International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 6 (2018) pp. 3150-3156

Numerical solution of Cahn-Hiliard Equation

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Abstract

In this paper, we use the reduced differential transform method (RDTM) and new iterative method (NIM) to present solution of the nonlinear Cahn-Hiliard equation (CHE) with initial conditions, which are able to solve linear and nonlinear partial equations and the use of a very simple and less than other methods in solutions and accuracy.

Keywords : Cahn-Hiliard equation; Reduced differental Transform Method (RDTM) ; New iterative method(NIM).

INTRODUCTION

The Cahn-Hiliard equation $([21],[22])$ finds applications in diverse fields.In complex flyids and soft matter (inter facial fluid flow, polymerscipnce and in industrial applications) we found some exact solutions of the equations by considering a modified extended tanh function method A numerical solution to Cahn-Hilliard equation is obtained using NIM method $([13]-[15],[21],[22])$ and RDTM method $([4],[7],[9],[25])$.

This equations is very crucial in matherials many articles have investigated mathematically and numerically this equation The authors in[20] using solutions of the Cahn-Hilliard equation $([21],[22])$.

We are interested in the Cahn-Hilliard equation in its simplest, one-dimensional form[2],

$$u_t = u_{xx} + u - u^3 \qquad u \in \mathbb{R} \tag{1}$$

The equation was originally introduced as a model for phase-separation in binary alloys, and has since been used to describe the formation and annihilation of patterns in many contexts, including phase transitions in material science [3], polymer- and protein dynamics $([1],[11])$, and pattern formation in uids [10]. Phenomenologically, this equation reproduces qualitatively and sometimes even quantitatively the spontaneous formation of patterns from homogeneous equilibrium and a subsequent evolution of characteristic wavelengths through a coarsening process. In bounded, onedimensional domains, equipped with Neumann boundary conditions $u_x = u_{xx} = 0$ at $x = 0;\ L$ or with periodic boundary conditions, the dynamics of the Cahn-Hilliard equation is fairly well understood.As $t \to \infty$, solutions converge to the global attractor, which consists of equilibria and heteroclinic orbits between them. Equilibria and their stability properties can be characterized completely, and, to some extent, existence of heteroclinic connections is known.

REDUCED DIFFERENTIAL TRANSFORMMETHOD (RDTM) [8]

Let, suppose that $u(x;t)$ can be represented two variable functions as a product of two single variable functions $f(x)$ and $g(t)$ to show following manner $([4]-[7])$

$$u(x;t) = f(x)g(t) \tag{2}$$

From the similar meaning of definition of Differential Transform Method and its properties,we can write the transforming form of RDTM $([4]-[7])$

$$u(x;t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} U_k(x)t^k, \tag{3}$$

where $U_k(x)$ is called t dimensional spectrum function of $u(x;t)$. If function $u(x;t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest,then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} \tag{4}$$

Thus, from (4), it can be written the inverse transform of a sequence $U_k(x)_k^{\infty} = 0$

$$u(x;t) = \sum_{k=0}^{\infty} U_k(x)t^k \tag{5}$$

then combining (4) and (5), we obtain the RDTM solution as

$$u(x;t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}t^k \qquad (6)$$

If we consider the expressions $(4),(5)$ and (6), it's clearly shown that the concept of the reduced di erential transform is derived from the power series expansion.So, we give a table which included fundamental transformation properties of RDTM in Table 1. The proofs of Table 1 and the basic de nitions of reduced di erential transform method can be found in [5]. For illustration of the proposed method, we write the Cahn-Hiliard Equation (CHE) (1) in the standard operator form $([4]-[7])$:

$$L(u(x,t))+Nu(x,t)=g(x,t) \qquad (7)$$

with initial condition

$$u(x;0)=f(x) \qquad (8)$$

where $L=\dfrac{\partial}{\partial t}$ is a linear operator, $Nu(x;t)$ is a nonlinear terms and $g(x;t)$ inhomogeneous term. According to the RDTM and Table 1, we can construct the following iteration formula [4-7]

$$(k+1)U_{k+1}(x)=G_k(x)-NU_k(x). \qquad (9)$$

Here, $U_k(x)$, $G_k(x)$ and $NU_k(x)$ are the transformations of the functions $L(u(x,t))$ $g(x;t)$ and $Nu(x;t)$ respectively. From the initial condition, we write

$$U_0(x)=f(x) \qquad (10)$$

Substituting (10) into (9) and by straightforward iterative calculations, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $U_k(x)_k^n=0$ gives the approximation solution as

$$\bar{u}_n(x,t)=\sum_{k=0}^{n}U_k(x)t^k \qquad (11)$$

where n is order of approximate solution. Therefore, the exact solution of the problem is given by $([4]-[7])$

$$u(x;t)=\lim_{x\to\infty}\bar{u}_n(x,t). \qquad (12)$$

Table 1. Basic transformations of RDTM for some functions

Functional Form	Transformed Form
$u(x,t)$	$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}$
$w(x,t)=u(x,t)\pm v(x,t)$	$W_k(x)=U_k(x)\pm V_k(x)$
$w(x,t)=\alpha u(x,t)$	$W_k(x)=\alpha U_k(x), \alpha\ constant$
$w(x,t)=x^m t^n$	$x^m\delta(k-n), \delta(k)=\begin{cases}1 & k=0\\ 0 & k\neq 0\end{cases}$
$w(x,t)=x^m t^n u(x,t)$	$W_k(x)=x^m U_{k-n}(x)$
$w(x,t)=u(x,t)v(x,t)$	$W_k(x)=\sum_{r=0}^{k}U_r(x)V_{k-r}(x)\ =\sum_{r=0}^{k}V_r(x)U_{k-r}(x)$
$w(x,t)=\frac{\partial^r}{\partial t^r}u(x,t)$	$W_k(x)=(k+1)(k+2)...(k+r)U_{k+r}(x)$
$w(x,t)=\frac{\partial}{\partial x}u(x,t)$	$W_k(x)=\frac{d}{dx}\ U_k(x)$
$w(x,t)=\frac{\partial^2}{\partial t^2}u(x,t)$	$W_k(x)=\frac{\partial^2}{\partial t^2}U_k(x)$

BASIC IDEA OF NIM

To describe the idea of the NIM, consider the following general functional equation $([12]-[22])$:

$$u(x)=f(x)+N(u(x)), \qquad (13)$$

where N is a nonlinear operator from a Banach space B $\to$ B and f is a known function. We are looking for a solution u of (13) having the series form :

$$u(x)=\sum_{i=0}^{\infty}u_i(x). \qquad (14)$$

The nonlinear operator N can be decomposed as follows :

$$N\left(\sum_{i=0}^{\infty}u_i\right)=N(u_0)+\sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty}u_j\right)-N\left(\sum_{j=0}^{\infty}u_j\right)\right\} \qquad (15)$$

From Eqs. (14) and (15), Eq. (13) is equivalent to:

$$\sum_{i=0}^{\infty}u_i=f+N(u_0)+\sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty}u_j\right)-N\left(\sum_{j=0}^{\infty}u_j\right)\right\} \qquad (16)$$

We define the recurrence relation

$$u_0=f, \qquad (17)$$

$$u_1=N(u_0), \qquad (18)$$

$$u_{n+1}=N(u_0+u_1+...+u_n)-N(u_0+u_1+...+u_{n-1}),\ n=1,2,3... \qquad (19)$$

then

$$(u_0+u_1+...+u_{n+1})=N(u_0+u_1+...+u_n),\ \ n=1,2,3...$$

$$u=\sum_{i=0}^{\infty}u_i=f+N\left(\sum_{i=0}^{\infty}u_i\right) \tag{20}$$

If N is a contraction, i.e.

$$\|N(x)-N(y)\|\le k\|x-y\|,0<k<1,$$

then

$$\|u_{n+1}\|=\|N(u_0+u_1+...+u_n)-N(u_0+u_1+...+u_{n-1})\|$$
$$\le k\|u_n\|\le...\le k^n\|u_0\| \quad n=0,1,2...,$$

and the series $\sum_{i=0}^{\infty}u_i$ absolutely and uniformly converges to a solution of (13) [23] which is unique, in view of the Banach fixed point theorem [24]. The k-term approximate solution of (13) and (14) is given by $\sum_{i=0}^{k-1}u_i$

IMPLEMENTATION OF PRESENTED METHOD

Reduced differential transform method (RDTM) for solving Cahn-Hiliard Equation (CHE) following [9] :

$$u_t=u_{xx}+u-u^3 \tag{21}$$

with initial condition

$$u(x,0)=1/(1+e^{\frac{x}{\sqrt{2}}}) \tag{22}$$

Let, $U_k(x)$ denotes transformation form of the function $u(x;t)$. Then, by using the basic properties of the reduced di erential transformation as shown in Table 1, we can write the transformed form of equation (22) as

$$(k+r)U_{k+1}(x)=\sum_{r=0}^{k}\frac{d^2}{dx^2}U_{k-r}(x)-\left(\sum_{r=0}^{\infty}U_r(x)\right)^3+\sum_{r=0}^{\infty}U_r(x) \tag{23}$$

and using the initial condition (22), we get the reduced transform form

$$u_0(x)=1/(1+e^{\frac{x}{\sqrt{2}}}) \tag{24}$$

Now, put (24) into place (23), from hence we have the $U_k(x)$ values following :

$$u_1(x,t)=e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t \tag{25}$$

$$u_2(x,t)=\frac{-9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}$$
$$-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}$$
$$-\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}$$
$$-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-$$
$$\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}$$
$$-\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+$$
$$\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}$$
$$-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)$$

Thus, if we continue this process and also the inverse transformation of the set of $U_k(x)_k^{\infty}=0$ values are written:

$$\sum_{k=0}^{\infty}U_k(x)t^k=1/(1+e^{\frac{x}{\sqrt{2}}})+e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t \tag{26}$$

$$+\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t-\frac{9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}$$

$$+\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}$$

$$-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}$$

$$-\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}$$

$$-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)+\ldots$$

Arranging (26) and from (5) and (6), we obtain RDTM solution of (21) as

$$u(x,t)=\lim_{n\to\infty}u_n(x,t)=\sum_{k=0}^{\infty}U_k(x)t^k=1/(1+e^{\frac{x}{\sqrt{2}}})+e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2} \quad (27)$$

$$-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t-\frac{9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-$$

$$3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]$$

$$\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]$$

$$\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}-\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}$$

$$\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)+\cdots$$

New iterative method (NIM) for solving Cahn-Hiliard Equation (CHE) ([21],[22]) :

$$u_t=u_{xx}-u^3+u \quad (28)$$

subject the initial condition

$$u(x,0)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}} \qquad (29)$$

from (17) and (29) ,we obtain

$$u_0=(x,t)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}$$

Therefore ,The initial value problem (28) and (29) is equivalent to the following integral equations:

$$u(x,t)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}+I_t\left(u_{xx}-u^3+u\right)$$

Taking

$$N(u)=I_t\left(u_{xx}-u^3+u\right)$$

Therefore from (17),(18) and (19),we can obtain easily the following first few components of the new iterative solution for the equation (28) and(29)

$$u_0(x,t)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}$$

$$u_1(x,t)=e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t$$

$$u_2=\frac{-9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-$$

$$\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-$$

$$\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}-\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+$$

$$\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)$$

And the rest of the components of iteration formula (20) are obtained. The approximate solution which involves few terms is given by

$$u=\sum_{i=0}^{2}u_i=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}+e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+$$

$$\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t-\frac{9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+$$

$$\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}$$

$$-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}-$$

$$\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}$$

$$-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)+\cdots$$

CONCLUSION

Cahn-Hiliard Equation is solved numerically by (RDTM) and (NIM). The solutions obtained by (RDTM) and (NIM) show that it has higher accuracy same time presented method are more quickly Additionally, we can say that (RDTM) and (NIM) are very simple and powerful numerical method to solve various nonlinear partial differential equations.

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