

Z-Transform for Deleting Components color Images

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Abstract

The paper discusses the effectiveness Z-Transform for Deleting Components color images .Three cases have been applied with linear filters to determine the importance and how to delete components from a database containing 30 color images and a selected extension.

Three filters were used for the conversion of each case where the results proved that the first case is the best in deleting the components of the color image and with any filter we use. Z-Transform was implemented in the Matlab environment and proved the success of the proposed method.

Keywords: Z-Transform, image Transform techniques, Structural Similarity Index, color Images, Components color Images

INTRODUCTION

Transformation is a very powerful mathematical tool so using it in mathematical treatment of problem is appear in many applications.

The idea of Z-transform back to 1730 when De Moivre introduced the concept of “generating functions” to probability theory [2]. In 1952 Ragazzini and Lotfi Zadeh get up the sampled-data control group at Columbia University.

Z-transform is transformation for discrete data and used to solve problems in discrete systems in a method similar to the use of Laplace transforms for continuous systems and its a generalization of discrete Fourier transform.

Z-transform converts a discrete time signal, which is a sequence of real or complex numbers, into a complex frequency domain representation and used in many areas of applied mathematics as digital signal processing, control theory, economics and some other fields.

Use mathematical transformations and their effect in digital images are to estimate the latent high quality images given the types quality Image is one of the basic more important research areas in the field of image processing and computer vision, the final goal of mathematical transformations are to reconstruct visually regions images suitable for human visual perception and to more interpretability of regions images for computer vision[1,2,3,4]

Paper contains Section 2 Structure Linear Transformation (Z-Transform). Section3 proposed method, Section 4. Results and Discussion and conclusions are given in section 5

STRUCTURE LINEAR TRANSFORMATION (Z-TRANSFORM)

The z-transform of a sequence $x_0, x_1, x_2, \dots, x_n, \dots$ is given by

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n} \text{ , Where } x_n \text{ is defined only } n \geq 0$$

The sequence $x_0, x_1, x_2, \dots, x_n, \dots$ is a function of an integer, however, its z-transform is a function of a complex variable z. The operation of taking the z-transform of the sequence x_n is represented by $Z\{x_n\} = X(z)$.

STRUCTURE LINEAR TRANSFORMATION (Z-Transform)[1,2,3]

In this section we constructor a new linear transformation (z-transform) and we rely it on Laplace transform , Where we took base of the constant number and the characteristics of sine and cos for Laplace transform. In the construction of the first part of the transform, We take the constant number base for Laplace transform Through which we find the value of the center for the new transform. For example.

In the second part of the structure, We find the direction of the angles of the new transform depending on the bases of sine and cos for laplace transform These two rules help to transfer the treatment perfectly without change the original image because it is orthogonal and compact. In the third part of the transform construction, We create the parallels based on linear transformation Laplace , Canny and Frei-Chen transformation Where it helps determine the direction of parallels.

And then using the two proposition:

Proposition 1: Let $M_{m,n}$ the set of all matrices of order $m \times n$ defined on the field F with two algebra in we mean by the known thus combining matrices:

$$M_1 \oplus M_2 = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ a_{12} & \dots & a_{2m} \\ \vdots & \dots & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} \oplus \begin{bmatrix} b_{11} & \dots & b_{1m} \\ b_{21} & \dots & b_{2m} \\ \vdots & \dots & \vdots \\ b_{n1} & \dots & b_{nm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} \oplus b_{11} & \dots & a_{1m} \oplus b_{1m} \\ a_{21} \oplus b_{21} & \dots & a_{2m} \oplus b_{2m} \\ \vdots & \dots & \vdots \\ a_{n1} \oplus b_{n1} & \dots & a_{nm} \oplus b_{nm} \end{bmatrix}$$

And the number of multiplication $r \in F$ and defined the relationship:

$$rM = r \begin{bmatrix} a_{11} & \dots & a_{1m} \\ a_{12} & \dots & a_{2m} \\ \vdots & \dots & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix}$$

Then $(M_{m,n}, \oplus, \otimes)$ be a space of matrices is defined R.

Proposition 2: The set of symmetric matrices $[A_{i,j}] = [A_{j,i}]$ defined on the field R with the processes of collection and multiplication matrices such as the number of known above then consist the sub space of the space of matrices on R[4].

From the structure steps above, It is created a new transform Which has the following properties linear, compact, orthogonal and continuous as well as contains complex value. This in turn gives us excellent results in image processing.

Structural Similarity Index (SSIM) for measuring image quality is index measuring the structural similarity between two images. Its value is calculated by the following equation if two images w1 and w2 at given pixel P:

$$SSIM(P) = \frac{2 * m1(P) * m2(P) + F1}{m1(P)^2 + m2(P)^2 + F1} \times \frac{2 * cov(P) + F2}{s1(P)^2 + s2(P)^2 + F2}$$

where : m1(P) and m2(P): mean value of w1 and w2 , q1(P) and q2(P):standard deviation of w1 and w2 , cov(P): covariance between w1 and w2 , F1 = (K1*L)^2: regularization constant , F2 = (K2*L)^2: regularization constant , K1, K2: regularization parameters must be >0 and L: dynamic range of the pixel values

THE PROPOSED METHOD

Z-transform components act on the sequence along vertical and horizontal directions in a sense with function of 2 complex numbers . It exists in a 4D space so is very Difficult to visualize of a sequence is related to the spectral content of the sequence that is why other processes have been used to determine effectiveness. Delete the color image components as shown in the algorithm.

Figure (1) showed portion of images in which the proposed paper method is applied

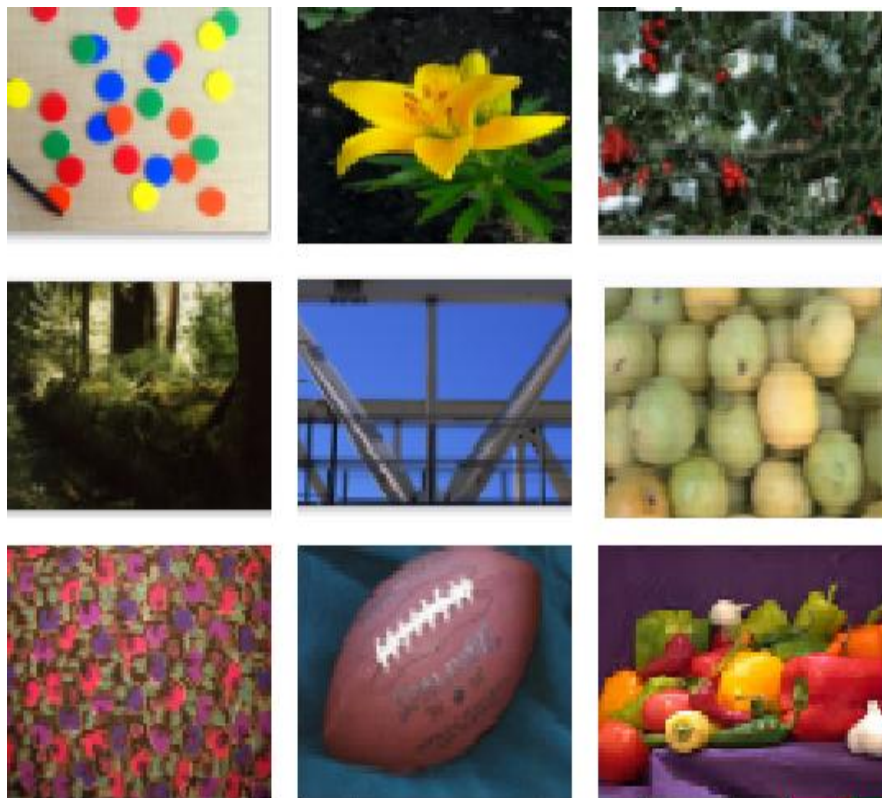


Figure 1: Some of the images in which the proposed method

Algorithm

Step1: Input the Color Image.

Step2: We will have taken set of square matrices 3×3

$$A_1 = \begin{bmatrix} 4 & 3 & 4 \\ 3 & \sqrt{2} & 3 \\ 4 & 3 & 4 \end{bmatrix}, A_2 = \begin{bmatrix} -\sqrt{3} & 1 & -\sqrt{3} \\ 1 & 8 & 1 \\ -\sqrt{3} & 1 & -\sqrt{3} \end{bmatrix}, A_3 = \begin{bmatrix} -1 & 4 & -1 \\ 4 & \sqrt{5} & 4 \\ -1 & 4 & -1 \end{bmatrix}$$

Step3: make set of square matrices 3×3 under the effect of Z-transform properties.

Step 4: multiply original image by square matrices

Step 5: Find edge detection for new square matrices

Step 6: Find all regions for three cases of square matrices under z-transform effect.

Step 7: Convert the resulting images after making the steps to gray scale

Step 8: Find the histogram for each output and determine the spread of the pixels

Step 9: Test the effect for each case used and compared to the effect of less in the deletion of the components of the image and any more effect in the deletion of the components of the image

Step 10: Retrieve all resulting color images into color images and test Structural Similarity Index (SSIM) for measuring image quality

To determine the numerical ratios of the match and determine the effectiveness Z-transform.

Step11: Output and print results

RESULTS AND DISCUSSION

We discuss all the steps of the algorithm in the second section of the proposed method to delete the components of the colored image choose three colored images from the database used first effect as shown in Figure(2).

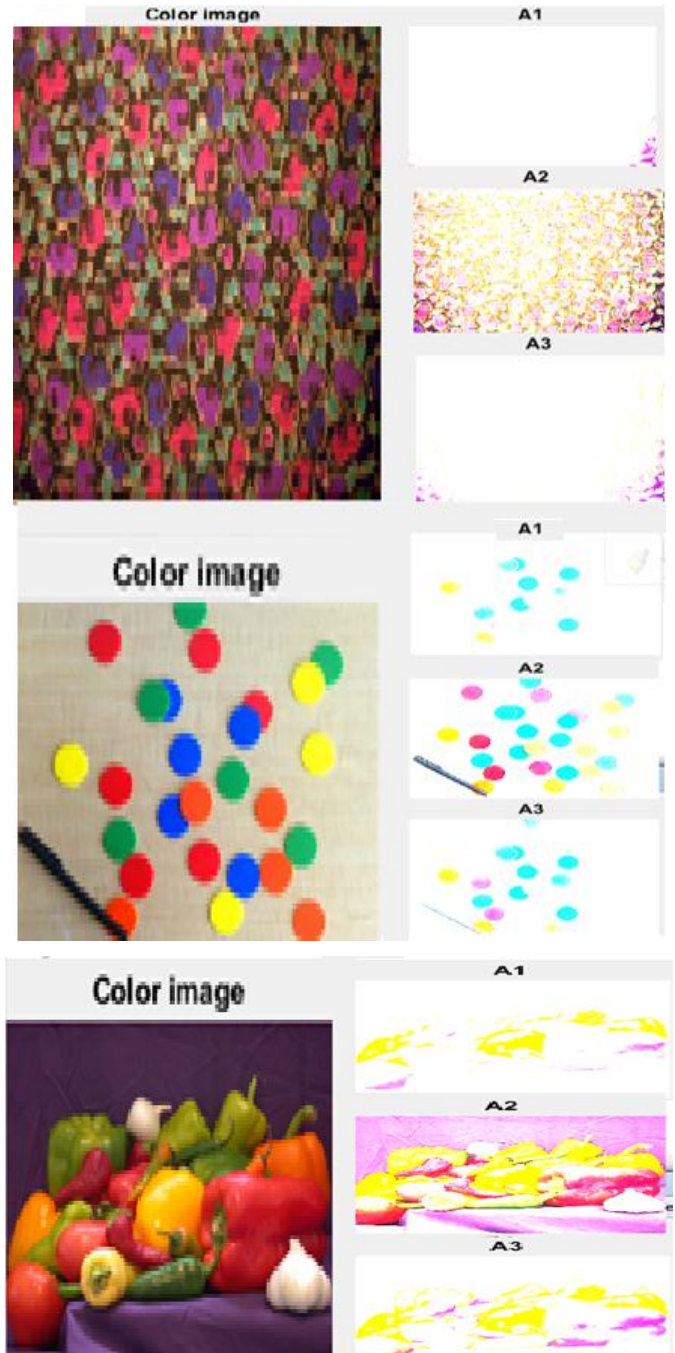


Figure 2: Set of square matrices 3×3 under the effect of Z-transform properties for three color images

Note that the second matrix is the most matrices to embrace the components of the colored image followed by the third matrix, while the first matrix is the most matrices that delete the components of the image according to the laws Z-Transform.

In Figure(3) showed Distribute pixels in the inner and outer areas of the colored image and effect laws Z-Transform In overlapping colors, indicating the lack of clarity of the colored image

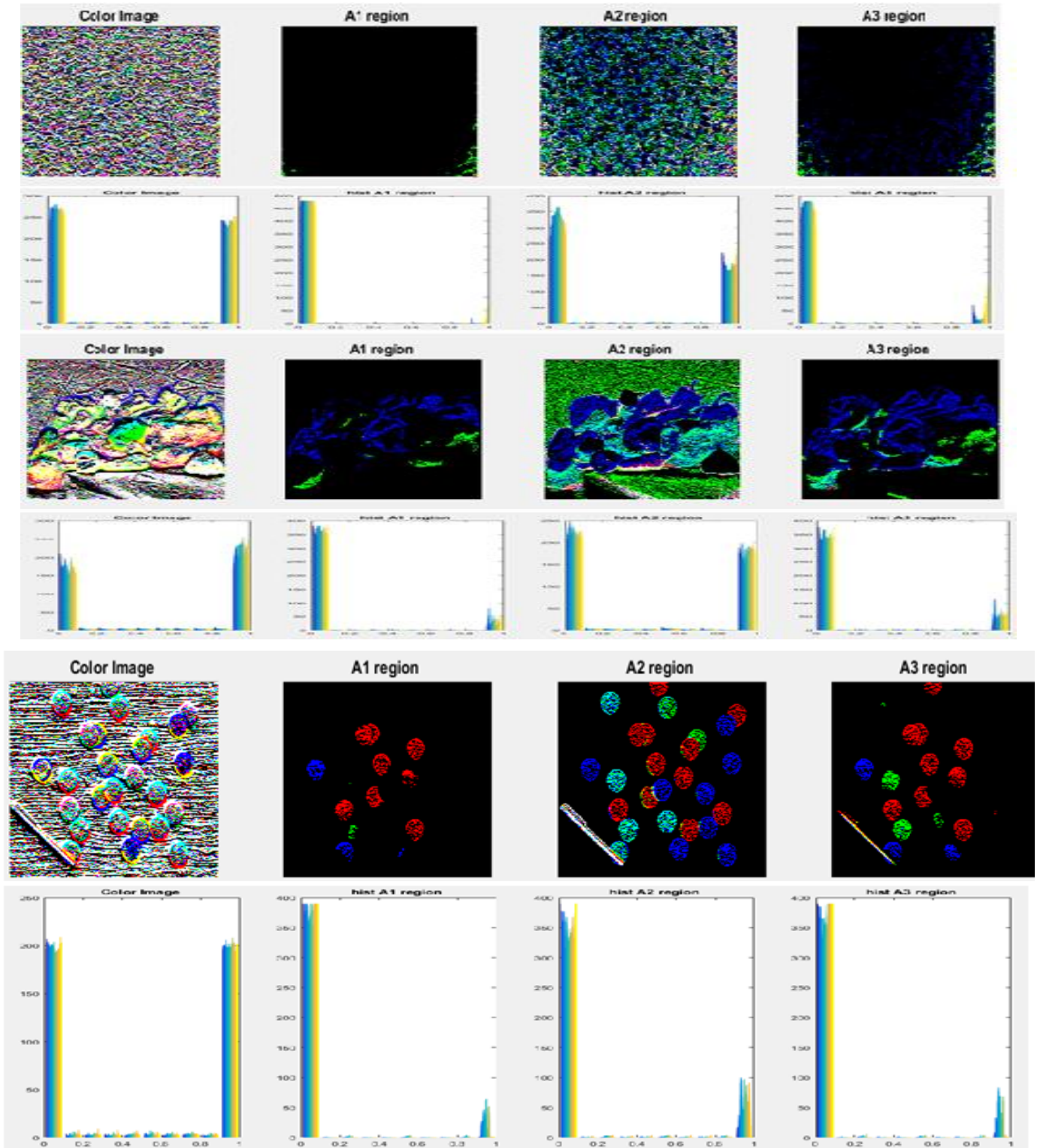
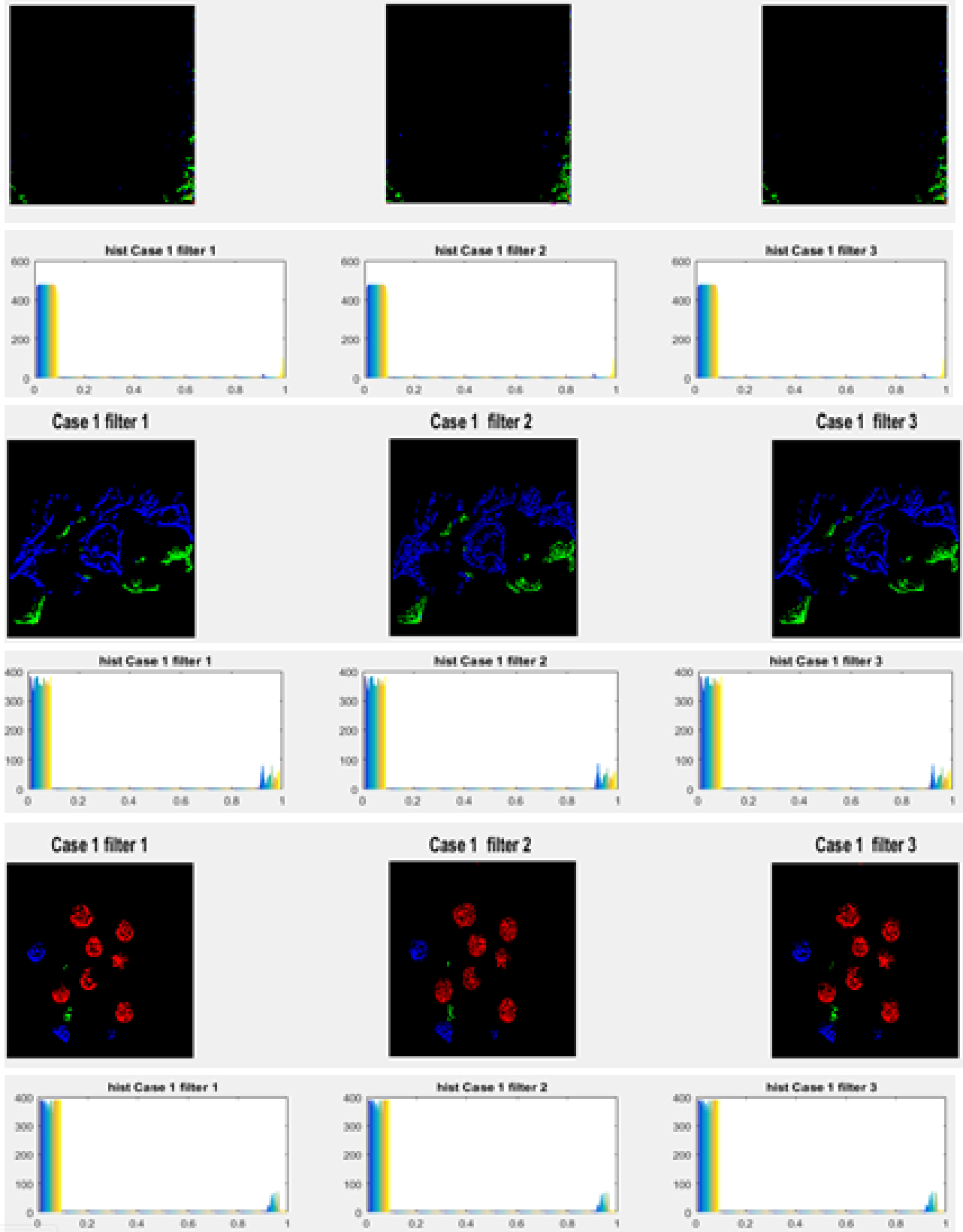
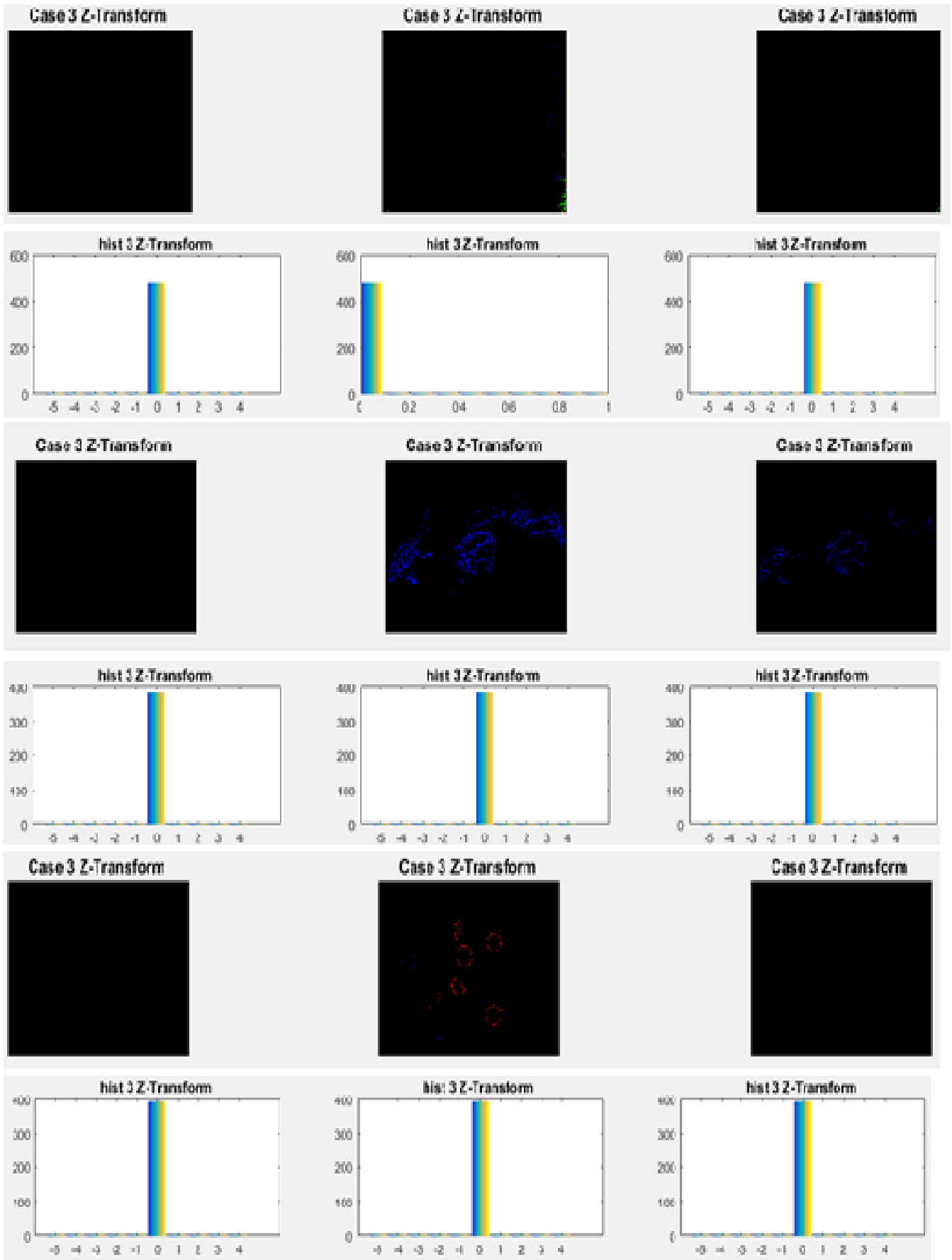


Figure 3 : Effect laws Z-Transform in overlapping colors, indicating the lack of clarity of the regions colored image

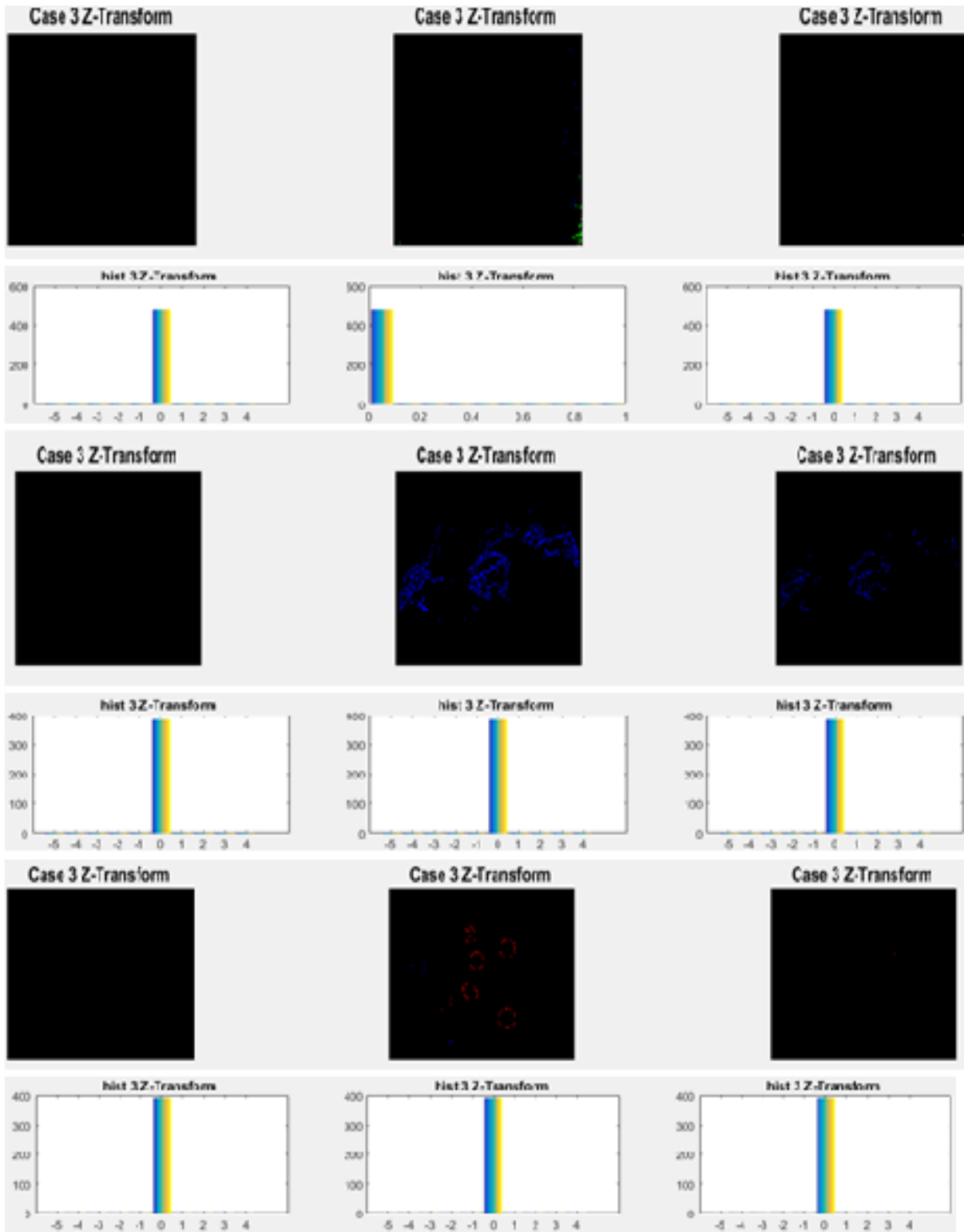
Figures (4, 5, 6) Three cases were identified using laws Z-Transform applied to three colored images and each case takes three filters used in the proposed algorithm for the paper.



Figures 4: Cases 1 using laws Z-Transform applied to three colored images with three filters



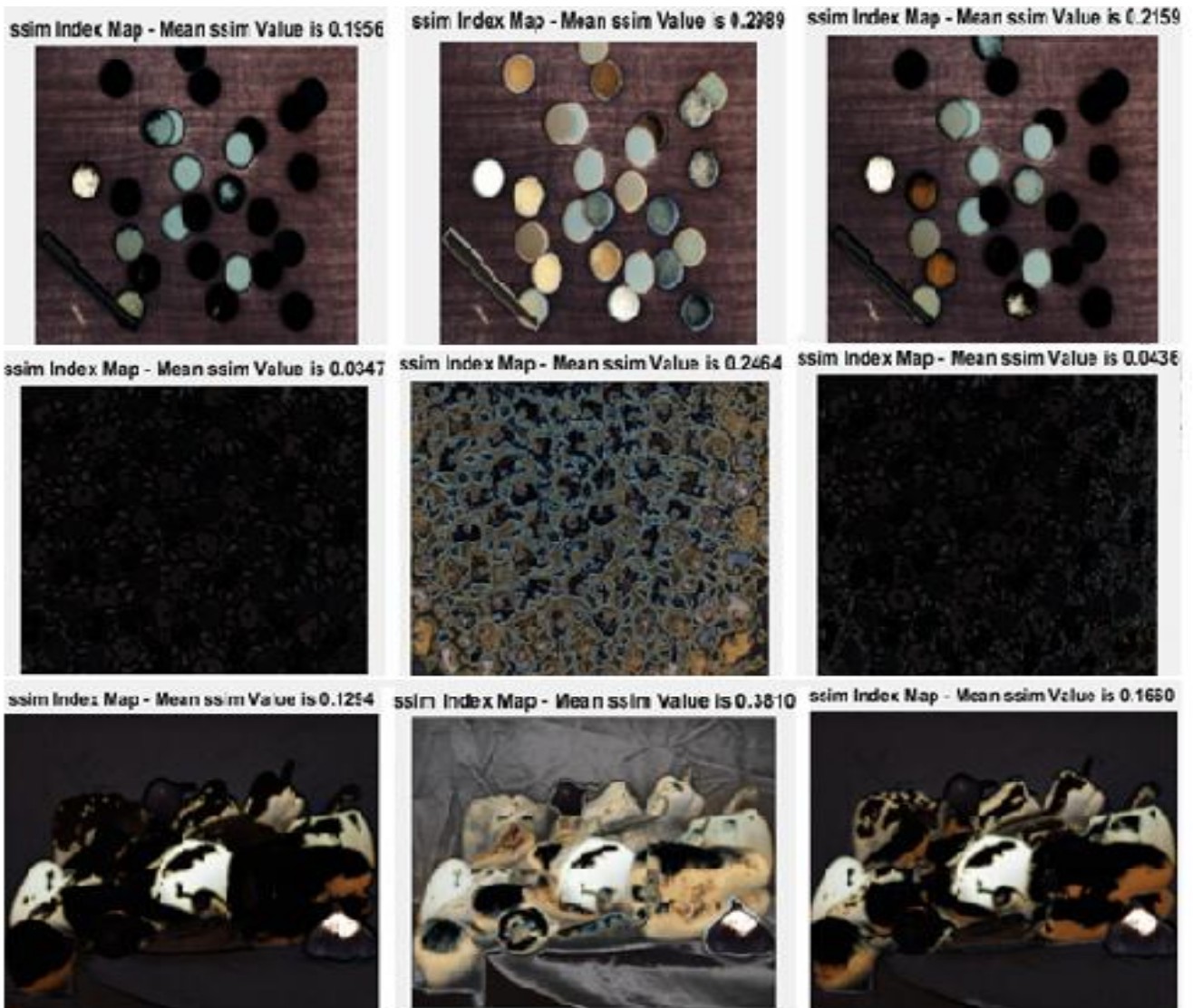
Figures 5: Cases 2 using laws Z-Transform applied to three colored images with three filters



Figures 6: Cases 3 using laws Z-Transform applied to three colored images with three filters

Figures (7) shown Structural Similarity Index (SSIM) for measuring image quality and determine the percentage of the highest convergence of the original color image and the

proportion of distance from the original image.



Figures 7: Structural Similarity Index (SSIM) for measuring image quality after determine the effectiveness Z-transform.

Table (1) Specifies the final values Structural Similarity Index (SSIM) for measuring image quality For the three cases where the results show that the second case is the closest to the original image and remains conservative on some ingredients next is the next case closest the first case is to delete most of the components of the original color image.

Table 1: Specifies the final values Structural Similarity Index (SSIM) for measuring image quality

Image No.	SSIM	Case 1	Case2	Case3
Image 1	SSIM	0.0347	0.2464	0.0436
Image 2	SSIM	0.1956	0.2989	0.2189
Image 3	SSIM	0.1294	0.3810	0.1660

CONCLUSION

Z-Transform It has proven successful in deleting components for color images and this application can be used in hiding or encoding As that Specifies the final values Structural Similarity Index (SSIM) for measuring image to determine the actual proportions of the difference between the original color images and the resulting images after the render Z-Transform and to find out what is suitable for the imported images.

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