Solving of the Task of Kolmogorov-Fisher Type Biological Population in the Regime with Aggravation

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Abstract

Computational investigated nonlinear processes of multicomponent systems competing biological populations, the analysis of the results based on the estimates of the solutions, which showed high performance algorithms and software complexes for finding new effects to solve the system of parabolic equations.

Keywords. B-spline, self-similar solutions, LS-regime, regime with aggravation.

INTRODUCTION

To date, the study of the linear mathematical models of biological processes are convenient for analysis, since the underlying linear differential equations developed General methods for their solution. In applied tasks of a real biological processes are nonlinear, and for their adequate description is the actual use of nonlinear mathematical models. Hyperbolic growth of the humankind, occurring in the regime with aggravation and surpassing tens of thousands of times all comparable processes, becomes the dominant feature in the solution of the differential equation of growth. For estimates of the population in the foreseeable future, the simulation results can be compared to calculations of the International Institute for applied systems analysis (NASA), the United Nations and other agencies [1].

Regime with aggravation is the process in which in one point or in some region of space or in all space temperature turns into infinity in a finite time, called exacerbations [2-3]. The interest in regimes with aggravation arose in the mid 70-ies of the last century in connection with the study of nonstationary processes in high temperature plasmas [4-5]. In the world wide distribution of mathematical models of processes described by quasilinear parabolic equations, due to the fact that they are derived from the fundamental conservation laws. Therefore, it is possible that the process of biological populations and physical process that does not have at first glance nothing in common, describe the same nonlinear diffusion equation, only with different numerical parameters [6-7]. Studies show that the nonlinearities change not only the quantitative characteristics of the processes, but the qualitative picture of their behavior. Interestingly, from the point of view of applications to study the following classes of nonlinear differential equations in which the unknown function and the derivative of this function consists of exponential way. Then, with the comparison theorems of solutions of this class can be extended. In this case, to find a suitable solution of the differential inequality is easier than any exact solution of parabolic equations describing nonlinear processes, biological populations.

In the last decade in connection with a growing interest in problems of structure formation the study of competing models of multicomponent biological systems in the class of systems of nonlinear equations of the type reaction-diffusion received a new impuls[8-9].

In [10] for single-population model, it was shown that nonlinear dependence of the migration flow from the local population density allows to describe adequately the characteristic behavior of population dynamics observed in the development of outbreaks of. In the description of the Malthusian demographic processes flash develops for large times exponentially both on local density and capture area of the range flash for any non-negative values of exponents migration flows. For all considered variants of the description of demographic processes development of flash on small time qualitatively coincides with Malthusian kinetics on the same time. However, at large time, her development is different. The key here is the relation between the exponents in members describing the migration and demographic processes.

Perhaps the emergence of both an unlimited outbreak of the Malthusian type of behavior, that is, the exponential growth of both the number and the flash area occupied, as well as a limited outbreak of the Ferhulist type, at which the local density increases to a value determined by the capacitance of the medium on an almost unchanged area of the range. After this, an increase in the range of the outbreak begins in the form of two settling waves propagating in directions opposite from the center of the flare. The peculiarity of the solutions obtained is that for any instant of time they are localized on a finite carrier. One of the main fundamental differences of the considered models from the well-known model of Kolmogorov-Petrovsky-Piskunov is the spatial localization of flashes for any time. In the model transmission solution only asymptotically tends to zero at infinite distance from the wave front. Introduction the exponential nonlinearity in demographic processes (Ally principle), as it turned out, in some cases, gives rise to the same spatial dynamics as the power factor of non-linearity in the migration of members.

In a model of competing populations were able to clarify the principle of the gauze to spatially distributed systems for direct [10]. It is shown that the interaction of two competing

populations with different mobility, it is possible to have multiple stationary distributions of numbers consisting of two types of alternating intervals. On one type of intervals is implemented in the stationary homogeneous solution corresponding to a point the principle of the gauze (there is only one kind), and another the coexistence of both species. Introduction nonlinear diffusion also resulted in model calculations the phenomenon of transit zones that are of great interest from the point of view of environmental geography. An extremely interesting phenomenon that occurs in models of competition with nonlinear diffusion, was the appearance of stable (in the numerical experiment) stationary inhomogeneous two-dimensional structures.

Introduction assumptions about the spatiality of the habitat of competing species allows for a different look at the process and results of competition. One of the peculiarities and difficulties of the study of mathematical models is the nonuniqueness of the solution that distinguishes them from the classical problems with a single solution. Therefore, there are problems such as to find a "good" approximation to each solution and construct an iterative method, which is: always converges to the desired solution (corresponding to the initial approximation), converges fast, provides sufficient accuracy.

 Currently studying various properties of solutions of nonlinear problems of large number of works and identify all new and new properties of their solutions. Solution of nonlinear boundary value problems always accompanied by considerable difficulties, because to solve them in an analytical form is possible only in exceptional cases and for establishing new properties of solutions require painstaking research. Standard methods of research of nonlinear problems depending on the type of nonlinearities are not yet available. Therefore, in each case to study the properties of solutions have resorted to various exact and approximate methods in leading universities and research centers around the world, including Oxford University, University of Cambridge, Department of Mathematical Sciences of University of Liverpool, Department of Applied Mathematics of University of Leeds, Faculty of Biological Sciences of University of Leeds, Department of Engineering Mathematics and School of Biological Sciences of University of Bristol (Great Britain), Department of Mathematics of Stanford University, Consortium of the Americas for Interdisciplinary Science and Department of Physics and Astronomy of University of New Mexico, Department of Biology of University of Louisiana, Departments of Entomology and Biology, Pennsylvania State University (USA),   Ecole des Hautes Etudes en Sciences Sociales, Institut de Math´ematiques de Toulouse, Universit´e Paul Sabatier (Франции), Universit`a degli Studi di Padova Dipartimento di Matematica (Италия), Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes (Chili), Centro Ato mico Bariloche, Instituto Balseiro and CONICET (Argentina), Department of Theoretical Ecology, Biology Centre all ASCR, Institute of Entomology (Czech Republic), School of Sciences, Jimei University, Xiamen (China), Moscow State University, Institute of applied mathematics, Institute of theoretical and experimental

Biophysics, Tomsk State University (Russia).

Review of foreign scientific research shows that since the work of Turing, the mathematical model of the formation and propagation of nonlinear waves and processes of structural self-organization in physical, chemical, biological, and social systems such as reaction-diffusion, where members describe nonlinear kinetics, and transport processes represented by isotropic diffusion (Oxford University, University of Cambridge) [11-12]. However, in many systems, no less important, more complex mechanisms diffusion is a nonlinear, anisotropic and cross-diffusion (cross-diffusion). Most studies have examined direct diffusion (self-diffusion) described by the equations, the diffusion coefficients, which are constant values (Ecole des Hautes Etudes en Sciences Sociales, Institut de Mathématiques de Toulouse, Universite Paul Sabatier (France), Universit'a degli Studi di Padova Dipartimento di Matematica (Italy)) [13-14]. In this class of systems, the processes of formation of spatial-temporal structures is determined by the diffusion coefficients and the specific forms of the kinetic function of reaction process. The case when the diffusion coefficients are not constants, but depend on dynamic variables, corresponds to the nonlinear diffusion. Examples of nonlinear diffusion in the processes of mass transfer in porous media, as well as in population models (Consortium of the Americas for Interdisciplinary Science and Department of Physics and Astronomy of the University of New Mexico) [15-16]. Mathematical models with diffusion coefficient depending on the density of bacteria, describes the processes of formation of complex spatial structures with the growth of bacterial colonies. Regimes with aggravation in the spatial localization in open dissipative systems described by models with nonlinear diffusion (Institute of applied mathematics, Institute of theoretical and experimental Biophysics, Tomsk State University) [17-18].

Different models of objectives of the population was considered by many scientists, and was revealed useful properties.

In [19] to a solution of the extended equations of Fisher-Kolmogorov with the assistance of the scheme of collocation quintic B-spline. The scheme is based on the formulation of crank-Nicholson for time integration functions and quintic B-spline for the integration in space.

As the authors described an algorithm applicable for a wide class of linear and nonlinear problems, the collocation method together with B-spline approximation represent an economical alternative since it only requires computing the unknown parameters of the grid. This scheme has convergence of the second order.

Model of evolution of the bacteria population on the basis of the equations of Fisher – Kolmogorov was considered in [20] .

For a one-dimensional type equation of the Fisher – Kolmogorov with quadratic nonlinear nonlocal kinetics and weak diffusion developed the overall design of the semiclassical asymptotic solutions concentrated on the basis of complex method of VKB – Maslov. A solution of the Cauchy problem in the class of semiclassical concentrated functions. Significant role in building solutions plays the

dynamic system equations of the Einstein – Ehrenfest (the system of equations for medium and centered moments). Found the symmetry operators of equations, nonlinear evolution operator and the class of private semiclassical asymptotic solutions.

In [21] developed a formalism of semiclassical asymptotics for a one-dimensional type equation of the Fisher – Kolmogorov with a nonlocal nonlinearity. Considered in the equation of FC generalizes the model of [22]: equation supplemented with a nonlocal drift term, able to account for the motion of the medium, which is a bacterial culture, in particular the flow of the substrate. Developed in the formalism extends the use of the model and allows to solve several interesting problems of population dynamics.

In [23] numerically investigated the influence of nonlocal effects on the population dynamics of microorganisms in the framework of the diffusion model with quadratic nonlinear nonlocal interaction. The basic equation of the model generalizes the known equation of Fisher-Kolmogorov-Petrovsky-Piskunov. To describe the nonlocal interaction kernel used in the Gaussian and uniform distributions. It is shown that in both cases, formed local maxima of the population, indicating the formation of population structure.

In the first case the characteristic is the consistent occurrence of local maxima in the distribution center and their subsequent division into two local maximum, moving in opposite directions. In the second case there is a periodic spatial structure with identical shape and parameters of the peaks. This dynamics indicates the formation process of population aggregation. Similar results were obtained in [24] for periodic boundary conditions and influence functions. These patterns and features related to the spatial heterogeneity of limiting growth factors (such as competition for substrates, products of metabolism), with the interaction of different types or groups of cells in the growth process.

In [25] for the equation was obtained by numerical solution of reaction-diffusion equations with a nonlocal nonlinearity, describing the formation of dissipative structures. Considered structures arising from initial distributions with one and several centers of localization . If you change the parameters of the equation solutions describe the formation of an expanding ring structures. The peculiarities of formation and interaction of the expanding ring-shaped structures depending on the nature of nonlocal interaction.

Development of self-similar solutions to the problem of biological population of Kolmogorov-Fisher in the regime with aggravation.

Regimes with aggravation in the spatial localization in open dissipative systems described by models with nonlinear diffusion. In [11] discusses self-similar solutions of the nonlinear heat conduction equation with volumetric heat source and a density dependent on the radius of the power law in planar, cylindrical, and spherical geometry. Self-similar solutions are investigated in the regime with aggravation and represent non-stationary dissipative structures. Discusses the range of functions of self-similar tasks, depending on

parameter values. The analysis of self-similar solutions, which allows you to set the number of eigenfunctions, their existence in the parameter and their evolution with parameter variation. We study the stability of self-similar solutions. It is shown that it depends on the values of the parameters, from a private function and its parity. Found a new structurally stable self-similar solution. Discusses self-similar solutions of the nonlinear heat conduction equation with volumetric heat source and a density dependent on the radius of the power law in planar, cylindrical and spherical geometry [11].

In the development of these works in the proposed thesis work is seen in the field Q={(t,x): 0< t < ∞, x∈R} parabolic system of two quasilinear equations of the reaction-diffusion problem of biological population of Kolmogorov-Fisher type

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{11}u_1^m + a_{12}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{11}u_1^m + b_{12}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_1(t)u_1(1-u_2^{\beta_1}) \\[2mm] \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{21}u_1^m + a_{22}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{21}u_1^m + b_{22}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_2(t)u_2(1-u_1^{\beta_2}) \end{cases} \quad (1)$$

a_{ij}, b_{ij} - positive real numbers, $\beta_1, \beta_2 \geq 0$, $u_1 = u_1(t,x) \geq 0$, $u_2 = u_2(t,x) \geq 0$ - desired solution.

At $a_{ij} \neq 0$, $b_{ij} = 0$ or $a_{ij} = 0$, $b_{ij} \neq 0$ mathematical model (1) is a system of the type reaction-diffusion with diffusion coefficients $a_{ij}u_i^m \geq 0$, $b_{ij}u_i^m \geq 0$. In the case where at least one of the coefficients and sign (can be any), the system (1) is the cross-diffusion (mutual diffusion for i,j=1,2).

There are three types of self-similar regimes with aggravation: HS, S, and LS [11-15].

At $0 < \beta_i < m_i$, $i = 1,2$ implements HS−mode. Studies have shown that self-similar task in this case is the only own function, monotonically decreasing on the interval with the maximum at the center of symmetry [11].

At $\beta_i = m_i$, $i = 1,2$ takes place S−mode. Self-similar solution is a non-stationary dissipative localized structure [11].

Self-similar solution in LS−mode, is a non-stationary dissipative structure; all points of which move to the center of symmetry, the solution at $T = -\tau$ turns into infinity in a single point – the center of symmetry [11]. Self-similar solutions can exist when $\beta_i > m_i$, $i = 1,2$.

System (1) has the approximate solution

$$\overline{f}_1 = A(a - b\xi^2)_+^{\eta_1}, \quad \overline{f}_2 = B(a - b\xi^2)_+^{\eta_2} \quad (y)_+ = \max(0, y)$$
.

Study of qualitative properties of the system (1) has allowed to perform numerical experiment depending on the values included in the numeric parameters. For this purpose, as the initial approximation was used constructed asymptotic solutions. The numerical solution of the problem for the

linearization of the system (1) were used linearization methods of Newton and Picard. To build self-similar system of equations for biological populations, we used the method of nonlinear splitting [4].

3. Numerical experiment. Based on qualitative research conducted numerical experiments.

Below are the algorithm numerical experiments.

Construct uniform mesh in t and x

$$\overline{\omega_{\tau h}} = \{t_j = j\tau, j = 0,1,...,m, \tau m$$

$$= T; x_i = a + ih, i = 0,1,...,n, h = \frac{b-a}{n}\}$$

and approximate task (1)

$$
\begin{cases}
\dfrac{y_i^{j+1} - y_i^j}{\tau} = \dfrac{1}{h}\left(a_{i+1}\dfrac{y_{i+1}^{j+1} - y_i^{j+1}}{h} - a_i\dfrac{y_i^{j+1} - y_{i-1}^{j+1}}{h}\right) + \\[4mm]
+ l_i^{j+1}\dfrac{y_{i+1}^{j+1} - y_{i-1}^{j+1}}{2h} + k_{1i}^{j+1}y_i^{j+1}\left(1 - \left(w_i^j\right)^{\beta_1}\right) \\[4mm]
\dfrac{w_i^{j+1} - w_i^j}{\tau} = \dfrac{1}{h}\left(b_{i+1}\dfrac{w_{i+1}^{j+1} - w_i^{j+1}}{h} - b_i\dfrac{w_i^{j+1} - w_{i-1}^{j+1}}{h}\right) + \\[4mm]
+ l_i^{j+1}\dfrac{w_{i+1}^{j+1} - w_{i-1}^{j+1}}{2h} + k_{2i}^{j+1}w_i^{j+1}\left(1 - \left(y_i^{j+1}\right)^{\beta_2}\right)
\end{cases}
$$

,

System diagrams nonlinear relative function y^{j+1} and w^{j+1}. For finding its solution using a method of iterations. The iterative process build as follows:

$$
\begin{cases}
\dfrac{\overset{s+1,j+1}{y_i} - y_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{a}_{i+1}\dfrac{\overset{s+1,j+1}{y_{i+1}} - \overset{s+1,j+1}{y_i}}{h} - \overset{s}{a}_i\dfrac{\overset{s+1,j+1}{y_i} - \overset{s+1,j+1}{y_{i-1}}}{h}\right) + l_i^{j+1}\dfrac{\overset{s+1,j+1}{y_{i+1}} - \overset{s+1,j+1}{y_{i-1}}}{2h} + \\[4mm]
+ k_{1i}^{j+1}\overset{s+1,j+1}{y_i}\left(1 - \left(w_i^j\right)^{\beta_1}\right) \\[4mm]
\dfrac{\overset{s+1,j+1}{w_i} - w_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{b}_{i+1}\dfrac{\overset{s+1,j+1}{w_{i+1}} - \overset{s+1,j+1}{w_i}}{h} - \overset{s}{b}_i\dfrac{\overset{s+1,j+1}{w_i} - \overset{s+1,j+1}{w_{i-1}}}{h}\right) + l_i^{j+1}\dfrac{\overset{s+1,j+1}{w_{i+1}} - \overset{s+1,j+1}{w_{i-1}}}{2h} + \\[4mm]
+ k_{2i}^{j+1}\overset{s+1,j+1}{w_i}\left(1 - \left(y_i^{j+1}\right)^{\beta_2}\right)
\end{cases}
\tag{2}
$$

Regarding the function $\overset{(s+1)j+1}{y}$ and $\overset{(s+1)j+1}{w}$ difference scheme (2) is linear. As the initial iteration taken from the function y and w previous time step: $\overset{(0)j+1}{y} = y^j$ and $\overset{(0)j+1}{w} = w^j$. For convergence of the iteration require the conditions

$$\max_i \left| \overset{(s+1)}{y_i} - \overset{(s)}{y_i} \right| \le \varepsilon \quad \text{и} \quad \max_i \left| \overset{(s+1)}{w_i} - \overset{(s)}{w_i} \right| \le \varepsilon.$$

In the development of work [1] proposed in the thesis to find the number of eigenfunctions of self-similar tasks in LS − mode, the analysis of solutions (1).

Analysis showed that the eigenfunction with the number $j = 2,3,4,...$ exists in the interval $m_i < \beta_i < \hat{\beta}_{ij}$, where

$$\hat{\beta}_{ij} = m_i + m_i /(j-1), \ i = 1,2, \ j = 2,3,4,....$$

At $\beta_i > \hat{\beta}_{i2} = 2m_i$, $i = 1,2$ self-similar task in LS mode has only one eigenfunction.

If the number of the eigenfunction is greater, then the interval over the parameter in which it exists β_i.

Number of eigenfunctions of the self-similar task for the given β_i and m_i defined by formula:

$$N_i = \begin{cases} [a_i], if \ a \ non-integer, \\ a_i - 1, \ if \ a \ integer. \end{cases}$$

where $a_i = \beta_i /(\beta_i - m_i)$, $i = 1,2$.

Right boundary, which are determined by the sequence of points in the region of existence of eigenfunctions is an infinite sequence converging to a point which is a common left border of the interval of existence of all native functions in LS–mode (Fig.1).

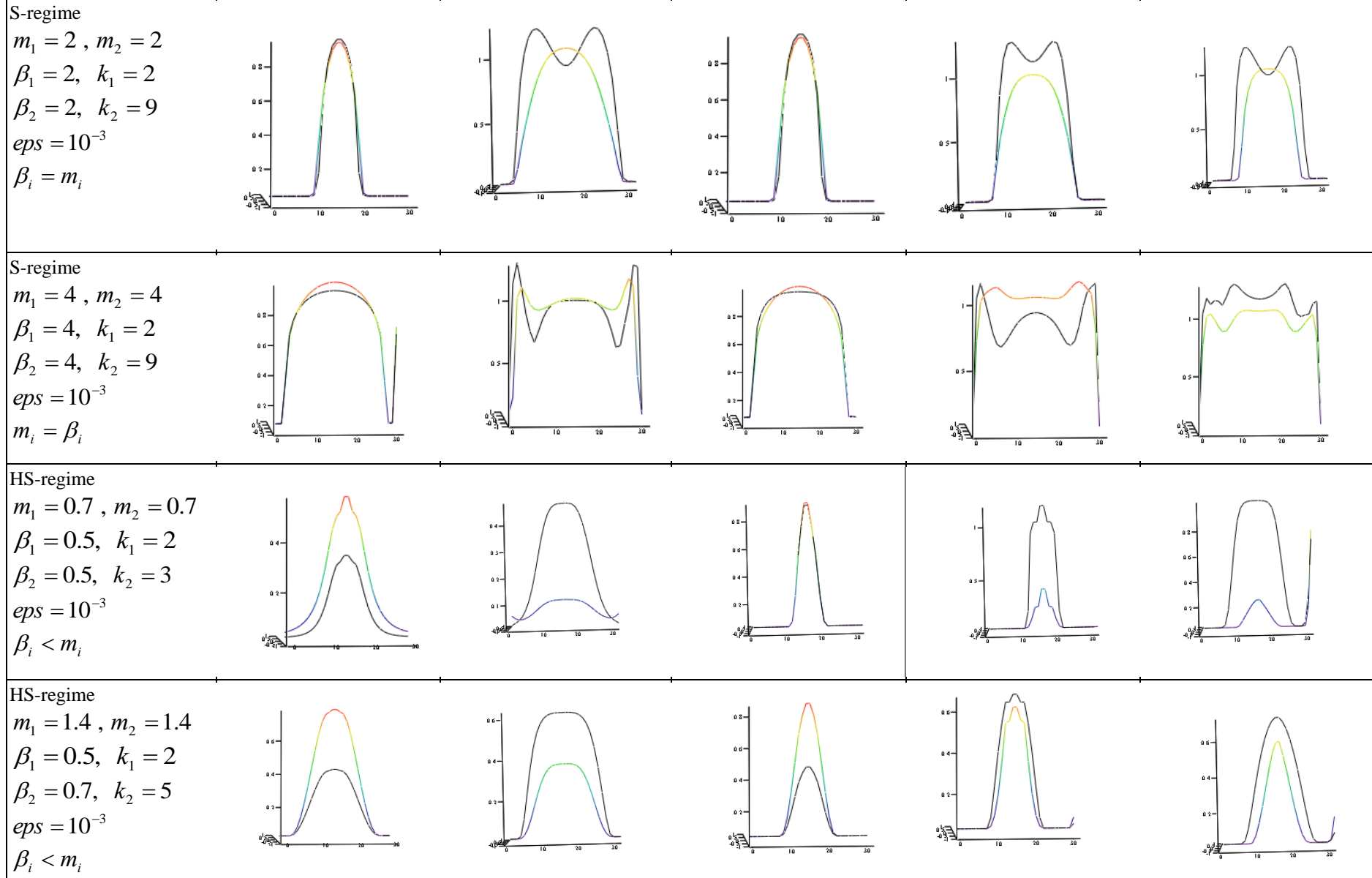| Parameter values	$t = 1$ $a_{11}=1; a_{12}=1; b_{11}=1; b_{12}=1$ $a_{21}=1; a_{22}=1; b_{21}=1; b_{22}=1$	$t = 40$ $a_{11}=1; a_{12}=1; b_{11}=1; b_{12}=1$ $a_{21}=1; a_{22}=1; b_{21}=1; b_{22}=1$	$t = 1$ $a_{11}=0; a_{12}=1; b_{11}=0; b_{12}=0$ $a_{21}=0; a_{22}=0; b_{21}=1; b_{22}=0$	$t = 40$ $a_{11}=0; a_{12}=1; b_{11}=0; b_{12}=0$ $a_{21}=0; a_{22}=0; b_{21}=1; b_{22}=0$	$t = 40$ $a_{11}=0; a_{12}=0; b_{11}=0; b_{12}=1$ $a_{21}=1; a_{22}=0; b_{21}=0; b_{22}=0$
LS-regime $m_1 = 2$, $m_2 = 2$ $\beta_1 = 4$, $k_1 = 2$ $\beta_2 = 4$, $k_2 = 9$ $eps = 10^{-3}$ $m_i < \beta_i < \hat{\beta}_{ij}$ $\hat{\beta}_{ij} = m_i + m_i/(j-1)$, $j = 2$					
LS-regime $m_1 = 2$, $m_2 = 2$ $\beta_1 = 3$, $k_1 = 2$ $\beta_2 = 3$, $k_2 = 9$ $eps = 10^{-3}$ $m_i < \beta_i < \hat{\beta}_{ij}$ $\hat{\beta}_{ij} = m_i + m_i/(j-1)$, $j = 3$					
LS-regime $m_1 = 2$, $m_2 = 2$ $\beta_1 = 2.6$, $k_1 = 2$ $\beta_2 = 2.6$, $k_2 = 9$ $eps = 10^{-3}$ $m_i < \beta_i < \hat{\beta}_{ij}$ $\hat{\beta}_{ij} = m_i + m_i/(j-1)$, $j = 4$					

S-regime $m_1 = 2$, $m_2 = 2$ $\beta_1 = 2$, $k_1 = 2$ $\beta_2 = 2$, $k_2 = 9$ $eps = 10^{-3}$ $\beta_i = m_i$	
S-regime $m_1 = 4$, $m_2 = 4$ $\beta_1 = 4$, $k_1 = 2$ $\beta_2 = 4$, $k_2 = 9$ $eps = 10^{-3}$ $m_i = \beta_i$	
HS-regime $m_1 = 0.7$, $m_2 = 0.7$ $\beta_1 = 0.5$, $k_1 = 2$ $\beta_2 = 0.5$, $k_2 = 3$ $eps = 10^{-3}$ $\beta_i < m_i$	
HS-regime $m_1 = 1.4$, $m_2 = 1.4$ $\beta_1 = 0.5$, $k_1 = 2$ $\beta_2 = 0.7$, $k_2 = 5$ $eps = 10^{-3}$ $\beta_i < m_i$	

Figure 1. Results of the computational experiment

CONCLUSION

Value $\beta_i = m_i$ and $\beta_i = \hat{\beta}_{ij}$ are the points at which ceases to be a private function. The first private function exists with any value $\beta_i > m_i, \quad i = 1,2$. At $\beta_i > \hat{\beta}_{i2} = 2m_i$, $i = 1,2$ self-similar task in LS mode can have only one native function.

Developed numerical schemes, algorithms and program complex allow to carry out computer simulation for the study of reaction-diffusion biological populations on the basis of qualitative properties of nonlinear mathematical models and determines the appearance of dissipative structures.

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