International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3210-3214
© Research India Publications. http://www.ripublication.com

Analysis of Symbol Error Rate in Amplify And Forward Nakagami-M Cooperative Networks

[1]Manar Al-Kali, [2] Ali A. Mohammed

[1]School of Computer Science, China University of Geosciences. 430070, Wuhan, China.
[2] Electronic Computer Center, AL-Karkh University of Sciences, Baghdad, Iraq.

Abstract

In this paper, cooperative wireless network over independent non-identical Nakagami-m fading channels is analyzed and evaluated under the consideration of Amplify- and- forward Mode (AF) with M-ary Phase Shift Keying (MPSK) modulation for different cases of the fading parameters m. The performance analysis of average Symbol Error Rate (SER) is considered to behave the network performance since that SER is determined by using Moment Generation Function (MGF) over various values of the Signal to Noise Ratio (SNR). The simulation results of our proposed mathematical model reveals that SER performance downgrades as either the m parameter or the number of relays increases.

Keywords: cooperative networks, amplify-and-forward, symbol error rate, Nakagami-m fading channels.

INTRODUCTION

The Cooperative networks and collaborative strategies protocol relay has been the subject of the research due to its great influence in the field of wireless communications and its applications. Many works state its role in advanced communications and many relaying strategies have been presented, and different relaying protocols could be employed. The received signals at any given relay node could be multiplied by a gain and forwarded, namely employing the amplify- and-forward (AF) relaying protocol. Or they could be decoded and the originally sent messages estimated, before being encoded again and forwarded. This second relaying protocol is known as decode-and-forward (DF). The third option is the compress-and-forward (CF) relaying protocol [1-3]. In these strategies, the users serve as information sources as well as relays. In AF, the relay solely amplifies the received signal from the source, and re-transmits it to the destination without doing any further processing on the source transmission [4],[5]. Therefore, AF relaying incurs low-cost hardware implementation, which represents an ingenious solution to the field of communications systems. For this reason, we investigate the performance of all participate cooperative networks with AF in Nakagami-m fading environments. Nakagami-m fading model has been an important subject of study in cooperative networks, because it provides the most appropriate information and realistic radio links [6-10].

For example, authors in [7] analyzed the performance of cooperative diversity wireless networks using amplify-and-forward relaying over independent, non-identical, Nakagami-m fading channels and derives the symbol error rate and the outage probability using MGF of the total SNR at the destination.

The authors in [8],[9] analyzed the performance SER of a cooperative communication wireless network with a single relay system over independent and identical Nakagami-m fading channels. The concept of the Altamonte code is transmitted through an amplify-and-forward (AF) relay. The exact SER is determined using the MGF of the total SNR for a particular signal in the case of M-ary phase shift keying M-PSK modulation schemes. In [10] the closed form expressions of the Cumulative Density Function (CDF) and MGF are used to find the Outage Probability. In [11][12] for a given bit error probability of a user, optimal-power-allocation strategy, considering both AF and DF, is proposed to minimize the total energy consumption based on the optimal power allocation between one source and one relay.

In this paper, the performance analysis for all participate AF (AP-AF) cooperative networks in Nakagami-m environment are presented. Also closed-form expressions for the MGF in independent none identically distributed Nakagami-m are derived. Then the obtained MGF is used to derive close-form expression of the outage probability, and SER of different M-PSK signals.

The rest of the paper is structured as follows: Section II describes the system model and the underlying assumption while Section III describes performance and analysis MGF, the outage Probability and SER. Section IV shows the simulation results and analytical results of SER. Section V is drawn the conclusions of this work.

SYSTEM MODEL

We consider the cooperative communications through channels of Nakagami-m fading. The cooperation network system model is composed of a single antenna source, ($1 \le i \le R$) single antenna relays and half duplex AF relays as well as a single destination, as shown in Fig. 1.

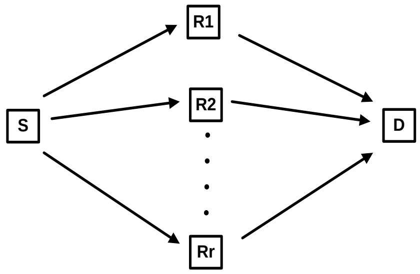

Figure 1: Cooperation Network System Model.

It is assumed that the channel state information is well known at the destination and time division multiplexing (TDM) is adopted in this network. In the first time slot, the source transmits a signal x to the relays then to the destination. The received signals from this transmission can be given:

$$y_{SR_i} = h_{SR_i} x + \eta_{SR_i} \tag{1}$$

$$y_{R_i D} = h_{R_i D} x + \eta_{R_i D} \tag{2}$$

Where h_{SR_i} and $h_{R_i D}$ are the channel gain between the source and the i_{th} relay terminals and the i_{th} relay and the destination terminals, respectively. The complex additive white Gaussian noise (AWGN) at the i_{th} relay is denoted by $\eta_{SR_i} \sim \mathrm{CN}(0, \mathrm{N}_o)$ where N_o is the noise variance.

In the second step of cooperation, the i_{th} relay terminal amplifies its received signal and forwards it to the destination through the $h_{R_i D}$ channel. The destination terminal receives the relay transmission according to:

$$y_{R_i D} = G_i h_{R_i D} y_{SR_i} + \eta_{R_i D} \tag{3}$$

Where $h_{R_i D}$ the channel is gain between the i_{th} relay terminal and the destination terminal and $\eta_{R_i D} \sim \mathrm{CN}(0, \mathrm{N}_o)$ is the complex additive white noise. The i_{th} relay gain denoted by G_i is chosen as $G_i = E_S / (E_S \left| h_{SR_i} \right|^2 + N_o)$ and E_S is the average energy per symbol [4].

$$\gamma_D = \sum_{i=1}^{R} \frac{\gamma_{SR_i} \gamma_{R_i D}}{1 + \gamma_{SR_i} + \gamma_{R_i D}} \tag{4}$$

where $\gamma_{SR_i} = \left| h_{SR_i} \right|^2 E_S / N_o$ and $\gamma_{R_i D} = \left| h_{R_i D} \right|^2 E_S / N_o$ are the instantaneous SNR of the $S - R_i$ and $R_i - D$ hops, respectively

$$\gamma_i^{up} = \min(\gamma_{SR_i}, \gamma_{R_i D}) \geq \frac{\gamma_{SR_i} \gamma_{R_i D}}{1 + \gamma_{SR_i} + \gamma_{R_i D}} \tag{5}$$

Therefore, the upper bound for the equivalent SNR can be written as:

$$\gamma_D^{up} \leq \sum_{i=1}^{R} \gamma_i^{up} \tag{6}$$

The upper bound SNR given by (6) is more suitable for analysis and is shown to be quite accurate at medium and high SNR values [5].

TABLES AND FIGURES

In order to find the moment generation function (MGF), We first drive the cumulative distribution function (CDF) for the end-to–end SNR, γ_D^{up} then we find Probability Density Function (PDF) as a derivative from the CDF. Then, the MGF of γ_D^{up} is calculated using the inverse Laplace transform. We present our analytical framework for the cooperative system with relay selection.

Outage Probability and MGF Analysis

The outage probability (P_{out}) is defined as the probability that the instantaneous total SNR falls below a given threshold. The outage probability of γ_D^{up} can be written as:

$$P_{out} = \int_0^{\alpha} f_{\gamma_D^{up}}(\gamma)\, d\gamma = F_{\gamma_D^{up}}(\gamma) \tag{6}$$

$$F_{\gamma_D^{up}}(\gamma) = \prod_{i=1}^{R} F_{\gamma_i^{up}}(\gamma) \tag{7}$$

where the γ_i^{up} upper bound of the end–to-end SNR. Then we derive the CDF of γ_i^{up} as follows:

$$F_{\gamma_i^{up}}(\gamma) = 1 - [1 - P_r(\gamma_{SR} \leq \gamma)][1 - P_r(\gamma_{RD} \leq \gamma)] \tag{8}$$

$$F_{\gamma_i^{up}}(\gamma) = 1 - [1 - F_{SR}(\gamma)][1 - F_{RD}(\gamma)] \tag{9}$$

$$F_{SR}(\gamma) = \Gamma(m, \frac{m}{\bar{\gamma}_{SR}^2} \gamma) \tag{10}$$

$$F_{RD}(\gamma) = \Gamma(m, \frac{m}{\bar{\gamma}_{RD}^2} \gamma) \tag{11}$$

Where (11) and (12) are the CDF of source to the i-th relay and from the i-th relay to the destination respectively.

Let us assume that:

$$\bar{\gamma}_{SR} = c_1 \bar{\gamma}_C \tag{12}$$

$$\bar{\gamma}_{RD} = c_2 \bar{\gamma}_C \tag{13}$$

where;

$$c_1 = \varepsilon \left| h_{SR} \right|^2 \times \sqrt{P_s} \tag{14}$$

$$c_2 = \varepsilon \left| h_{RD} \right|^2 \times \sqrt{P_R} \tag{15}$$

We consider assuming $c_1 = c_2 = c$, therefore CDF γ_D^{up} can be expressed as:

$$F_{\gamma_i^{up}}(\gamma) = \left(-\frac{2\Gamma(m, \frac{m}{c\bar{\gamma}_C}\gamma)}{\Gamma(m)} + \frac{\Gamma(m, \frac{m}{c\bar{\gamma}_C}\gamma)^2}{(\Gamma(m))^2} \right) \quad (16)$$

$$F_{\gamma_D^{up}}(\gamma) = \prod_{i=1}^{R} \left(-\frac{2\Gamma(m, \frac{m}{c\bar{\gamma}_C}\gamma)}{\Gamma(m)} + \left(\frac{\Gamma(m, \frac{m}{c\bar{\gamma}_C}\gamma)}{\Gamma(m)} \right)^2 \right) \quad (17)$$

Equation (18) represents the outage probability in AP-AF Nakagami-m system.

In order to derive the MGF of γ_D^{up}, we derive the PDF of γ_i^{up} from its CDF equation as follows:

$$f_{\gamma_i^{up}}(\gamma) = \left(-\underbrace{\frac{2(m-1)!e^{-\gamma}\sum_{q=0}^{m-1}\frac{m^q\gamma^{2q}}{c\bar{\gamma}_C}}{a}}_{a1} + \underbrace{\left(\frac{((m-1)!)^2 e^{-2\gamma}\left(\sum_{q=0}^{m-1}\frac{m^q\gamma^{2q}}{c\bar{\gamma}_C}\right)^2}{a} \right)}_{a2} \right) \quad (18)$$

$$= -\underbrace{\frac{2(m-1)!e^{-\gamma}\sum_{q=0}^{m-1}\frac{m^q\gamma^{2q}}{c\bar{\gamma}_C}}{a}}_{a1} + \underbrace{\frac{((m-1)!)^2 e^{-2\gamma}\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k}e^{-2\gamma}\prod_{q=1}^{m-1}\sum_{q=0}^{m-1}\frac{m^q\gamma^{2q}}{c\bar{\gamma}_C}}{a}}_{a2} \quad (19)$$

a_1 and a_2 are solved by using the following relationships:

$$\frac{\partial F_{\gamma_i^{up}}}{\partial\gamma} = ((m-1)!)^2 e^{-2\gamma}\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k}\prod_{q=1}^{m-1}\left(2q\frac{m^q\gamma^{2q-1}}{c\bar{\gamma}_C} - 2\frac{m^q\gamma^{2q}}{c\bar{\gamma}_C}\right) \quad (20)$$

$$\frac{\partial F_{\gamma_i^{up}}}{\partial\gamma} = (m-1)!e^{-\gamma}\sum_{q=0}^{m-1}(2q\frac{m^q\gamma^{2q-1}}{c\bar{\gamma}_C} - \frac{m^q\gamma^{2q}}{c\bar{\gamma}_C^q}) \quad (21)$$

$$f_{\gamma_i^{up}}(\gamma) = (m-1)!e^{-\gamma}\sum_{q=0}^{m-1}\frac{m^q}{c\bar{\gamma}_C^q}(\gamma^{2q-1} - \gamma^{2q}) + ((m-1)!)^2 \times$$

$$e^{-2\gamma}\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k} \times \prod_{q=1}^{m-1}\frac{m^q}{c\bar{\gamma}_C^q}(2q\gamma^{2q-1} - \gamma^{2q}) \quad (22)$$

Equation (22) represents the pdf of AP-AF Nakagami-m system. The moment generating function can be defined as follows:

$$Mg(s) = E\{e^{-s\gamma}\} = \int_0^\infty e^{-s\gamma} f_\gamma d\gamma \quad (23)$$

The MGF γ_D^{up} can be written from γ_i^{up} as follows:

$$M_{\gamma_D^{up}}(s) = \prod_{i=1}^{R} M_{\gamma_i}(s) \quad (24)$$

Substituting (22) in (23) we can obtain the MGF of γ_i^{up} as follows:

$$M_{\gamma_i^{up}}(s) = (m-1)!e^{-\gamma}\sum_{q=0}^{m-1}\frac{m^q}{c\bar{\gamma}_C^q}[\gamma^{2q-1} - \gamma^{2q}] + [(m-1)!]^2 e^{-2\gamma}\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k}$$

$$\times\prod_{q=1}^{m-1}\frac{m^q}{c\bar{\gamma}_C^q}\int_0^\infty[2q\gamma^{2q-1} - \gamma^{2q}]e^{-s\gamma}d\gamma \quad (25)$$

Solving (24) with using the [13, (3.351.3)]:

$$\int_0^\infty x^n e^{-\mu x} = n!\mu^{-n-1} \quad (26)$$

We get the MGF of γ_i^{up} as follows:

$$M_{g_{\gamma_i^{up}}}(s) = 2(m-1)!\sum_{q=0}^{m-1}\phi_1[N!(s+1)^{-N} - (N-1)!(s+1)^{-N-1}] - [m-1]^2!$$

$$\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k}\prod_{q=0}^{m-1}\phi_1[N!(s+2)^{-N} - (N-1)!(s+2)^{-N-1}] \quad (27)$$

where $N = 2q$, $\phi_1 = \frac{m^q}{c\bar{\gamma}_C^q}$ substituting (27) in (24) to obtain MGF of as follows:

$$M_{g_{\gamma_D^{up}}}(s) = \prod_{i=1}^{R} 2(m-1)!\sum_{q=0}^{m-1}\phi_1[N!(s+1)^{-N} - (N-1)!(s+1)^{-N-1}]$$

$$-(m-1)^2!\sum_{|k|}^{2}\binom{2}{k_1,\ldots,k_k}\prod_{q=0}^{m-1}\phi_1\left(N!(s+2)^{-N} - (N-1)!(s+2)^{-N-1}\right) \quad (28)$$

Symbol Error Rate

In this section we derive the SER MGF derived in previously with the help of partial fraction expansion. Using the MGF in (28) we estimate the SER for M-PSK signals as follows:

i. Binary Signals: The average SER for binary signals is given by [5]

$$P_{SER} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{g_{\gamma_D^{up}}}\left(\frac{g}{\sin^2\theta}\right)d\theta,$$

where $g = \sin^2(\frac{\pi}{M_o})$

As g = 1 for BPSK and g = 0.5 for orthogonal BFSK. By substituting (28) into (29) and after some manipulations the average SER in this case can be expressed as:

$$P_{SER} = \prod_{i=1}^{R}\Big[2(m-1)^2!\sum\phi\Big|\frac{N}{\pi}!\int_0^{\frac{\pi}{2}}\sum_{j=0}^{N}\binom{N}{j}\left(\frac{\sin^2\theta}{c+\sin^2\theta}\right)^N d\theta$$

$$-\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{c+\sin^2\theta}\right)^{N+1}\frac{(N-1)}{\pi}!d\theta\Bigg]-[M-1]^2\,!\sum_{|k|}^{2}\binom{2}{k_1,\ldots\ldots,k_k}\times$$

$$\prod_q^{m-1}\phi\left[\frac{N}{\pi}!\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta}{\left(c+2\sin^2\theta\right)}\,d\theta-\frac{(N-1)}{\pi}!\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{c+2\sin^2\theta}\right)^{N+1}d\theta\right]\Bigg] \quad (30)$$

Using partial fraction expansions and after some manipulations, (30) can be written for the BPSK signaling as:

$$P_{SER}=\prod_{i=1}^{R}\Bigg[2(m-1)^2!\sum\phi\left[N!\sum_{j=0}^{N}\binom{N}{j}I_1(c)^N-(N-1)!\sum_{n=0}^{N}I_1(c)^{N+1}\right]-$$

$$[M-1]^2\,!\sum_{|k|}^{2}\binom{2}{k_1,\ldots\ldots,k_k}\prod_q^{m-1}\phi\left[N^2!I_1(c)^N-(N-1)^2!I_1(c)\right]^{N+1}\Bigg] \quad (31)$$

where the closed-form expression for $I_1(c)$ is given by [14 , 5A.9]:

$$I_1(c)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta+c}\right)d\theta=\frac{1}{2}(1-\sqrt{\frac{c}{1+c}}) \quad (32)$$

ii. M-PSK Signals: The average SER for M-PSK signals can be written as [5]:

$$P_{SER}=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}M_{g_{\gamma_B^{up}}}\left(\frac{g_{M-PSK}}{\sin^2\theta}\right)d\theta \quad (33)$$

where $g_M-PSK=\sin^2(\frac{\pi}{M})$ and a closed-form expression for $I_2(c)$ is given by [14, 5A.15]:

$$I_2(c)=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(\frac{\sin^2\theta}{\sin^2\theta+c}\right)d\theta$$

$$=\frac{(M-1)}{M}\left\{1-\sqrt{\frac{c}{1+c}}\left(\frac{M}{(M-1)\pi}\right)\left[\frac{\pi}{2}+\tan^{-1}\left(\sqrt{\frac{c}{1+c}}\cot(\frac{\pi}{M})\right)\right]\right\} \quad (34)$$

NUMERICAL RESULT

In this Section, we provide numerical results of the illustrate analytical expressions of SER for BPSK and 8-PSK modulation signals in coherent cases for different values of m. Our Analytical expressions are verified with Monte Carlo Simulations.

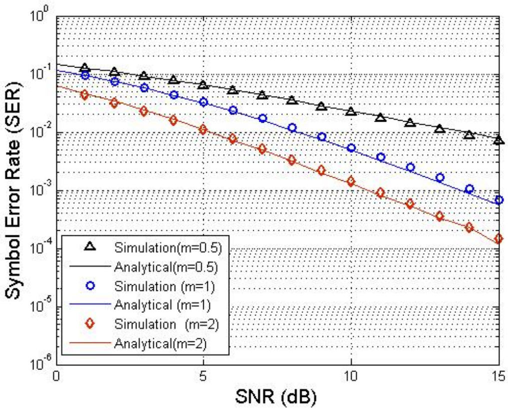

Figure 2

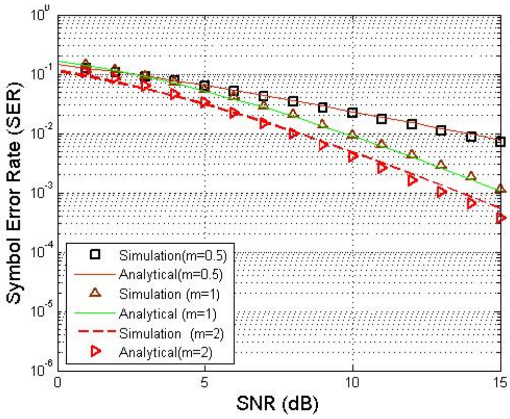

Figure 3

In fig.2, simulation results show that a comparison description of SER for different values of m through using BPSK simulated signals. These results give a clear impact of increasing the values of m on SER values, so that SER decreases as m increases as compared to SNR values.

While fig. 3 shows the performance evaluation of 8PSK simulated signals under Nakagami-m fading model consideration for different values of m. The results of this figure prove that SER values can be decreased as m value increases as compared to SNR values under our proposed Nakagami-m fading model consideration.

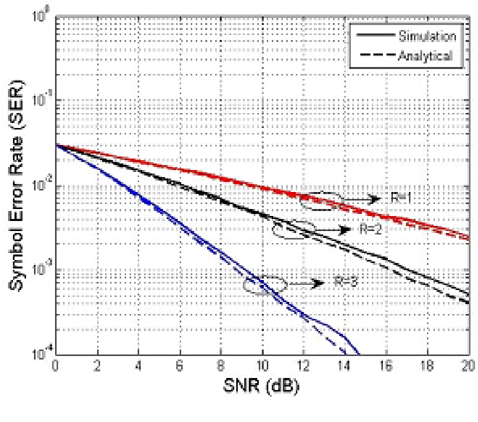

Figure 4

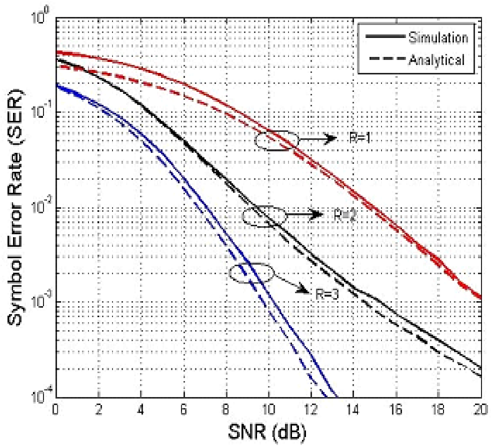

Figure 5

Fig. 4 shows the differences between analytical and simulation results of our proposed Nakagami-m distribution with BPSK modulation taking into account the number of antenna arrays (R). The results of this figure give a good behaviour of our proposed model since SER value is decreased as the number of Antenna arrays (R) is increased with respect to SNR value. While fig. 5 shows the same behaviour for 8PSK modulated signals.

REFRENCES

[1] A. Sendonaris, E. Erkip, User cooperation diversity part i: System description and use of cooperative diversity, IEEE Transaction on Information Theory, 51(11), pp.1927-1938. 2003.

[2] J. N. Laneman, D. N. C. Tse and G. W. Womell, Cooperative diversity in wireless networks: efficient protocols and outage behavior, IEEE Transaction on Information Theory, 50(12): 3062-3080.2004.

[3] Ali A. Mohammed, Li Yu, and Manar Al-Kali. Performance Analysis of Amplify-and-Forward Cooperative Networks in Non-Identically Distributed Nakagami-m Channels with Best RelayingSelection. Journal of Communications. 2015; 10(6), pp.396-402.

[4] Abbaspour-Asadollah, M. and Soleimani-Nasab, E. Performance analysis of cooperative wireless body area networks over Gamma fading channels. IEEE International Symposium on Telecommunications (IST),380-385,2016.

[5] Torabi, Mohammad, W. Ajib, Performance analysis of amplify-and-forward cooperative networks with relay selection over Rayleigh fading channels. IEEE 69th Conference on Vehicular Technology. pp:1-5, 2009.

[6] M. Nakagami, "The m-distribution–A general formula of intensity of rapid fading," in Statistical Methods in Radio Wave Propagation, W. C. Hoffman, Ed. Oxford, U.K.: Pergamon, pp. 3–36, 1960.

[7] S. Ikki and M. H. Ahmed, "Performance Analysis of Cooperative Diversity Wireless Networks over Nakagami-m Fading Channel," IEEE Communication Letters, vol.1, no. 4, pp.334–336, April 2007.

[8] Atapattu, Saman, and Nandana Rajatheva. "Performance evaluation of Alamouti STC through amplify-forward cooperative relay network over Nakagami-m fading channels IEEE coference on Future Generation Communication and Networking (FGCN), vol. 2, pp.258-263, 2007.

[9] Atapattu, Saman, and Nandana Rajatheva. "Exact SER of Alamouti code transmission through amplify-forward cooperative relay over Nakagami-m fading channels. IEEE International Symposium on Communications and Information Technologies. pp.1429-1433, 2010.

[10] M.Abdel-Hafez, "Exact analysis of cooperative relay wireless communication systems over Nakagami-m fading channels", in IEEE PIMRC , Japan, September 2010.

[11] Soleimani-Nasab, Ehsan, Michail Matthaiou, and Mehrdad Ardebilipour. "On the performance of multi-antenna AF relaying systems over Nakagami-m fading channels." IEEE International Conference on Communications (ICC), pp. 3041-3046, 2013.

[12] Veluppillai Mahinthan, Lin Cai, Jon W. Mark, and Xuemin Shen,"Partner selection based on optimal power allocation in cooperativediversity Systems," IEEE Transaction on Vehicular Technology, vol.57. No.1, Jan 2008.

[13] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products. Academic Press, 2007.

[14] M.K. Simon and M.S. Alouini, Digital Communications over Fading Channels: A Unified Approach to Performance Analysis. Wiley, 2000.