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Heat and Mass Transfer in MHD Unsteady Flow of a Viscous Fluid Past an Infinite Vertical Porous Plate Under Oscillatory Suction Velocity in Presence of a Heat Source

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Abstract

The study of heat and mass transfer in MHD flow of a viscous, incompressible, electrically conducting fluid past an infinite vertical porous plate is done. The plate is embedded with porous medium with time dependent permeability $\{K = \overline{K}_0\,(1 + \varepsilon\, e^{int})\}$ under the oscillatory suction velocity normal to the plate. A uniform magnetic field is applied normal to the flow and permeability of the porous medium fluctuates with time. The discussion is confined to the small Eckert number E.

Keywords: Mass Transfer, MHD Flow, electrically conducting, vertical plate, oscillatory, suction velocity

INTRODUCTION:

The study of heat and mass transfer has been widely done by many researchers during last few decades due to application in science and technology. Such phenomena are observed in many physical conditions like buoyancy induced motions in the atmosphere, quasi solid bodies like earth and many more. Unsteady oscillatory free convective flow plays an important role in the field of chemical engineering, aerospace technology etc. such type of flow arise due to unsteady motion of a boundary or boundary temperature.

Several authors have studied free convection and mass transfer flow of a viscous fluid through porous medium. In these studies the permeability of the medium was considered to be constant. But the porosity of a medium not necessarily is constant because the porous material containing the fluid is a non-homogeneous medium. In this context, Sreekanth et.al. have studied the effect of variation of permeability on a free convective flow past a vertical wall in a porous medium. Here permeability was considered as a function of time. Singh et.al. discussed the effect of variation of permeability on MHD free convective and mass transfer flow of a viscous fluid. Acharya et.al also studied free convection and mass transfer in a steady flow through porous medium with constant suction velocity.

But in these studies, oscillation of the suction velocity was not considered. Singh *et.al*. investigated the effect of permeability variation and oscillatory suction velocity on free convective and mass transfer flow of a viscous flow past an infinite, vertical plate with a uniform transverse magnetic field. This study is an extension of the study done by Singh et.al. In this study, we try to investigate the effect of permeability variation and oscillatory suction velocity on free convective and mass transfer flow of a viscous flow in presence of a heat source. Here also the plate is assumed to be vertical, porous and infinite. The results are represented graphically for numerical values of the parameters involved.

MATHEMATICAL FORMULATION:

We have considered the flow of an incompressible, electrically conducting, and viscous fluid past an infinite vertical porous plate in a porous medium with variable permeability and suction velocity. We have considered the x-axis along the plate in the direction of the flow and y-axis normal to it. A uniform magnetic field has been applied in a direction normal to the flow. In this study, we consider the Reynolds' Number **R** to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. Also, all the fluid properties are assumed to be constant except the density variation with temperature. Therefore, the flow is due to the buoyancy force caused by temperature difference between wall and the medium. The governing equations for momentum, energy and concentration are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0\,(1 + \varepsilon\, e^{int})\,\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta\,(\overline{T} - \overline{T}_\infty) + g\beta^*\,(\overline{C} - \overline{C}_\infty) + \nu\,\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\nu \bar{u}}{\overline{K}_0(1+\varepsilon\, e^{int})} - \frac{\sigma B_0^2 \bar{u}}{\rho} \qquad (1)$$

$$\frac{\partial \overline{T}}{\partial \bar{t}} - v_0\,(1 + \varepsilon\, e^{int})\,\frac{\partial \overline{T}}{\partial \bar{y}} = \frac{\lambda}{\rho C_p}\frac{\partial^2 \overline{T}}{\partial \bar{y}^2} + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - \frac{\sigma B_0^2 \bar{u}}{\rho C_p} + \overline{Q}(\overline{T} - \overline{T}_\infty) \qquad (2)$$

$$\frac{\partial \overline{C}}{\partial \bar{t}} - v_0\,(1 + \varepsilon\, e^{int})\,\frac{\partial \overline{C}}{\partial \bar{y}} = D\frac{\partial^2 \overline{C}}{\partial \bar{y}^2} \qquad (3)$$

U*n*der boundary conditions:

At $y = 0$: $\bar{u} = 0, \overline{T} = \overline{T}_w + \varepsilon\,(\overline{T}_w - \overline{T}_\infty)e^{int}, \overline{C} = \overline{C}_w + \varepsilon\,(\overline{C}_w - \overline{C}_\infty)e^{int}$

At $y \to \infty$: $\bar{u} \to 0, \overline{T} \to 0, \overline{C} \to 0$

We introduce the following non-dimensional quantities:

$$y = \frac{v_0}{\nu}\bar{y}, t = \frac{v_0^2}{4\nu}\bar{t}, \; n = \frac{4\nu\bar{n}}{v_0^2}, u = \frac{\bar{u}}{v_0}, \; T = \frac{(\bar{T}-\bar{T}_\infty)}{(\bar{T}_w-\bar{T}_\infty)}, C = \frac{(\bar{C}-\bar{C}_\infty)}{(\bar{C}_w-\bar{C}_\infty)}$$

$$G_r = \frac{g\nu\beta(\bar{T}_w-\bar{T}_\infty)}{v_0^3}, G_m = \frac{g\nu\beta^*(\bar{C}_w-\bar{C}_\infty)}{v_0^3}, P = \frac{\mu C_p}{\lambda}\left(=\frac{\nu}{\alpha}\right) \text{ [Prandtle Number]}$$

$$S = \frac{\nu}{D}, \; M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, E = \frac{v_0^2}{C_p(\bar{T}_w-\bar{T}_\infty)} \text{ [Eckert No.]}, S = \frac{\bar{Q}\nu}{v_0^2}, \; K_0 = \frac{\bar{K}_0 v_0^2}{\nu^2}$$

Where u is the velocity along the x-axis, ν the kinematic coefficient of viscosity, g is the acceleration due to gravity, β is the coefficient of volume expansion for heat transfer, β^* is the volumetric coefficient for expansion with species concentration, $\bar{T}$ is the fluid temperature, $\bar{T}_\infty$ is the fluid temperature at infinity, $\bar{C}$ is the species concentration, $\bar{C}_\infty$ is the species concentration at infinity, D is the molecular diffusivity, K_0 is the constant permeability of the medium, μ is the coefficient of viscosity, C_p is the specific heat at constant pressure, n is the frequency of oscillation, t is the time and ρ is the density of the medium.

Using above non-dimensional quantities, the equations (1), (2) and (3) becomes:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1+\varepsilon e^{int}\right)\frac{\partial u}{\partial y} = G_r T + G_m C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_0(1+\varepsilon e^{int})} - Mu \quad (4)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \left(1+\varepsilon e^{int}\right)\frac{\partial T}{\partial y} = \frac{1}{P}\frac{\partial^2 T}{\partial y^2} + E\left(\frac{\partial u}{\partial y}\right)^2 + MEu^2 + QT \quad (5)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1+\varepsilon e^{int}\right)\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (6)$$

Under boundary conditions:

$$\left.\begin{array}{l} \text{At } y=0: \; u=0, T=(1+\varepsilon e^{int}), C=(1+\varepsilon e^{int}) \\ \text{At } y=\infty: \; u\to 0, T\to 0, C\to 0 \end{array}\right\} \quad (7)$$

METHOD OF SOLUTION:

In order to solve the equations (4), (5) and (6) under the boundary consitions (7), we put

$$\left.\begin{array}{l} u(y,t) = u_0(y) + \varepsilon u_1(y)e^{int} \\ T(y,t) = T_0(y) + \varepsilon T_1(y)e^{int} \\ C(y,t) = C_0(y) + \varepsilon C_1(y)e^{int} \end{array}\right\} \quad (8)$$

Substituting (8) in (4), (5) and (6) and equating harmonic and Non-harmonic parts we have

$$u_0'' + u_0' - a_0 u_0 = -G_r T_0 - G_m C_0 \quad (9)$$

$$u_1'' + u_1' - a_1 u_1 = -G_r T_1 - G_m C_1 - u_0' - \frac{1}{K_0} u_0 \quad (10)$$

$$T_0'' + P T_0' + PQ T_0 = -EPu_0'^2 - MEPu_0^2 \quad (11)$$

$$T_1'' + P T_1' + PQT_1 = -2EPu_0' u_1' - 2MEPu_0 u_1 \quad (12)$$

Where

$$a_0 = M + \frac{1}{K_0}; \; a_1 = M + \frac{1}{K_0} + \frac{in}{4}; \; a_2 = SP - \frac{iPn}{4}$$

Under boundary conditions:

$$\left.\begin{array}{l} \text{At } y=0: \; u_0 = u_1 = 0, \; T_0 = T_1 = 1, \; C_0 = C_1 = 1 \\ \text{when } y\to\infty: \; u_0 \to 0, \; u_1 = 0; T_0 = T_1 \to 0, \; C_0 = C_1 \to 0 \end{array}\right\} \quad (15)$$

To solve the equations from (9) to (14), we assume the following for E<1

$$\left.\begin{array}{l} u_0(y) = u_{00}(y) + Eu_{01}(y) \\ u_1(y) = u_{10}(y) + Eu_{11}(y) \\ T_0(y) = T_{00}(y) + ET_{01}(y) \\ T_1(y) = T_{10}(y) + ET_{11}(y) \\ C_0(y) = C_{00}(y) + EC_{01}(y) \\ C_1(y) = C_{10}(y) + EC_{11}(y) \end{array}\right\} \quad (16)$$

Substituting (15) in equations from (9) to (14) and equating the coefficients of E^0 and E^1 (neglecting E^2), we get the following equations:

$$u_{00}'' + u_{00}' - a_0 u_{00} = -G_r T_{00} - G_m C_{00} \quad (17)$$

$$u_{01}'' + u_{01}' - a_0 u_{01} = -G_r T_{01} - G_m C_{01} \quad (18)$$

$$u_{10}'' + u_{10}' - a_1 u_{10} = -G_r T_{10} - G_m C_{10} - u_{00}' - \frac{1}{K_0} u_{00} \quad (19)$$

$$u_{11}'' + u_{11}' - a_1 u_{11} = -G_r T_{11} - G_m C_{11} - u_{01}' - \frac{1}{K_0} u_{01} \quad (20)$$

$$T_{00}'' + PT_{00}' + PS\, T_{00} = 0 \quad (21)$$

$$T_{01}'' + PT_{01}' + PS\, T_{01} = -Pu_{00}'^2 - MPu_{00}^2 \quad (22)$$

$$T_{10}'' + PT_{10}' + a_2 T_{10} = -P\, T_{00}' \quad (23)$$

$$T_{11}'' + PT_{11}' + a_2 T_{11} = -P\, T_{01}' + 2Pu_{00}' u_{01}' - 2MPu_{00}u_{10} \quad (24)$$

$$C_{00}'' + S_c C_{00}' = 0 \quad (25)$$

$$C_{01}'' + S_c C_{01}' = 0 \quad (26)$$

$$C_{10}'' + S_c C_{10}' - \frac{in\, S_c}{4} C_{10} = -S_c C_{00}' \quad (27)$$

$$C_{11}'' + S_c C_{11}' - \frac{in\, S_c}{4} C_{11} = -S_c C_{01}' \quad (28)$$

And the boundary conditions (15) become

$$\left.\begin{array}{ll} \text{At } y=0: & u_{00} = u_{01} = 0; \; u_{10} = u_{11} = 0, \; T_{00} = T_{10} = 1, \\ & T_{01} = T_{11} = 0; \; C_{00} = C_{10} = 1; \; C_{01} = C_{11} = 0 \\ \text{when } y\to\infty: & u_{00} = u_{01} \to 0; \; u_{10} = u_{11} \to 0, \; T_{00} = T_{10} \to 1, \\ & T_{01} = T_{11} \to 0; \; C_{00} = C_{10} \to 1; \; C_{01} = C_{11} \to 0 \end{array}\right\} (29)$$

Now solving equations (17) to (28) under boundary conditions (29) we have:

$$C_{00} = e^{-S_c y}$$

$$C_{01} = e^{-S_c y}$$

$C_{10} = \left(1 - \frac{i4S_c}{n}\right) e^{-m_1 y} + \frac{i4S_c}{n} e^{-S_c y}$

$C_{11} = \left(1 - \frac{i4S_c}{n}\right) e^{-m_1 y} + \frac{i4S_c}{n} e^{-S_c y}$

$T_{00} = e^{-m_2 y}$

$T_{01} = S_0 e^{-m_2 y} + S_2 e^{-2m_3 y} + S_1 e^{-2m_2 y} + S_3 e^{-2S_c y} + S_4 e^{-(m_2+m_3) y}$
$+ S_5 e^{-(m_2+S_c)y} + S_6 e^{-(m_3+S_c)y}$

$T_{10} = \left(1 - \frac{Pm_2}{m_2^2 - mP + a_2}\right) e^{-m_4 y} + \frac{Pm_2}{m_2^2 - mP + a_2} e^{-m_2 y}$

$T_{11} = R_1 e^{-m_2 y} + R_2 e^{-m_4 y} + R_3 e^{-2m_2 y} + R_4 e^{-2m_3 y} + R_5 e^{-2S_c y} + R_6 e^{-(m_1+m_3) y} + R_7 e^{-(m_1+S_c) y} + R_8 e^{-(m_2+m_4) y} + R_9 e^{-(m_2+m_5) y} + R_{10} e^{-(m_2+S_c) y} + R_{11} e^{-(m_3+m_4) y} + R_{12} e^{-(m_3+m_5) y} + R_{13} e^{-(m_3+S_c) y}$

$u_{00} = (a_3 + a_4) e^{-m_3 y} - a_3 e^{-m_2 y} - a_4 e^{-S_c y}$

$u_{01} = A_1 e^{-m_3 y} + A_2 e^{-m_2 y} + A_3 e^{-2m_2 y} + A_4 e^{-2m_3 y} + A_5 e^{-2S_c y} + A_6 e^{-(m_2+m_3) y} + A_7 e^{-(m_2+S_c)y} + A_8 e^{-(m_3+S_c)y} + A_9 e^{-S_c y}$

$u_{10} = B_1 e^{-m_1 y} + B_2 e^{-m_2 y} + B_3 e^{-m_3 y} + B_4 e^{-m_4 y} + B_5 e^{-m_5 y} + B_6 e^{-S_c y}$

$u_{11} = M_1 e^{-m_1 y} + M_2 e^{-m_2 y} + M_3 e^{-m_3 y} + M_4 e^{-m_4 y} + M_5 e^{-m_6 y} + M_6 e^{-S_c y} + M_7 e^{-2m_2 y} + M_8 e^{-2m_3 y} + M_9 e^{-2S_c y}$
$+ M_{10} e^{-(m_1+S_c) y} + M_{11} e^{-(m_2+S_c) y} + M_{12} e^{-(m_3+S_c) y} + M_{13} e^{-(m_4+S_c) y} + M_{14} e^{-(m_5+S_c) y} + M_{15} e^{-(m_1+m_3) y}$
$+ M_{16} e^{-(m_1+m_5) y} + M_{17} e^{-(m_2+m_3) y} + M_{18} e^{-(m_2+m_4) y} + M_{19} e^{-(m_2+m_5) y} + M_{20} e^{-(m_3+m_4) y}$

Where

$m_1 = \frac{1}{2}\left[S_c + \sqrt{S_c^2 + inS_c}\right] = C_1 + iD_1$, $m_2 = \frac{P + \sqrt{P^2 - 4PS}}{2}$, $m_3 = \frac{1 + \sqrt{1 + 4a_0}}{2}$,

$m_4 = \frac{P + \sqrt{P^2 - 4a_2}}{2} = C_2 + iD_2$, $m_5 = \frac{1 + \sqrt{1 + 4a_1}}{2} = C_3 + iD_3$

$a_3 = \frac{-G_r}{m_2^2 - m_2 + a_0}$, $a_4 = \frac{-G_r}{S_c^2 - S_c + a_0}$, $S_1 = \frac{-Pa_3^2(m_2^2 + M)}{4m_2^2 - 2m_2 P + PS}$, $S_2 = \frac{-P(a_3 + a_4)^2(m_3^2 + M)}{4m_3^2 - 2m_3 P + PS}$,

$S_3 = \frac{-Pa_4^2(S_c^2 + M)}{4S_c^2 - 2S_c P + PS}$, $S_4 = \frac{2P(a_3 + a_4)a_3(m_2 m_3 + M)}{(m_2 + m_3)^2 - (m_2 + m_3)P + PS}$, $S_5 = \frac{-2Pa_3 a_4(m_2 S_c + M)}{(m_2 + S_c)^2 - (m_2 + S_c)P + PS}$,

$S_6 = \frac{2P(a_3 + a_4)a_4(m_3 S_c + M)}{(m_3 + S_c)^2 - (m_3 + S_c)P + PS}$, $S_0 = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$

$R_1 = \frac{PT_1 m_2}{m_2^2 - Pm_2 + a_2}$, $R_2 = \frac{2PT_3 m_2 - 2PB_4 m_2^2}{4m_2^2 - 2Pm_2 + a_2}$, $R_3 = \frac{2PT_2 m_3 + 2MPB_3(a_3 + a_4) + 2PB_3(a_3 + a_4)m_3^2}{4m_3^2 - 2Pm_3 + a_2}$,

$R_4 = \frac{2PT_4 - 2MPa_4 B_6 - 2Pa_4 B_6 S_c^2}{4S_c^2 - 2PS_c + a_2}$, $R_5 = \frac{2MPB_1(a_3 + a_4) - 2PB_1 m_3 m_5(a_3 + a_4)}{(m_3 + m_5)^2 - (m_3 + m_5)P + a_2}$,

$R_6 = \frac{2MPB_2(a_3 + a_4) - 2PB_1 m_3 m_5(a_3 + a_4)}{(m_3 + m_5)^2 - (m_3 + m_5)P + a_2}$, $R_7 = \frac{2MPB_4(a_3 + a_4) - 2MPB_3 a_3 - 2PB_3 m_3 m_2 + 2PB_4 m_3 m_2(a_3 + a_4)}{(m_2 + m_3)^2 - (m_2 + m_3)P + a_2}$

$R_8 = \frac{-2MPB_2 a_3 - 2PB_2 m_2 m_4 a_3}{(m_2 + m_4)^2 - (m_2 + m_4)P + a_2}$, $R_9 = \frac{-2PB_1 m_2 m_5 a_3}{(m_2 + m_5)^2 - (m_2 + m_5)P + a_2}$,

$R_{10} = \frac{2MPB_5(a_3 + a_4) + 2PB_5 m_1 m_3(a_3 + a_4)}{(m_1 + m_3)^2 - (m_1 + m_3)P + a_2}$, $R_{11} = \frac{-2PB_5 m_1 m_2 a_3}{(m_1 + m_5)^2 - (m_1 + m_5)P + a_2}$,

$R_{12} = \frac{-2MPB_5 a_4 - 2PB_5 S_c m_1 m_4}{(m_1 + S_c)^2 - (m_1 + S_c)P + a_2}$, $R_{13} = \frac{-2MPB_1 a_3 - 2MPB_4 a_4 - 2PB_6 m_2 a_3 S_c + PT_5(m_2 + m_3) + PT_6(m_2 + S_c)}{(m_2 + S_c)^2 - (m_2 + S_c)P + a_2}$

$R_{14} = \frac{-2MPB_3 a_4 - 2PB_3 m_3 a_4 S_c + 2PB_6 m_2 a_3 S_c + 2PMB_6 + PT_7(m_3 + S_c)}{(m_3 + S_c)^2 - (m_3 + S_c)P + a_2}$,

$R_{15} = \frac{-2PB_1 m_4 S_c a_4}{(m_4 + S_c)^2 - (m_4 + S_c)P + a_2}$, $R_{16} = \frac{-2PB_5 m_1 m_2 a_3}{(m_5 + S_c)^2 - (m_5 + S_c)P + a_2}$

$R_{17} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 + R_{10} + R_{11} + R_{12} + R_{13} + R_{14} + R_{15} + R_{16}$

$A_1 = \frac{-G_r S_1}{m_2^2 - m_2 - a_0}$, $A_2 = \frac{-G_r S_2}{4m_2^2 - 2m_2 - a_0}$, $A_3 = \frac{-G_r S_3}{4m_3^2 - 2m_3 - a_0}$, $A_4 = \frac{-G_r S_4}{4S_c^2 - 2S_c - a_0}$

$A_5 = \frac{-G_r S_5}{4(m_2 + m_3)^2 - 2(m_2 + m_3) - a_0}$, $A_6 = \frac{-G_r S_6}{4(m_2 + S_c)^2 - 2(m_2 + S_c) - a_0}$, $A_7 = \frac{-G_r S_5}{4(m_3 + S_c)^2 - 2(m_3 + S_c) - a_0}$

$A_8 = \frac{-G_m}{S_c^2 - S_c - a_0}$, $A_9 = -(A_1 + A_2 + A_3 + A_4 + A_5 + A_7 + A_8)$

$B_1=\frac{T_1}{m_1^2+m_1-a_1}$, $B_2=\frac{T_2}{m_2^2+m_2-a_1}$, $B_3=\frac{T_3}{m_3^2+m_3-a_1}$, $B_4=\frac{T_4}{m_4^2+m_4-a_1}$, $B_5=B_1+B_2+B_3+B_4$,

$B_6=\frac{T_5}{S_c^2+S_c-a_1}$, $T_4=-G_m\left(1-\frac{i4S_c}{n}\right)$, $T_3=\frac{G_r m_2 P}{m_2^2-Pm_2+a_2}-m_2a_3+\frac{a_3}{K_0}$,

$T_4=m_3(a_3+a_4)-\frac{(a_3+a_4)}{K_0}$, $T_1=-G_r\left(1-\frac{Pm_2}{m_2^2-Pm_2+a_2}\right)$, $T_5=-\frac{i4G_mS_c}{n}-S_ca_4+\frac{a_4}{K_0}$

$M_1=\frac{-G_m\left(1-\frac{i4S_c}{n}\right)}{m_1^2+m_1-a_1}$, $M_2=\frac{-G_rR_2+2m_3A_4-\frac{A_4}{K_0}}{m_2^2+m_2-a_1}$, $M_3=\frac{m_3A_1-\frac{A_1}{K_0}}{m_3^2+m_3-a_1}$, $M_4=\frac{-G_rR_1}{m_4^2+m_4-a_1}$,

$M_5=\frac{S_cA_9-\frac{A_9}{K_0}}{S_c^2+S_c-a_1}$, $M_6=\frac{-G_rR_3+2m_2A_3-\frac{A_3}{K_0}}{4m_2^2+2m_2-a_1}$, $M_7=\frac{-G_rR_4+2m_3A_4-\frac{A_4}{K_0}}{4m_3^2+2m_3-a_1}$, $M_8=\frac{-G_rR_5+2S_cA_5-\frac{A_5}{K_0}}{4S_c^2+2S_c-a_1}$, $M_9=\frac{-G_rR_{12}}{(m_1+S_c)^2+(m_1+S_c)-a_1}$, $M_{10}=\frac{-G_rR_{10}+(m_2+S_c)A_7-\frac{A_7}{K_0}}{(m_2+S_c)^2+(m_2+S_c)-a_1}$, $M_{11}=\frac{-G_rR_8+(m_3+S_c)A_8-\frac{A_8}{K_0}}{(m_3+S_c)^2+(m_3+S_c)-a_1}$,

$M_{12}=\frac{-G_rR_{15}}{(m_4+S_c)^2+(m_4+S_c)-a_1}$, $M_{13}=\frac{-G_rR_{14}}{(m_5+S_c)^2+(m_5+S_c)-a_1}$, $M_{14}=\frac{-G_rR_9}{(m_1+m_3)^2+(m_1+m_3)-a_1}$,

$M_{15}=\frac{-G_rR_{16}}{(m_1+m_5)^2+(m_1+m_5)-a_1}$, $M_{16}=\frac{(m_2+m_3)A_6-\frac{A_6}{K_0}}{(m_2+m_3)^2+(m_2+m_3)-a_1}$, $M_{17}=\frac{-G_rR_{11}}{(m_2+m_4)^2+(m_2+m_4)-a_1}$, $M_{18}=\frac{-G_rR_{13}}{(m_2+m_5)^2+(m_2+m_5)-a_1}$, $M_{19}=\frac{-G_rR_7}{(m_3+m_4)^2+(m_3+m_4)-a_1}$, $M_{20}=\frac{-G_rR_6}{(m_3+m_5)^2+(m_3+m_5)-a_1}$

Now with the convection that the real parts of complex quantities have physical significance in the problem, the velocity and temperature fields can be expressed as follows:

$$u(y,t)=u_0(y)+\varepsilon(u_r\cos nt-u_i\sin nt) \quad (30)$$

$$T(y,t)=T_0(y)+\varepsilon(T_r\cos nt-T_i\sin nt) \quad (31)$$

$$C(y,t)=C_0(y)+\varepsilon(C_r\cos nt-C_i\sin nt) \quad (32)$$

Where:

$u_r=\mathrm{Re}(u_1)$, $u_i=\mathrm{Im}(u_1)$,

$T_r=\mathrm{Re}(T_1)$, $T_i=\mathrm{Im}(T_1)$,

$C_r=\mathrm{Re}(C_1)$, $C_i=\mathrm{Im}(C_1)$,

The expressions for u_r, u_i, T_r, T_i, C_r and C_i are obtained but not presented here for the brevity and transient velocity, temperature and concentration field are obtained for $nt=\frac{\pi}{2}$

1. Skin Friction and Rate of Heat transfer:

The Skin friction coefficient (τ_0) at the plate in terms of amplitude and phase is given by

$$\tau_0=\left(\frac{\partial u}{\partial y}\right)_{y=0}=u_0'(0)+\varepsilon e^{int}u_1'(0)$$

Splitting this equation into real and imaginary parts and taking the real parts only we get:

$$\tau_0=\tau_0^0+\varepsilon|B|\cos(nt+\theta)$$

Where

$$\left.\begin{array}{c} B=B_r+iB_i=u_1'(0) \\ \tau_0^0=u_0'(0) \\ |B|=\sqrt{B_i^2+B_i^2} \\ \tan\theta=\frac{B_i}{B_r}\end{array}\right\} \quad (30)$$

Also the Heat Transfer coefficient (Nu_0) at the plate in terms of the amplitude and phase is given by

$$Nu_0=-\left(\frac{\partial T}{\partial y}\right)_{y=0}=T_0'(0)+\varepsilon e^{int}T_1'(0)$$

Splitting this equation into real and imaginary parts and taking the real parts only we have-

$$Nu_0=Nu_0^0+\varepsilon|H|\cos(nt+\varphi)$$

Where

$$\left.\begin{array}{c} H=H_r+iH_i=T_1'(0) \\ Nu_0^0=T_0'(0) \\ |H|=\sqrt{H_i^2+H_i^2} \\ \tan\varphi=\frac{H_i}{H_r}\end{array}\right\} \quad (31)$$

The expressions for $\tau_0^0, Nu_0^0, B_r, B_i, H_r$ and H_i are obtained but not presented here for the sake of brevity.

The Mass Transfer coefficient (Sh_0) at the plate in terms of its amplitude and phase is given by

$$Sh_0=-\left(\frac{\partial C}{\partial y}\right)_{y=0}=C_0'(0)+\varepsilon e^{int}C_1'(0)$$

Splitting this equation into real and imaginary parts and taking the real parts only we have-

$$Sh_0=Sh_0^0+\varepsilon|D|\cos(nt+\delta)$$

Where

$$\left.\begin{array}{c} D=D_r+iD_i=C_1'(0) \\ Sh_0^0=C_0'(0) \\ |D|=\sqrt{D_i^2+D_i^2} \\ \tan\delta=\frac{D_i}{D_r}\end{array}\right\}$$

DISCUSSION AND CONCLUSION:

In the following section, we have studied the velocity field, temperature, concentration field, coefficient of skin-friction, rate of heat and mass transfer by assigning numerical values to the various parameters involved in the mathematical formulation of the problem.

The values of the different parameters involved are taken as follows:

The values of Prandtl Number (P) are taken for P=0.025 (for Mercury), P=0.71 (for Air), P=7.0 (for Water). The values of Schmidt Number (S_c) are taken as S_c=0.22 (for Hydrogen), S_c=0.30 (for Helium), S_c=0.6 (for Water Vapor), S_c=0.66 (for Oxygen) and S_c=0.78 (for Ammonia). The Grashof Number (G_r) is considered for two cases-for heating of the plate ($G_r < 0$) and for cooling of the plate ($G_r > 0$).

For transient velocity profile we have considered S_c=0.22, 0.66, the Magnetic parameter (M) is taken as M=0.5, 1.0, permeability parameter K_0=10.0, 20.0, frequency parameter n=5.0, Prandtl Number P=0.71, perturbation parameter ε=0.005 and $nt = \frac{\pi}{2}$. The Grashof Number (G_r) are taken to be G_r=10.0, 20.0 for cooling of the plate and G_r= -10.0, -20.0 for heating of the plate. The other parameters are taken as shown in the figures and tables.

Fig. 1 depicts the behaviour of the transient velocity u due to change in values of G_r , G_m, S_c, M and K_0 at n=5.0, E= 0.04, ε=0.005, P=0.71, $nt = \frac{\pi}{2}$ for cooling of the plate. Here, it is observed that an increase of G_r or G_m or K_0 results a decrease of the transient velocity while an increase of S_c or M results an increase of it. We also observe that, at the vicinity of the plate there is a sudden deplete of the velocity and thereafter, it steadily increases.

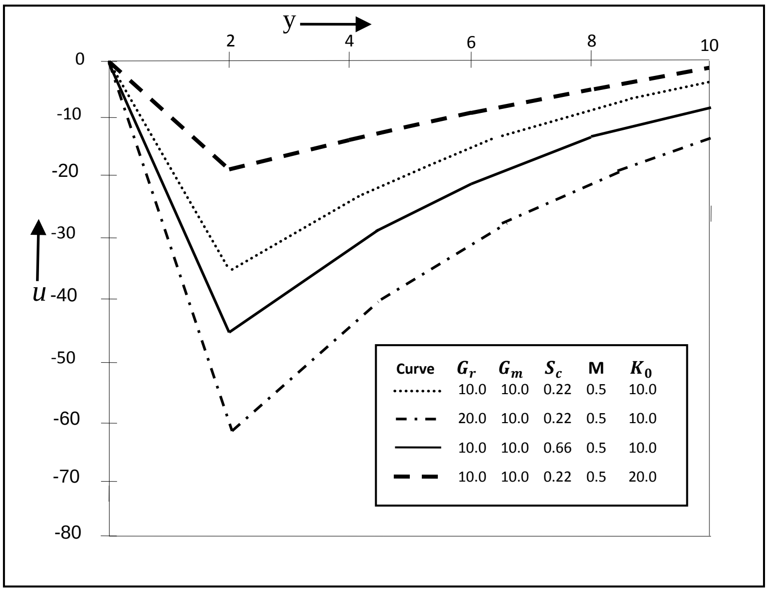

Fig. 1 Velocity Profile due to cooling of plate (n=5.0, E= 0.04, ε=0.005, P=0.71, $nt = \frac{\pi}{2}$)

Fig. 2 shows the change of temperature field T due to the variation of Prandtl number P for Mercury (P=0.025), Air (P=0.71), Water (P=7.0) and Water at 4^0C (P=11.4) at n=5.0, E= 0.04, ε=0.005 and $nt = \frac{\pi}{2}$. It has been observed that, an increase of the Prandtl number P yields a decrease of the temperature and it falls more rapidly for water in comparison to air. A very interesting observation is seen in the figure that the temperature for mercury remains almost constant from which we can conclude that mercury is more useful for maintaining temperature difference and it can be easily used for laboratory purpose.

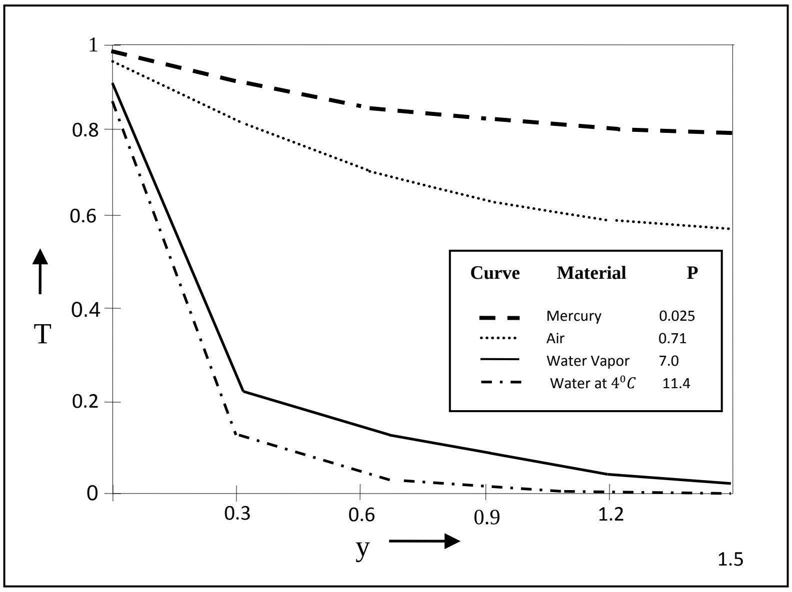

Fig.2 Temperature with variation of Prandtl Number (n=5.0, E= 0.04, ε=0.005, $nt = \frac{\pi}{2}$)

Fig. 3 shows the transient concentration field due to the variation of Schmidt Number S_c for Hydrogen (S_c=0.22), Helium (S_c=0.30), Water Vapour (S_c=0.60) and Oxygen (S_c=0.66) at n=5.0, E= 0.04, ε=0.005, P=0.71 and $nt = \frac{\pi}{2}$. Here we notice that the concentration field decreases gradually for Hydrogen and Helium, but rapidly for Water Vapour and Oxygen.

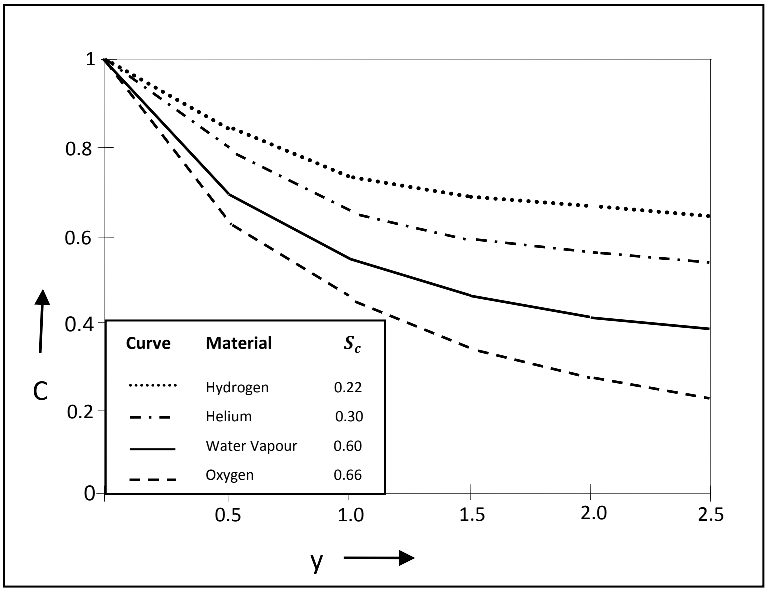

Figure 3. Variation of Concentration with Schmidt Number S_c (n=5.0, E= 0.04, ε=0.005, $nt = \frac{\pi}{2}$)

Table I and II represent the numerical values of the Skin-friction τ_0 in terms of the amplitude $|B|$ and phase $\tan\theta$ for different values of G_r, G_m, E, S_c, M, P, K_0 and n corresponding to cooling of the plate and heating of the plate respectively.

TABLE I

Values of amplitude |B| and phase $\tan\theta$ of Skin-friction due to cooling of the plate

G_r	G_m	E	S_c	M	K_0	n	P	|B|	$\tan\theta$
10.0	4.0	0.004	0.22	0.5	10.0	5.0	0.71	46.2315	-0.26958
20.0	4.0	0.004	0.22	0.5	10.0	5.0	0.71	77.2569	-0.31582
10.0	6.0	0.004	0.22	0.5	10.0	5.0	0.71	49.2231	-0.36124
10.0	4.0	0.004	0.66	0.5	10.0	5.0	0.71	25.3695	-0.25631
10.0	4.0	0.004	0.22	1.0	10.0	5.0	0.71	10.2365	-0.14783
10.0	4.0	0.004	0.22	0.5	20.0	5.0	0.71	78.2584	-0.27369
10.0	4.0	0.004	0.22	0.5	10.0	10.0	0.71	37.0025	-0.33125
10.0	4.0	0.004	0.22	0.5	10.0	5.0	7.00	09.2350	-0.19333

In Table I, we have seen that with the increase of G_r, G_m or K_0, there is moderate increase of the values of the amplitude |B|, whereas an increase of the values of S_c, M, P or n results a decrease in the values of |B|. The values of $\tan\theta$ decrease with the increase of G_r, K_0 or n while it increases with the increase of G_m, S_c, M or P.

TABLE II

Values of amplitude |B| and phase $\tan\theta$ Skin-friction due to heating of the plate

G_r	G_m	E	S_c	M	K_0	n	P	|B|	$\tan\theta$
-10.0	-4.0	0.004	0.22	0.5	10.0	5.0	0.71	51.0236	-0.29415
-20.0	-4.0	0.004	0.22	0.5	10.0	5.0	0.71	79.2589	-0.12567
-10.0	-6.0	0.004	0.22	0.5	10.0	5.0	0.71	49.1256	-0.53678
-10.0	-4.0	0.004	0.66	0.5	10.0	5.0	0.71	23.4780	-0.22665
-10.0	-4.0	0.004	0.22	1.0	10.0	5.0	0.71	08.2376	-0.47890
-10.0	-4.0	0.004	0.22	0.5	20.0	5.0	0.71	66.9865	-0.33749
-10.0	-4.0	0.004	0.22	0.5	10.0	10.0	0.71	30.2891	-0.29731
-10.0	-4.0	0.004	0.22	0.5	10.0	5.0	7.00	12.0037	-0.99858

In Table II, we observe that, the amplitude |B|, increases with the increase of G_r, G_m or K_0 while an increase of S_c, M, n or P leads to decrease of the values of |B|. Also we see that the values of the phase $\tan\theta$ decreases due to the increase of G_r, K_0 or n but it increases with the increase of G_m, S_c, M or P.

TABLE III

Values of amplitude |H| and phase $\tan\varphi$ of Heat Transfer Coefficient (Nu_0) for $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

E	n	P	|H|	$\tan\varphi$	Nu_0
0.004	5.0	0.025	01.2026	0.6258	00.0347
0.004	5.0	0.710	05.4485	0.3152	00.3654
0.004	5.0	7.000	13.2991	0.1036	05.1490
0.004	5.0	11.40	23.3279	0.0631	13.3505
0.004	10.0	0.710	05.4690	0.4783	00.3344

Table III represents the numerical values of the amplitude |H| and phase $\tan\varphi$ and the Heat Transfer Coefficient (Nu_0) due to the variation of Prandtl Number (P) and frequency n. Here we have noticed that values of |H| and Nu_0 are least for P=0.025 (Mercury) but these are highest for P=11.40 (Water at 4^0C) but we see the opposite trend in case of the phase $\tan\varphi$. Also, an increase of the frequency n results an increase of the |H| and $\tan\varphi$ but decrease in the values of Nu_0

TABLE IV

Values of amplitude |D| and phase $\tan\delta$ of Mass Transfer Coefficient (Sh_0) for $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$

E	n	S_c	|D|	$\tan\delta$	Sh_0
0.004	5.0	0.22	01.2026	0.6258	0.3265
0.004	5.0	0.30	01.4015	0.6123	0.3452
0.004	5.0	0.60	01.5129	0.5907	0.3946
0.004	5.0	0.78	01.6203	0.5877	0.4268
0.004	10.0	0.22	01.3690	0.7780	0.3112

Table IV gives the variation of the numerical values of the amplitude |D| and the phase $\tan\delta$ of the rate of mass transfer coefficient Sh_0 with Schmidt Number S_c and frequency n corresponding to $\varepsilon = 0.005$ and $nt = \frac{\pi}{2}$. Here, we observe that the values of |D| and Sh_0 increase but $\tan\delta$ decreases with the increase of S_c. An increase of n leads to an increase in the values of |D| and $\tan\delta$ but decrease in the values of Sh_0.

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