

# Investigating the Non-Uniform Boundary Conditions Effects on MHD Free and Mass Convection along a Semi-Infinite Inclined Flat Plate Solar Captor Subjected to Chemical Reaction, Radiation Heat Flux and Internal Heat Generation or Absorption

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## Abstract

The convergence results between physical and industrial applications for hydro-magnetic free convection and mass transfer studies along a semi-infinite flat horizontal or inclined plate require analysis of non-uniform conditions at the wall (temperature, concentration) and presence of several parameters such as magnetic field, chemical reaction, radiation heat flux, internal heat generation or absorption, without neglecting the static pressure difference induced by the buoyancy force. Non local similarity transformation of boundary layers equations which used enable us to transform the partial differential equations to no-linear ordinary differential equations that are solved numerically by finite difference method expressed in commode forms. The impact of these parameters on the velocity, temperature, and concentration is explained in graphs. The main results are that the non uniform boundary conditions of the temperature and concentration affect substantially the heat and mass rates of the exchange.

**Keywords:** Non-uniform boundary, Similarity, Inclined semi-infinite plate, MHD, natural convection, heat and mass transfer.

## INTRODUCTION

Multiple processes used in industry are very often the seat exchanges of heat and mass transfer by natural convection over a semi-infinite horizontal or inclined flat plate, in which the variety of the studies in this field are explained. The domains of applications of this phenomenon are very vast such-that: exchangers, petroleum reservoir, cooling of nuclear reactor.

In the development of renewable energy, some devices as inclined flat plate or collector solar work under radiation flux and other various conditions. Among these conditions, the uniform temperature or concentration do not reflected the reality. It is now interesting to adjust to the observed operating systems. Moreover, the presence of a magnetic field is more viewed as a monitoring tool and should incorporate in any solar system to control the process.

Among the first initiatives of research in the domain of heat and mass' transfer along a vertical and inclined plate we find Gebhart and Pera [1], Chen et al. [2] who have accepted the uniform wall conditions.

The industrial developments imposed an improvement of studies related to the MHD heat and mass transfer (solar physics, meteorology...), Ferraro and Plumpton [3], Cramer and Pai [4] among the firsts who made research on it. Research was improved by including the effects of electrically conducting fluid, which explains many publications such as Michiyoshi et al. [5], Gray [6]. An examination of laminar heat and mass transfer over a semi-infinite horizontal plate with the existence of chemical reaction was done by Anjalidevi and Kandasamy [7].

Chamka and Khaled [8] analyzed the phenomenon of hydro-magnetic heat and mass transfer by free convection of electrically conducting around an inclined plate adding heat source to which temperature and space are related, the boundary conditions of the temperature and concentration are in function of space.

The MHD free convection and mass transfer equations past an infinite vertical porous plate taking into consideration the combined effect of heat source and thermal diffusion were solved by Singh et al. [9] using similarity method. A research of free heat and mass transfer over a permeable inclined surface with variable wall temperature and concentration was carried out by Chen [10].

Phenomenon of unsteady natural convection flow of a viscous incompressible electrically conducting fluid over an inclined plate with variable heat and mass flux' was the subject of Ganesan and Palani's study [11].

Afify [12] performed an investigation about the effect of chemical reaction on magnetic free heat and mass transfer of a viscous, incompressible and electrically conducting fluid around a stretching sheet.

In the presence of a uniform magnetic field, Alam et al. [13] published their study concerning the Hall effects on the steady MHD free-convective flow and mass transfer over an inclined stretching sheet. Ali et al. [14] worked on the MHD free convection and mass transfer flow past an inclined semi-infinite plat with the existence of heat generation with uniform boundary conditions of temperature and concentration.

The impact of magnetic field and chemical reaction on unsteady combined heat and mass transfer by free convection along an impulsively started semi- infinite vertical plate was the main aim of Al-Odat and Al-Azab [15] using the numerical method . Abd el-aziz and Salem [16]

aimed the influences of coupled natural convection and mass transfer with chemical reaction and electrically conducting fluid past a vertical and linearly stretched permeable sheet with variable surface temperature and mass flux. The effects of chemical reaction, heat source, radiation absorption and mass diffusion's parameters of heat generating fluid past a vertical porous plate with variable suction have been studied by Ibrahim et al. [17]. Shateyi [18] examined the buoyancy and radiation effects on heat and mass transfer around a semi-infinite stretching surface.

Chamkha et al. [19] developed the previous researches by including the effects of heat generation or absorption, thermal radiation, and chemical reaction on unstable free convective heat and mass transfer along an infinite vertical porous plate in the presence of a transverse magnetic field and Hall current.

Unsteady free convection and mass transfer through an infinite vertical plate with exponentially accelerated in the presence of magnetic field and thermal radiation was treated by Rajesh et al. [20]. Rajeswari et al. [21] studie's were based on the chemical reaction effect on the magnetic free -forced convection and mass transfer over a semi-infinite vertical porous plate with electrically conducting viscous incompressible fluid. Bég et al. [22] achieved the study of natural convection and mass transfer of a Newtonian fluid along vertical and inclined plates with the existence of chemical reaction parameter. Seddeek [23] performed his study on steady free convection heat and mass transfer over a stretching sheet by including variable viscosity on hydro-magnetic without neglecting the effect of radiation and chemical reaction. Ibrahim and Makinde [24] based his studies on what had been done by Makinde [25] and Beg et al. [26], concerning the effects of chemical reaction on a MHD free convection and mass transfer of a continuously moving porous vertical surface using numerically method. Alharbi et al. [27] analyzed the heat and mass transfer of incompressible MHD visco-elastic fluid embedded in a porous medium over a stretching sheet under a chemical reaction. Numerical solution of steady MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface in the presence of heat generation and a porous medium was carried out by Reddy and Reddy [28]. Jashim Uddin et al. [29] used the thermal convective boundary condition in the study of free convection and mass transfer along a moving vertical flat plate.

In the current document, our attention is focused on the collection of various parameters such as magnetic field, chemical reaction, heat flux radiation, and internal heat generation or absorption in the study of MHD free and mass transfer along a semi-infinite flat horizontal or inclined plate, taking into account the variation of the temperature and concentration in the wall and the presence of the static pressure difference induced by the buoyancy forces. In order to unveil the impact of parameters previously mentioned on the velocity, temperature, and concentration.

## MATHEMETICAL ANALYSIS

Consider unsteady hydro-magnetic free convection and mass transfer flow of viscous incompressible fluid with heat generation/absorption, chemical reaction, and radiative heat flux along a semi-infinite flat plate inclined from the horizontal with an acute angle  $\gamma$ ; admittedly a linear variation of surface temperature and concentration  $T_w$  and  $C_w$ , radiative heat flux value following  $x$  direction is very small with that in the  $y$  direction what explains the negligence of this value, A uniform magnetic field of strength  $B_0$  is imposed along the  $y$  axis, Boussinesq approximation is retained. Boundary layers equations can be write according to these assumptions:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g\beta(T - T_\infty) \sin \gamma + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g\beta^*(C - C_\infty) \sin \gamma \quad (2)$$

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g\beta(T - T_\infty) \cos \gamma \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} u^2 + \frac{v}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q_0}{\rho C_p} (T - T_\infty) - \frac{1}{\rho C_p} \left( \frac{\partial q_r}{\partial y} \right) \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_r (C - C_\infty) \quad (5)$$

Where  $a$  and  $b$  are constants,  $n$  and  $m$  are exponents.  $u$ ,  $v$ ,  $T$  and  $C$  are velocity component in  $x$  direction, velocity component in  $y$  direction, temperature and concentration respectively.  $g$  is the acceleration due to gravity,  $T_w$  and  $C_w$  are the wall temperature and concentration respectively,  $T_\infty$  and  $C_\infty$  are the temperature and concentration of the uniform flow respectively,  $\alpha$  is thermal conductivity,  $\nu$  is the kinematic viscosity,  $C_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity of the fluid,  $\rho$  is density of the ambient fluid,  $\sigma$  is the electrical conductivity,  $P$  is the static pressure difference induced by the buoyancy force,  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of thermal expansion with concentration,  $B_0$  is the externally imposed magnetic field in the  $y$  direction,  $Q_0$  is heat generation or absorption constant,  $q_r$  is the component of radiative heat flux,  $D$  is the molecular diffusivity,  $K_r$  is chemical reaction parameter.

Combination of continuity and moment's equations (2) and (3) gives:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta \cos \gamma \frac{\partial}{\partial x} \int_y^\infty (T - T_\infty) dy + v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \sin \gamma - \frac{\sigma B_0^2}{\rho} u + g\beta^*(C - C_\infty) \sin \gamma \quad (6)$$

Roseland's approximation about the radiative heat flux  $q_r$  is mentioned below:

$$q_r = -\frac{4\sigma^* \partial T^4}{3K^* \partial y} \quad (7)$$

Where  $\sigma^*$  and  $K^*$  are the Stefan-Boltzman constant and the Roseland mean absorption coefficient respectively. To get  $T^4$  we apply Taylor series without taking into consideration upper order terms:

$$T^4 \approx 4T_\infty T - 3T_\infty^4 \quad (8)$$

Using the above approximations (7) and (8), the energy equation (4) becomes:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p} u^2 + \frac{v}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{1}{\rho C_p} \left( \frac{16T_\infty \sigma^* \partial^2 T}{3K^* \partial y^2} \right) \quad (9)$$

The boundary conditions are defined as follow:

$$\begin{aligned} \text{At } y = 0: u = v = 0, T = T_w = T_\infty + ax^n, \text{ and} \\ C = C_w = C_\infty + bx^m \end{aligned} \quad (10)$$

For  $y \rightarrow \infty: T \rightarrow T_\infty$  and  $C \rightarrow C_\infty$

The system of Eq. (5), Eq. (6), (9) and Eq. (10) will be transformed from  $(x, y)$  coordinates to the dimensionless coordinate's  $\xi(x), \eta(x, y)$ , by introducing the following transformations used by Chen [30] for  $0^\circ \leq \gamma < 90^\circ$ :

$$\xi = \xi(x), \eta = \frac{y}{\xi(x)} = \frac{y}{x} \left( \frac{Gr_x \cos \gamma}{5} \right)^{1/5} \quad (11)$$

Where,  $\xi$  depending only on  $x$ , is the non-similar parameter and  $\eta$  is a pseudo-similarity variable. For a similar boundary layer,  $\xi = 0$  and  $\eta$  reduces to a true similarity variable. One also introduces a reduced stream function  $f(\xi, \eta)$ , a dimensionless temperature  $\theta(\xi, \eta)$ , and a dimensionless concentration  $H(\xi, \eta)$  defined, respectively, by:

$$f(\xi, \eta) = \frac{\psi(x, y)}{5v(Gr_x \cos \gamma/5)^{1/5}}; \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}; H(\xi, \eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (12)$$

Here,  $Gr_x$  is the local Grashof number and  $\psi$  is the stream function which satisfies the continuity equation and is related to the velocity components in the usual way as

$u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$ . Using new variables  $F, L$  and  $E$  such as:

$$F = \frac{\partial f}{\partial \xi}; L = \frac{\partial \theta}{\partial \xi}; E = \frac{\partial H}{\partial \xi}$$

The equations (1-5) are transformed and become:

$$\begin{aligned} f'''' + (3+n)ff'''' - (3n-1)f'f'' + \left[ \frac{(2-n)}{5}\eta + \xi \right] \theta' - \\ n\theta - \frac{(n+3)}{5}\xi L - Mf''\xi^{(4-2n)/(3+n)} + \xi NH' - \\ (n+3)\xi[f'F'' - f''F] = 0 \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{1}{Pr}(1+R)\theta'' + (n+3)f\theta' - 5nf'\theta - \\ (n+3)\xi(f'L - \theta'F) + ME_c f'^2 \xi^{(4-2n)/(3+n)} + \\ E_c f''^2 + S\theta \xi^{(4-2n)/(3+n)} = 0 \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{1}{Sc}H'' + (3+n)fH' - 5mf'H + \\ (n+3)\xi(H'F - f'E) - JH = 0 \end{aligned} \quad (15)$$

In the second step, we derivate the equations (13-15), with respect to  $\xi$ . Neglecting the derivatives terms related to  $\xi$  of the new variables  $F, L$  and  $E$  which are minute, additional equations are then derived and we get:

$$\begin{aligned} F'''' + (3+n)(Ff'''' + fF'''' + \theta' - (3n-1)(F'f'' + \\ f'F'') - \frac{(6n+3)}{5}L + \left[ \frac{(2-n)}{5}\eta + \xi \right] L' + N(\xi E' + H') - \\ (n+3)[(f'F'' - f''F) + \xi(F'F'' - F''F)] - \\ MF''\xi^{(4-2n)/(3+n)} - Mf''\frac{(4-2n)}{(3+n)}\xi^{(1-3n)/(3+n)} = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{1}{Pr}(1+R)L'' + (n+3)(F\theta' + fL') - 5n(F'\theta + f'L) - \\ (n+3)\xi(f'L - \theta'F) - (n+3)\xi(F'L - L'F) + 2E_c F''f'' + \\ ME_c f'^2 \frac{(4-2n)}{(3+n)}\xi^{(1-3n)/(3+n)} + 2ME_c f'F'\xi^{(4-2n)/(3+n)} + \\ SL\xi^{(4-2n)/(3+n)} + S\theta \frac{(4-2n)}{(3+n)}\xi^{(1-3n)/(3+n)} = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{1}{Sc}E'' + (3+n)(FH' + fE') - 5m(F'H + f'E) - JE + \\ (3+n)[(H'F - f'E) + \xi(E'F - F'E)] = 0 \end{aligned} \quad (18)$$

Where

$M = \sigma B_0^2 (\tan \gamma)^2 / \mu \alpha_1^{10/(3+n)}$ : Magnetic parameter with

$$\alpha_1 = \left[ \frac{ag\beta}{v^2} (\cos \gamma/5) \right]^{1/5} \tan \gamma$$

$$Ec = \left[ \frac{5\nu}{x} (Gr_x \cos \gamma / 5)^{2/5} \right]^2 / C_p (T_w - T_\infty): \text{ Eckert number}$$

$Sc = \frac{\nu}{D}$ : Schmidt number;  $S = Q_0 (\tan \gamma)^2 / \mu C_p \alpha_1^{10/(3+n)}$ : heat generation or absorption parameter,  $R = 16\sigma^* T_\infty / 3K^* k$ : Thermal radiation parameter,  $J = K_r \rho (\tan \gamma)^2 / \mu \alpha_1^{10/(3+n)}$ : Chemical reaction parameter.  $N = \beta^* (C_w - C_\infty) / \beta (T_w - T_\infty)$ : Buoyancy ratio-indicating the relative importance of species and thermal diffusion,  $Pr = \frac{\mu C_p}{k}$ : Prandtl number, ( )' prime indicates differentiation with respect to  $\eta$ .

Boundary layers become:

$$\begin{aligned} f(\xi, 0) = f'(\xi, 0) = f'(\xi, \infty) = 0 \\ F(\xi, 0) = F'(\xi, 0) = F'(\xi, \infty) = 0 \\ \theta(\xi, 0) = 1, \theta(\xi, \infty) = 0 \\ L(\xi, 0) = L(\xi, \infty) = 0 \\ H(\xi, 0) = 1, H(\xi, \infty) = 0 \\ E(\xi, 0) = E(\xi, \infty) = 0 \end{aligned} \quad (19)$$

The local Nusselt number, the skin-friction coefficient, and the local Sherwood number are important physical parameters. These can be defined as:

$$Nu_x = \frac{hx}{k} = -(Gr_x \cos \gamma / 5)^{3/5} \theta'(\xi, 0) \quad (20)$$

$$\tau_w = \mu (\partial u / \partial y)_{y=0} = \frac{5\nu\mu}{x^2} (Gr_x \cos \gamma / 5)^{3/5} f''(\xi, 0) \quad (21)$$

$$Sh_x = \frac{m_w}{(C_w - C_\infty)} \left( \frac{x}{D} \right) = -(Gr_x \cos \gamma / 5)^{1/5} H'(\xi, 0) \quad (22)$$

With  $m_w = -D(\partial C / \partial y)_{y=0} =$

$-D \frac{(C_w - C_\infty)}{x} (Gr_x \cos \gamma / 5)^{1/5} H'(\xi, 0)$  is the mass flux

Ordinary differential equations (13-15) and (16-18), which are valid for  $0^\circ \leq \varphi < 90^\circ$  subject to the boundary conditions (19) and developed previously by the non-local similarity transformation are non-linear. They must be solved by an efficient numerical method, using finite difference method via Lobatto III approach.

A real congruence was observed in the Fig. 1 through a comparison of results of friction factor and heat transfer rates with the one of Chen et al. [30]. It is demonstrated that the second level approach derived here and used for the non similar problem is appropriate and accurate method.

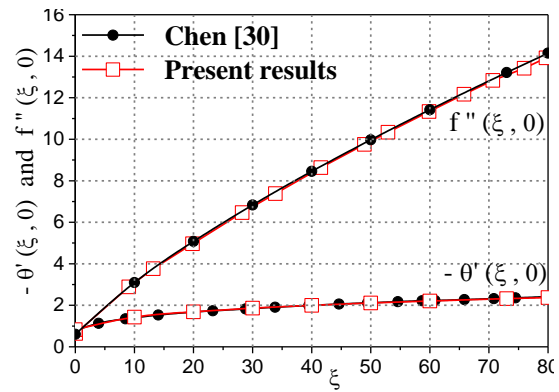
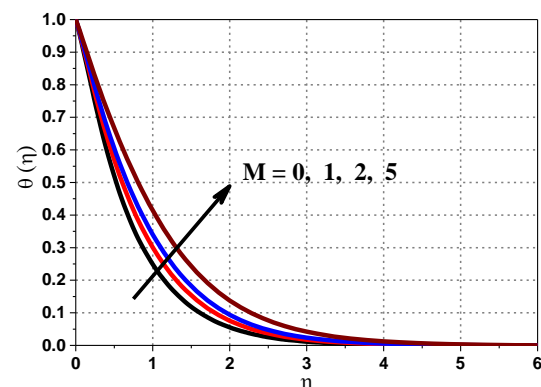
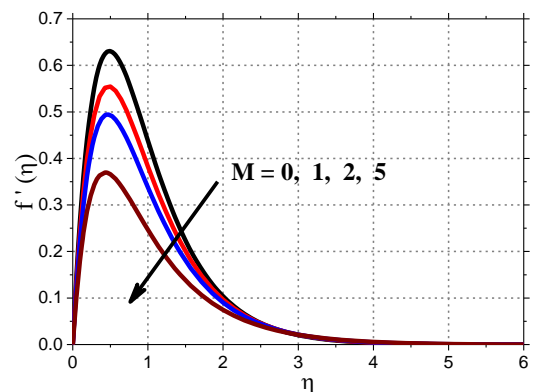


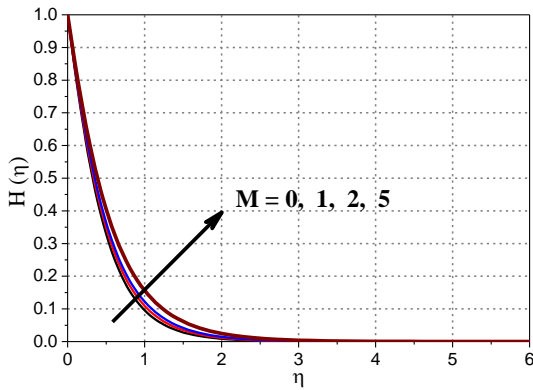
Figure 1: Comparison of  $-\theta'(\xi, 0)$  and  $f''(\xi, 0)$  with those of Chen for  $Pr=0.7, n=1, M=0, N=0, R=0, J=0$ .

## RESULTS AND DISCUSSION

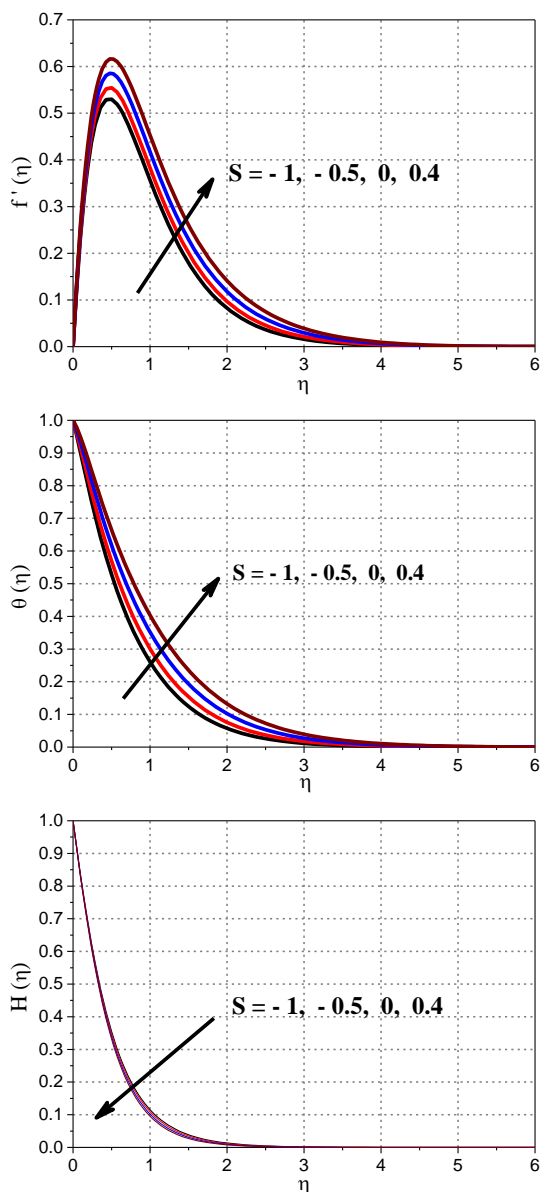
Several figures were presented to explain the impact of some parameters on the velocity, temperature, and concentration. The chosen parameters are: Magnetic parameter  $M$ , Eckert number  $Ec$ , heat generation/absorption parameter  $S$ , thermal radiation parameter  $R$ , Schmidt number  $Sc$ , chemical reaction parameter  $J$ , buoyancy ratio-indicating the relative importance of species and thermal diffusion, exponent in the power-law variation of the wall temperature  $n$ , exponent in the power-law variation of the wall concentration  $m$ , Prandtl number  $Pr$ , and non-similar parameter  $\xi$ .

Physically, the magnetic parameter  $M$  is regarded as a resistance force to fluid flow which causes the slowdown of the fluid's movement in the boundary that is why  $M$  is inversely proportional with velocity but proportional with temperature and concentration, as it is illustrated on Fig. 2





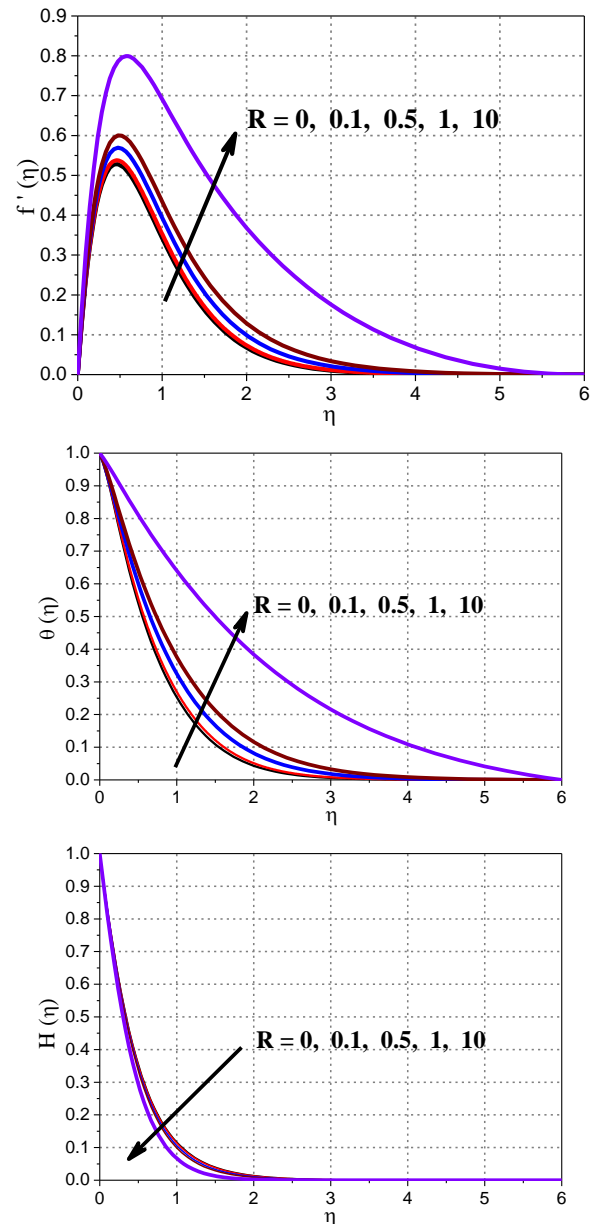
**Figure 2:** Velocity a), temperature b), and concentration c) profiles for various values of  $M$  with  $n=1$ ;  $Pr=0.72$ ;  $\zeta=5$ ;  $Ec=1$ ;  $S=-0.5$ ;  $R=1$ ;  $Sc=0.5$ ;  $m=2$ ;  $J=2$ ;  $N=1$ .



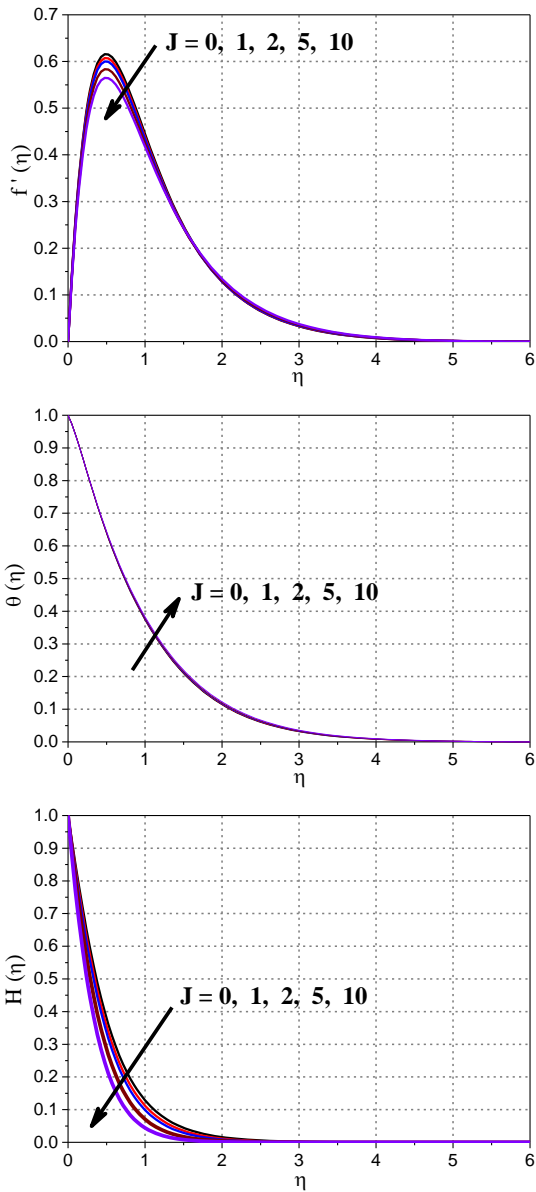
**Figure 3:** Velocity a), temperature b), and concentration c) profiles for various values of  $S$  With  $n=1$ ;  $Pr=0.72$ ;  $\zeta=5$ ;  $M=1$ ;  $Ec=1$ ;  $R=1$ ;  $Sc=0.5$ ;  $m=2$ ;  $J=2$ ;  $N=1$ .

Fig. 3 illustrates the effect of the heat generation/absorption parameter  $S$  on the velocity, temperature, and concentration. The presence of heat generation ( $S > 0$ ) causes the increase of the thermal state of the fluid and the generation of upper thermal boundary layer as well. Contrary, the existence of the heat absorption ( $S < 0$ ) leads to the decrease of the thermal state of the fluid, thus produce lower thermal boundary layers.

Fig. 4 clearly shows the impact of the thermal radiation parameter  $R$  on the velocity, temperature, and concentration. The increase of this parameter  $R$  causes an important augmentation in the thermal condition of the fluid and its thermal boundary layer, as result of which, the velocity and temperature of the fluid increase, however the concentration decreases slightly.

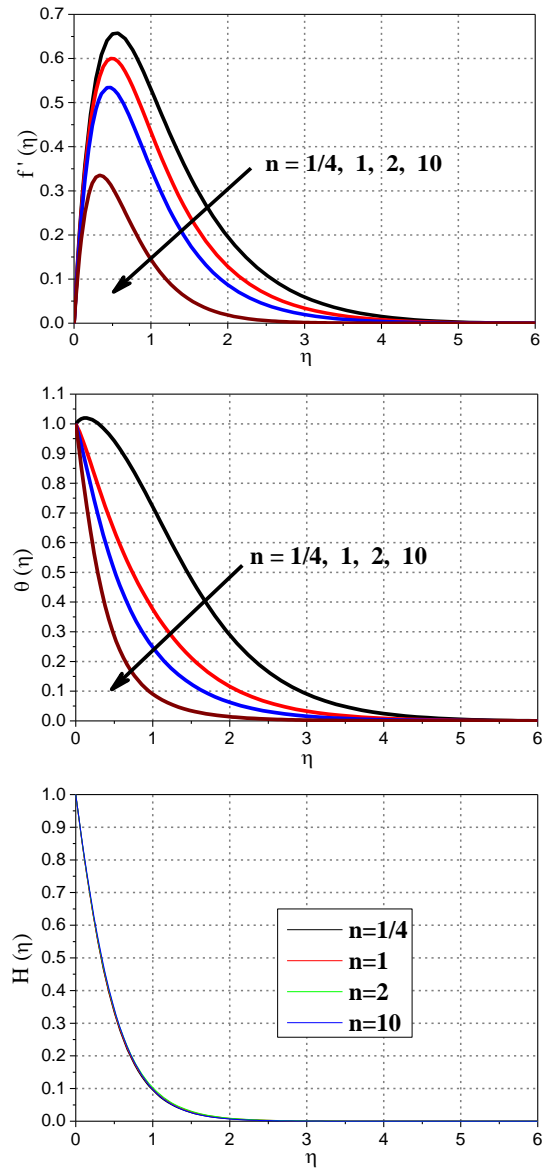


**Figure 4:** Velocity a), temperature b), and concentration c) profiles for various values of  $R$  With  $n=1$ ;  $Pr=0.72$ ;  $\zeta=5$ ;  $M=1$ ;  $Ec=1$ ;  $S=0.2$ ;  $Sc=0.5$ ;  $m=2$ ;  $J=2$ ;  $N=1$ .

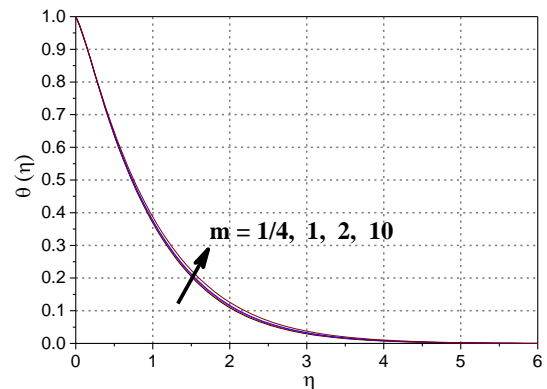


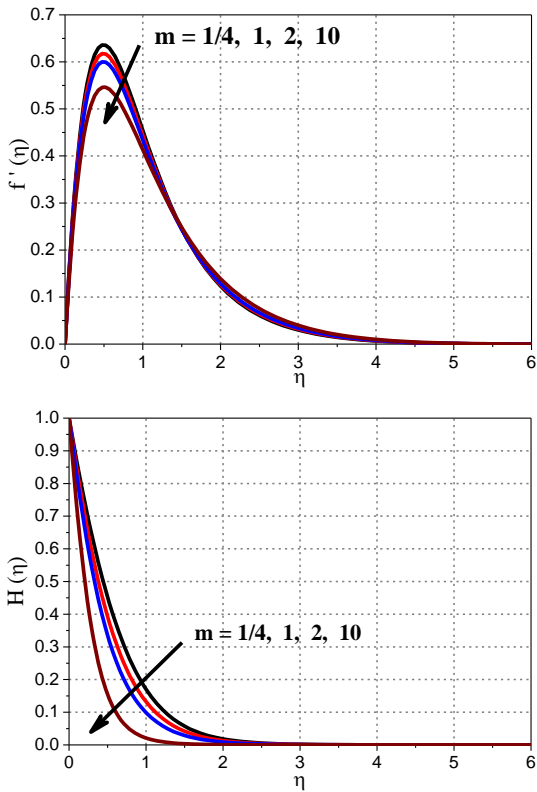
**Figure 5:** Velocity a), temperature b), and concentration c) profiles for various values of  $J$  With  $n=1$ ;  $Pr=0.72$ ;  $\zeta=5$ ;  $M=1$ ;  $Ec=1$ ;  $S=0.2$ ;  $R=1$ ;  $m=2$ ;  $Sc=0.5$ ;  $N=1$ .

Due to the diminution of the boundary layer's thickness caused by the destructive chemical, the chemical reaction parameter  $J$  is inversely proportional with velocity and concentration firstly, and proportional with the temperature secondly, Fig. 5.



**Figure 6:** Velocity a), temperature b), and concentration c) profiles for various values of  $n$  With  $Pr=0.72$ ;  $\zeta=5$ ;  $M=1$ ;  $Ec=1$ ;  $S=0.2$ ;  $R=1$ ;  $m=2$ ;  $Sc=0.5$ ;  $J=2, N=1$



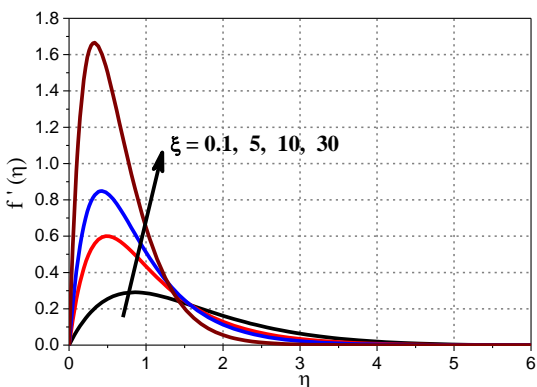


**Figure 7:** Velocity a), temperature b), and concentration c) profiles for various values of  $m$  With  $Pr=0.72$ ;  $\zeta=5$ ;  $M=1$ ;  $Ec=1$ ;  $S=0.2$ ;  $R=1$ ;  $m=2$ ;  $Sc=0.5$ ;  $J=2, N=1$ .

Since we supposed that the temperature at the wall is proportional to exponent  $n$ , so the temperature and velocity decrease when exponent  $n$  goes up, no variation of the concentration is observed as Fig. 6 shows.

The concentration at the wall is relative to exponent  $m$  as we supposed on the boundary layers conditions, what leads to the decrease of the concentration and velocity, the temperature still without important change, Fig. 7.

It is clear from Fig. 8 that all the profiles are different noticeably for the several values of the non similar parameter interpreted as the coordinate along the plate.



**Figure 8:** Velocity a), temperature b), and concentration c) profiles for various values of  $\zeta$  With  $n=1$ ;  $Pr=0.72$ ;  $M=1$ ;  $Ec=1$ ;  $S=0.2$ ;  $R=1$ ;  $m=2$ ;  $Sc=0.5$ ;  $J=2, N=1$ .

Table 1 shows the impact of the parameters  $\zeta$ ,  $M$ ,  $S$ ,  $R$ ,  $Sc$  and in Table 2 the impact of  $\zeta$ ,  $J$ ,  $N$ ,  $n$ ,  $m$ , on the rate of heat transfer coefficient -  $\theta(\zeta, 0)$  and, the skin friction coefficient  $f''(\zeta, 0)$ , and the rate of mass transfer coefficient and  $-H'(\zeta, 0)$ . It is clearly seen that:

- The local Nusselt number  $Nu_x$  increase by the increase of  $Sc$ ,  $J$ ,  $n$ ,  $m$ ,  $Pr$ , and decrease by the increase of  $\zeta$ ,  $M$ ,  $Ec$ ,  $S$ ,  $R$ ,  $N$ .
- The skin friction  $\tau_w$  is proportional to  $\zeta$ ,  $Ec$ ,  $S$ ,  $R$ ,  $N$ ; and inversely proportional to  $M$ ,  $Sc$ ,  $S$ ,  $J$ ,  $n$ ,  $m$ ,  $Pr$

The increase of  $\zeta$ ,  $Ec$ ,  $S$ ,  $R$ ,  $N$  allows an increases of the local Sherwood number  $Sh_x$ , but it decreases by the increase of  $M$ ,  $n$ , and  $Pr$ .

**Table 1.** The effect of parameters M, S and R on  $-\theta'(\xi, 0)$ ,  $f''(\xi, 0)$ , and  $-H'(\xi, 0)$

$\xi$	M	S	R	J	n	m	$-\theta'(\xi, 0)$	$f''(\xi, 0)$	$-H'(\xi, 0)$
<b>0.1</b>	1	0.2	1	2	1	2	0.6339	0.8479	1.3954
							0.6309	1.3115	1.5025
							0.4712	3.1498	1.8206
5	<b>0</b>	0.2	1	2	1	2	0.5769	3.3720	1.8754
							0.4712	3.1498	1.8206
							0.2174	2.5717	1.6617
5	1	<b>-1</b>	1	2	1	2	0.9401	2.9180	1.7607
		<b>0</b>					0.5609	3.1010	1.8084
		<b>0.4</b>					0.3755	3.2040	1.8340
5	1	0.2	<b>0.1</b>	2	1	2	0.5149	2.9634	1.7691
			<b>0.5</b>				0.4952	3.0577	1.7958
			<b>1</b>				0.4712	3.1498	1.8206

**Table 2.** The effect of parameters J, n and m on  $-\theta'(\xi, 0)$ ,  $f''(\xi, 0)$ , and  $-H'(\xi, 0)$

$\xi$	M	S	R	J	n	m	$-\theta'(\xi, 0)$	$f''(\xi, 0)$	$-H'(\xi, 0)$
5	1	0.2	1	<b>1</b>	1	2	0.4672	3.1747	1.6971
				<b>2</b>			0.4712	3.1498	1.8206
				<b>10</b>			0.4926	3.0131	2.6332
5	1	0.2	1	2	<b>1</b>	2	0.4712	3.1498	1.8206
				<b>2</b>			0.9106	3.0306	1.7994
				<b>10</b>			1.9527	2.5965	1.7655
5	1	0.2	1	2	1	<b>1</b>	0.4616	3.2089	1.5890
						<b>2</b>	0.4712	3.1498	1.8206
						<b>10</b>	0.5023	2.9386	2.8376

**CONCLUSION**

Non local similarity transformation of MHD free and heat mass transfer equations over a horizontal or inclined plate associated to chemical reaction, radiation heat flux, internal heat generation or absorption enable us to convert Partial Differential Equations to non linear Ordinary Differential Equations which are solved numerically. We suppose that the temperature and the concentration at the wall are linearly varied by the space.

The below results culminate the study:

- The velocity is inversely proportional with Magnetic field, chemical reaction, exponents n and m, Schmidt and Prandtl numbers, and consequently proportional with Eckert number, heat generation/absorption parameter, thermal radiation parameter, and Buoyancy ratio-indicating.
- The temperature increases whenever Magnetic field, Eckert number, heat generation / absorption parameter, thermal radiation parameter, Schmidt and

Prandtl numbers, chemical reaction, exponent m increase, and decreases when Buoyancy ratio-indicating, exponent n, Prandtl number decrease.

- The concentration is reduced when Eckert number, heat generation/absorption parameter, thermal radiation parameter, Schmidt number, chemical reaction, Buoyancy ratio-indicating, exponent m reduce, and contrary follows the Magnetic field and Prandtl number.

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