

Effect of a New Class of Circulant Matrices on the Colorimetric Classes of the Color Images

Rasha Najah Mirza

Department of mathematics/Faculty of Computer Science and mathematics/ University of Kufa, Iraq.

Professor Hind Rustum Mohammed

Computer Science Department, Faculty of Computer Science and mathematics/ University of Kufa, Iraq.

Abstract:

In this paper, we have introduce new class of Circulant matrices specifically, the complex symmetric Circulant matrix , we give general formula and found that this matrix have limited behavior at the high power and discuss the effect of this behavior on the color images.

Keywords: Special matrix, power of matrix, Square matrix, Circulant Matrices, Coloring of image.

INTRODUCTION

Circulant matrix was one of the most important matrices in the field of Mathematical computation especially image processing, we will see the effect of the high power of this matrix, exactly those for dimension $n > 1$ and n is an odd number. [3,2]

Most common method to deal with variable acquisition conditions consists of applying a color constancy algorithm [8], while to obtain device-independent color description a color characterization procedure is applied. Color balancing model is composed of two stages first discounts the illuminant color, second maps the image colors from the device-dependent RGB space into a standard device independent color space. More effective pipelines have been proposed [4,5] that deal between the two processing stages.

Color model and color space are characterized by defections many algorithm and used to represent pixel color, color layers based color spaces were introduced when there was a need for the user to specify color properties numerically [1].

The paper is organized in the beginning by Definitions of Circulant Matrices, then the analytic of proposed matrix, after that explain the proposed method, then results comes and finally the conclusion.

Basic Definitions and Properties of Circulant Matrices

An $n \times n$ Circulant matrix X_n is defined as

$$X_n = \begin{pmatrix} a_0 & a_1 & \dots & a_{n-2} & a_{n-1} \\ a_{n-1} & a_0 & \dots & a_{n-3} & a_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & \dots & a_0 & a_1 \\ a_1 & a_2 & \dots & a_{n-1} & a_0 \end{pmatrix}$$

Where C_n of dimension $n \times n$.

The matrix X_n is called symmetric Circulant if $a_j = a_{n-j}$, where $j = 1, \dots, n$. [7]

This type of matrix accure with many applications like digital image, image restoration, signal processing, coloring image, coding theory and numerical solution of some kinds of partial differential equations. [3,2]

The Eigen value and Eigen vectors of C_n are given as

$$\lambda_r = \sum_{m=0}^{n-1} c_m \rho^{im}, r = 0, 1, \dots, n-1$$

Where $\rho = \exp\left(\frac{2\pi i}{n}\right)$ and $i = \sqrt{-1}$, the Eigenvectors corresponding this Eigenvector is $v_r = (1, \rho^m, \rho^{2m}, \dots, \rho^{(n-1)m})^T$, $m = 0, 1, \dots, n-1$. [7]

3. The New Class of Complex Symmetric Circulant matrix

We will deal with symmetric complex Circulant matrix K_n of dimension where n is an odd number grater than 1 .

$n = 2k + 1$; $k = 1, 2, \dots$ There entries are all $\mp i$ and ∓ 1 , where $i = \sqrt{-1}$, have the form

$$Z_n = \begin{pmatrix} i & 1 & \dots & 0 & 0 & 0 \\ 1 & i & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & i & 1 & 0 \\ 0 & 0 & \dots & 1 & i & 0 \\ 0 & 0 & \dots & 0 & 0 & i \end{pmatrix}$$

We can rewrite Z_n as the following block matrix,

$$Z_n = \begin{pmatrix} D_2 & O_2 & O_2 & \dots & O_2 & \vdots & 0 \\ O_2 & D_2 & O_2 & \dots & O_2 & \vdots & 0 \\ O_2 & O_2 & \ddots & & O_2 & \vdots & 0 \\ O_2 & O_2 & O_2 & & D_2 & \dots & 0 \\ \dots & & \dots & & \dots & & 0 \\ 0 & 0 & 0 & & 0 & 0 & i \end{pmatrix}$$

where $D_2 = \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$ and O_2 is 2x2 zero matrix

Its easy to show that the Eigen value of D_2 is $\pm 1 + i$

We have four cases they are:

Example 1. $K_{7 \times 7} = \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i \end{pmatrix}$

Case 1, If $K_n = Z_n$,

Lemma 1:

The Eigen value, of the matrix of Case 1 will be $\lambda_{1s} = 1 + i$, $-1 + i, \dots, i$, $s = 1, 2, \dots$ and 1s refer to case1.

Proof : Since, Z_n is diagonal block matrix, so that the Eigen value of K_n will be the Eigen value of the block diagonal matrices [6].

Case 2, If $K_n = -Z_n$,

Lemma 2:

The Eigen value, of the matrix of Case 2 will be $\lambda_{2s} = -\lambda_{1s}$, i.e $\lambda_{2s} = 1 - i, -1 - i, \dots, -i$, $s = 1, 2, \dots$ and 2s refer to case2

Proof : from lemma 1 we get that $K_n X = \lambda_{1s} X$ and since $K_n = -Z_n$

Thus, $-K_n X = -\lambda_{1s} X$

Case 3, If $K_n = \bar{Z}_n$

Lemma 3:

The Eigen value, of the matrix of Case 3 will be $\lambda_{3s} = \bar{\lambda}_{1s}$, i.e $\lambda_{3s} = 1 - i, -1 - i, \dots, -i$

Proof :

from lemma 1 we get that $K_n X = \lambda_{1s} X$ and since $K_n = \bar{Z}_n$

Thus, $\bar{Z}_n X = \bar{\lambda}_{1s} X$

Case 4, If $K_n = -\bar{Z}_n$

Lemma 4:

The Eigen value, of the matrix of Case 4 will be $\lambda_{4s} = -\bar{\lambda}_{1s}$ i.e

$\lambda_{4s} = 1 + i, -1 + i, \dots, i$

Proof: from lemma 1 we get that $K_n X = \lambda_{1s} X$ and since $K_n = -\bar{Z}_n$

Thus, $\bar{Z}_n X = -\bar{\lambda}_{1s} X$

THE PROPOSED METHOD

Now, we will expound the stage of applying the proposed system agencies

We propose that this matrix of order $s = 4k, k = 1, 2, 3, \dots$ is

For instance if

$(C_n)^s = \begin{pmatrix} (-4)^k I_{n-1} & o \\ o & 1 \end{pmatrix}$ where, I_{n-1} is the identity matrix and o is zero vector .

For instance, if C_n is the matrix of case 1 or case 2 or case 3 or case 4, then :

$$(C_{7 \times 7})^4 = \begin{pmatrix} -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (C_{7 \times 7})^8 = \begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(C_{7 \times 7})^{12} = \begin{pmatrix} -64 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -64 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -64 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -64 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -64 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -64 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (C_{7 \times 7})^{16} = \begin{pmatrix} 256 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 256 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 256 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 256 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 256 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 256 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

EXPERIMENTAL RESULTS

The results will be discussed here .The system was done on a database matlab library images any format .Figure (1) shown sample of gray images.

Figure (2) shown the effect of a C_n matrix on the images in both cases $(C_{3 \times 3})^4$ and $(C_{3 \times 3})^{12}$, Note that the illumination is to the hand of darkness and this leads to non - show components and areas in the images.



Figure 1. sample of gray images



Figure 2. Effect of a C_n matrix on the images in both cases $(C_{3 \times 3})^4$ and $(C_{3 \times 3})^{12}$



Figure 3. Effect of a C_n matrix on the images in both cases $(C_{3 \times 3})^8$ and $(C_{3 \times 3})^{16}$

Figure (4) shown the effect Propagation chromatography layers of color layers red and blue and green Where we find that the spread of chromosomes very converges in $(C_{3 \times 3})^{16}$.

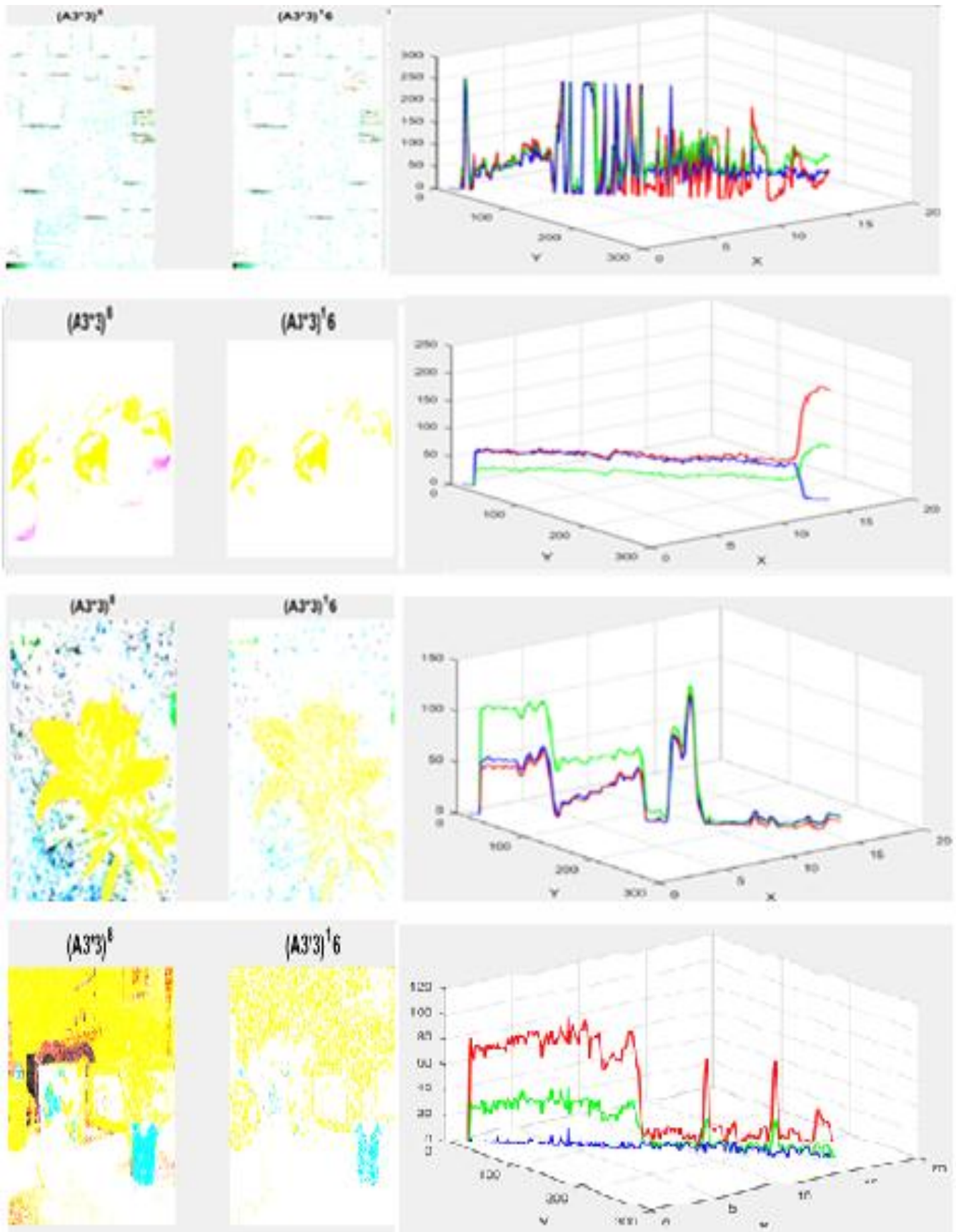


Figure 4: shown the effect Propagation chromatography layers

Figure (5) shown the effect of a C_n matrix on the images in both cases $(C_{5 \times 5})^4$, $(C_{5 \times 5})^{12}$, $(C_{7 \times 7})^4$ and $(C_{7 \times 7})^{12}$. Note that the illumination of $(C_{7 \times 7})$ is to the hand of darkness compare with $(C_{5 \times 5})$.

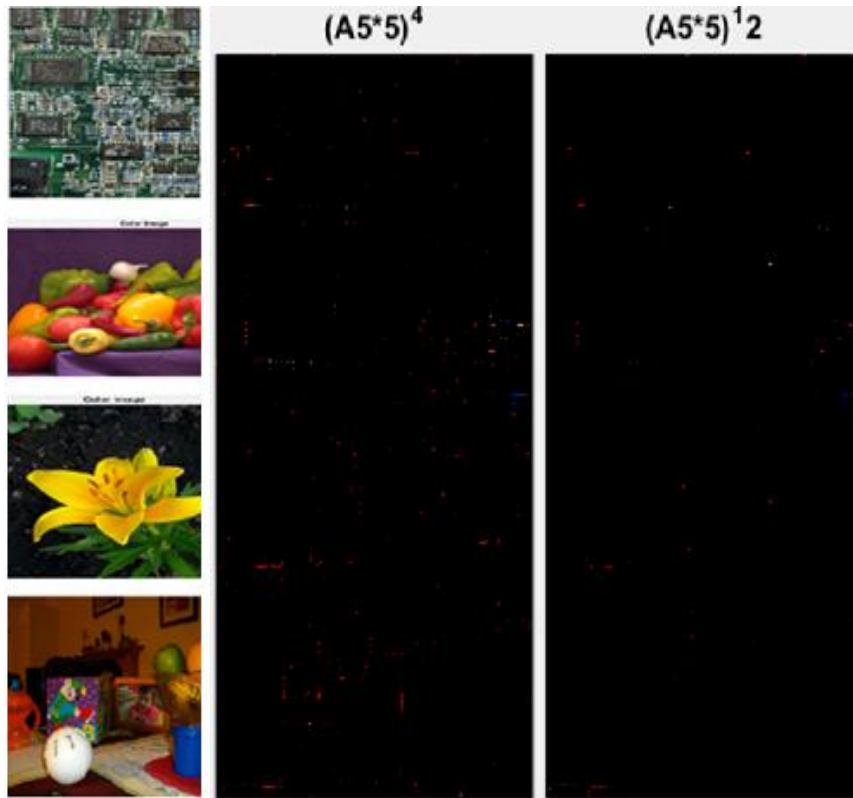


Figure 5. :Effect of a C_n matrix on the images in both cases $(C_{5 \times 5})^4$, $(C_{5 \times 5})^{12}$, $(C_{7 \times 7})^4$ and $(C_{7 \times 7})^{12}$

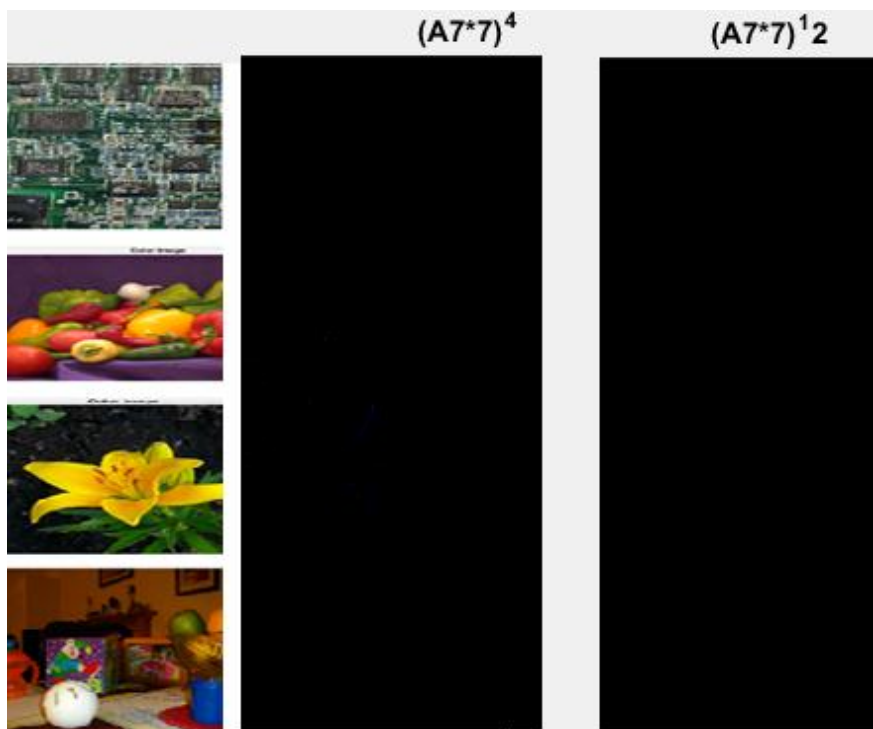


Figure 6. shown the effect of a C_n matrix on the images in both cases $(C_{5 \times 5})^8$ and $(C_{5 \times 5})^{16}$. Note that the illumination is to high light and Hide most micro - components of images

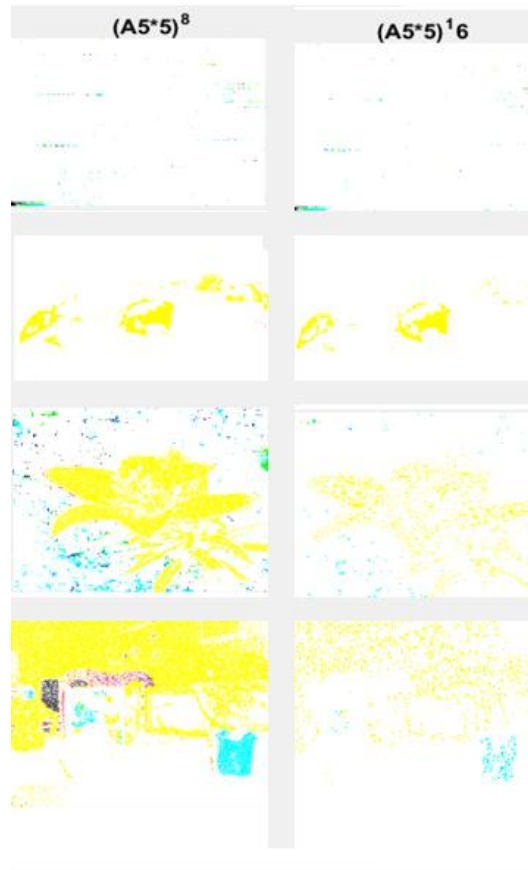


Figure 6: Effect of a C_n matrix on the images in both cases $(C_{5 \times 5})^8$ and $(C_{5 \times 5})^{16}$

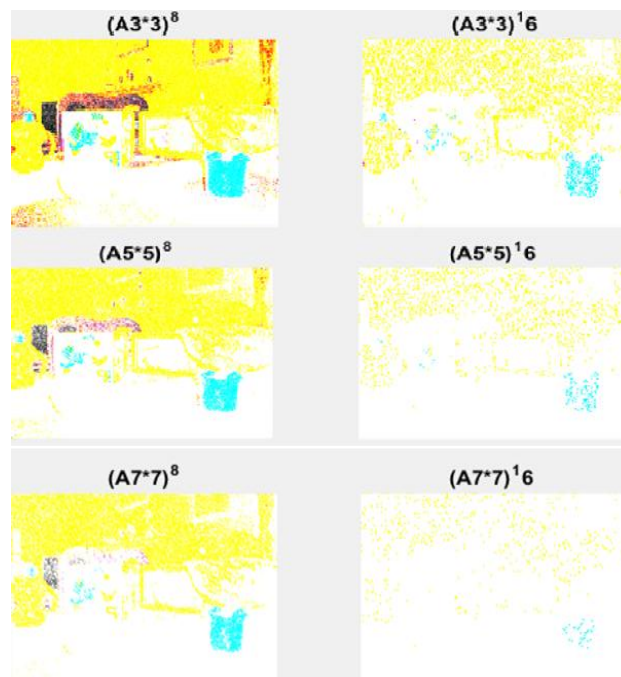


Figure 7: Compared to show images of the components of the cases

Most of the exact components of images are hidden in case $(C_{7 \times 7})$, While the exact components of the images appear in

the case $(C_{3 \times 3})$, The situation remains moderate in determining the appearance of ingredients in $(C_{5 \times 5})$, Figure (7)

shown Compared to show images of the components of the cases .

Note that the blue color layer is more layered which shows the features and components of the images, Followed by the red

color that shows the part that shows the large components either green color is the worst layer of chromatic images with the disappearance of features and components of basic images, whether large or small. Figure (8) shown Effect of a New Class of Circulant Matrices on the Colorimetric Classes.

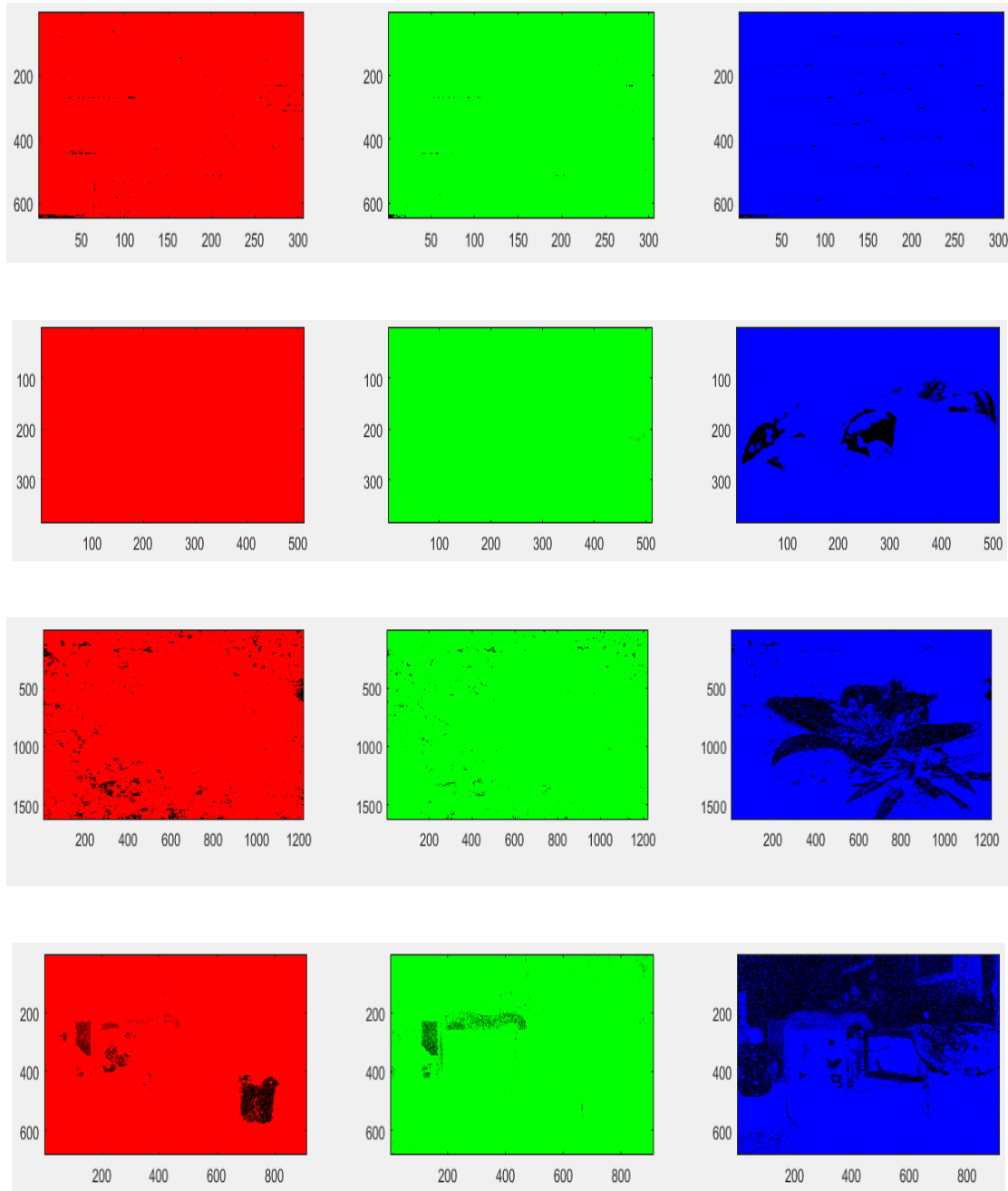


Figure 8: Shown Effect of A New Class of Circulant Matrices On the Colorimetric Classes

CONCLUSION

A new class of complex symmetric Circulant matrix have special behavior in the high order and this effect directly on the result of the Colorimetric Classes in colors images. Can be used The proposed method complex symmetric Circulant matrix in encoding the basic and small components of color

images if we use the green layer . The proposed method can also be used to determine the basic components and lighting if use the second and fourth cases If want to reduce lighting we can use the first and third mode of the proposed method The results demonstrated the effectiveness of the method in chromatography.

REFERENCES

- [1] Ameer A. Mohammed Baqer, Hind Rostom Mohammed, Color Based Segmentation Iris image for Secure Distributed Systems, International Journal of Scientific & Engineering Research, Volume 4, Issue 12,,pp.708-713, 2013.
- [2] A. Andrew, Eigenvector of Certain Matrices , 1973,,linear Algebra Appl, pp.157-162,
- [3] A. Gorsho, Adaptive Equalization of Highly Dispersive Channels for Data Transmission, , January, 1969, BSTJ, 48 , PP. 55-70.
- [4] Bianco, S.; Schettini, R. Error-tolerant Color Rendering for Digital Cameras. J. Math. Imaging Vis. 2014,50, 235–245.
- [5] Bianco, S.; Schettini, R.; Vanneschi, L. Empirical modeling for colorimetric characterization of digital cameras.In Proceedings of the 2009 16th IEEE International Conference on Image Processing (ICIP), Cairo, Egypt,7–10, 2009.
- [6] Ch. Zhang, ch. Luo, A. Huang & J. Lu , The Eigenvalue Distribution of Block Diagonally Dominant Matrices and Block H-Matrices , ELA, issn 1081-3810, September, 2010, volume 20, pp.621-639.
- [7] Davis P.R, Circulant Matrices, (1979) John Wiley, New York.
- [8] Gijssenij, A.; Gevers, T.; Van DeWeijer, J. Computational color constancy: Survey and experiments. IEEE Trans.Image Process. 2011, 20, 2475–2489.