588	3.6045	2.0456	0.932
$CS5$		15	2.6896	0.2116	1.8228	3.6413	1.8185	0.938
$CS6$		20	2.8762	0.1306	2.0691	3.7644	1.6954	0.966
$CS7$		30	3.1418	0.1281	2.3857	3.9547	1.5691	0.978
$CS8$	60	20	2.8574	0.132	2.065	3.7233	1.6582	0.956
$CS9$		30	3.0361	0.1025	2.3022	3.8309	1.5287	0.98
					$\hat{\beta}$			
$CS1$	40	10	1.3416	0.2533	0.4704	2.9453	2.4749	0.99
$CS2$		15	1.6466	0.1001	0.7239	3.1478	2.4238	0.994
$CS3$		20	1.9028	0.0986	0.9349	3.3472	2.4123	0.98
$CS4$	50	10	1.2925	0.3192	0.4276	2.9418	2.5142	0.974
$CS5$		15	1.5416	0.1586	0.6597	2.9993	2.3396	0.964
$CS6$		20	1.7931	0.0848	0.8871	3.1676	2.2805	0.994
$CS7$		30	2.4161	0.4762	1.3533	3.9229	2.5696	0.99
$CS8$	60	20	1.7199	0.1081	0.838	3.0598	2.2219	0.986
$CS9$		30	2.088	0.1835	1.1691	3.39	2.2209	1

for **c**, β, S and H are computed. All results are obtained at $c = 3$; $\beta = 1.8$; $T = 0.9$; $t = 0.4$; $\alpha = 0.5$; $a = 0.5$; $b = 1$; $\gamma = 0.8$; $\xi = 1$; $N = 11000$; $Q = 1000$; $\eta = 0.05$ and different censoring schemes (see Table 1).

Remarks: From Tables 2 - 5, observe that:

- When n and m increase, the mean squared error (MSE) decrease.

Table 5. Mean, mse, LL, UL, IL, interval length and cov for **S** and **H** using Bayesian method under mcmc approximation

CS	n	m	$mean$	mse	LL	UL	IL	Cov
				$\hat{S}$				
$CS1$	40	10	0.7938	0.048	0.6721	0.9155	0.2434	0.998
$CS2$		15	0.7853	0.0533	0.6709	0.8997	0.2288	0.99
$CS3$		20	0.7756	0.0574	0.6626	0.8886	0.226	0.98
$CS4$	50	10	0.7937	0.0463	0.677	0.9105	0.2335	0.99
$CS5$		15	0.7887	0.0478	0.6817	0.8957	0.214	0.99
$CS6$		20	0.789	0.0512	0.6863	0.8918	0.2055	0.956
$CS7$		30	0.7774	0.0566	0.68	0.8747	0.1947	0.932
$CS8$	60	20	0.7943	0.0477	0.6983	0.8903	0.1919	0.96
$CS9$		30	0.7875	0.0492	0.6974	0.8776	0.1802	0.948
				$\hat{H}$				
$CS1$	40	10	1.3195	0.246	0.3795	2.2595	1.88	0.994
$CS2$		15	1.4856	0.2356	0.691	2.2802	1.5893	0.994
$CS3$		20	1.6217	0.2902	0.8814	2.3621	1.4807	1
$CS4$	50	10	1.296	0.2734	0.2714	2.3206	2.0492	0.988
$CS5$		15	1.4252	0.2233	0.6434	2.207	1.5636	0.986
$CS6$		20	1.5263	0.2461	0.828	2.2246	1.3966	0.984
$CS7$		30	1.7839	0.4018	1.1137	2.4541	1.3404	0.982
$CS8$	60	20	1.4777	0.2395	0.7972	2.1581	1.3609	0.988
$CS9$		30	1.6376	0.2955	1.0265	2.2488	1.2223	0.996

- In most cases, mean squared errors and interval lengths calculated for Bayesian procedure are smaller than calculated for maximum likelihood, so Bayesian procedure is better than the maximum likelihood as expected.
- Coverage probabilities in the two methods nearly so close.

CONCLUSION

In this paper, the maximum likelihood and Bayes methods are used to make interval estimation for the unknown parameters of EBXIID. Markov Chain Monte Carlo approximation is used in Bayesian procedure to solve the hard integrations. Some numerical computations and comparisons are presented to illustrate the methods of inference developed here.

REFERENCES

[1] Agresti, A. (2002). *Categorical Data Analysis.* 2nd Edition. New York: Wiley.

[2] Ali Mousa, M. A. M. and Jaheen, Z. F. (2002). Statistical inference for the Burr model based on progressively censored data. *Computers & Mathematics with Application*, Vol. 43, No. 10–11, pp 1441–1449.

[3] Balakrishnan, N. and Aggarwala R. (2000). *Progressive Censoring: Theory, Methods, and Applications*, Birkhauser, Boston, Berlin.

[4] Childs A., Chandrasekar B., Balakrishnan N., Kundu D. (2003). Exact Likelihood inference based on type-I and type-II hybrid censored samples from the exponential distribution, *Ann. Insti. Statistical Mathematics* 55, pp: 319–330.

[5] Gharib, M., Mohie El - Din, M.M. and Mohammed, B.E. (2010). An extended Burr (XII) distribution and its application to censored data; *Journal of Advanced Research in Statistics and Probability*; Vol: 2; Issue 1; 37–50:

[6] Greene, W.H. (2000). *Econometric Analysis*, 4th Ed. New York: Prentice Hall.

[7] Hastings, W.K. (1970). Monte Carlo sampling methods using Markovchains and their applications. *Biometrika* 57, 97–109.

[8] Jaheen, Z. F. and Okasha, H. M. (2011). E-Bayesian estimation for the Burr Type-XII model based on type-2 censoring. *Applied Mathematical Modelling*, 35, 4730–4737.

[9] Jong-Wuu Wu, Wen-Chuan Lee, Ching-Wen Hong and Sung-Yu Yeh. (2014). Implementing Lifetime Performance Index of Burr-XII Products With Progressively

Type II Right Censored Sample. *International Journal of Innovative Computing, Information and Control*, Volume 10, Number 2, pp. 671–693.

[10] Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H. and Teller, E. (1953). Equations ofnstate calculations by fast computing machines. *Journal Chemical Physics*21, 1087–1091.

[11] Moore D. and Papadopoulos A.S. (2000). The Burr Type-XII distribution as a failure model under various loss functions. *Microelectronics Reliability*, Vol. 40, Number 12, pp. 2117–2122(6).

[12] Soliman, A.A. (2005). Estimation of parameters of life from progressively censored data using Burr-XII model. *IEEE Transactions on Reliability*, Vol. 54, No. 1, pp. 34–42.

[13] Wingo, D. R. (1993). Maximum likelihood methods for fitting the Burr type-XII distribution to multiply (progressively) censored life test data. Metrika, 40, 203–210.

[14] Wu, J. W. and Yu, H. Y. (2005). Statistical inference about the shape parameter of the Burr Type-XII distribution under the failure-censored sampling plan. *Applied Mathematics and Computation* , Vol. 163, No. 1, pp. 443–482.

[15] Xiuchun L., Yimin S., Jieqiong W. and Jian C. (2007). Empirical Bayes estimators of reliability performances using LINEX loss under progressively Type-II censored samples. *Mathematics and Computers in Simulation*, Vol. 73, No. 5, pp. 320–326.