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Reliability and Optimum Analysis for Number of Standby Units in a System Working with One Operative Unit

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Abstract

Reliability and Optimum analysis for number of standby units in a system which works with one operative unit is carried out. Standby units are the units kept for the standby (spare) mode. Failure in a system may occur due to various causes which may include electric loads, energy loads and fuse replacement problems etc which occur frequently and hence there is a need of standby units as otherwise user may suffer many losses. One may also face a problem as to how many standby units there should be for a system working with single operative unit. The present paper deals with the optimization of number of standby units for a system working with one operative unit by making use of Semi-Markov processes and regenerative point technique. The results and discussion reveal as to whether no or one or two standby units be used for a system working with one operative unit.

Keywords: Standby, Semi-Markov process, Regenerative point technique, Operative, Optimization and Availability.

INTRODUCTION

Though there is an improvement in technology every day, yet the failure in the systems is an unavoidable phenomenon. Such failures may lead to heavy losses. One of the ways to minimize losses is to use redundancy. Standby systems often find applications in various industrial and other setups. In the literature of reliability, lot of models have been developed introducing redundancy by various researchers. Osaki and Asakura (1972) introduced reliability analysis of a two-unit standby redundant system with preventive maintenance. Nakagawa (1980) studied an inspection policy for a standby unit by taking a standby electric generator as an example. Venkateswarlu and Kochar (1982) discussed about the cost effectiveness of introducing standby into a production system. Nakagawa (1984) determined the problem about the number of units minimizes the expected cost per unit time. Yun (1989) also discussed the problem of i) Optimal number of units with inspection ii) Optimal number of units without inspection. In a supplement the work has been done on the systems: a) System without standby, b) Standby system without preference, and c) Standby system with preference. Tuteja et al. (1991) analysed a two-unit system (one operative and other standby) with partial failures and two types of repairmen and subject to random inspection. Kumar et al. (1996) introduced the idea of patience time together with instruction time and investigated two-unit cold standby system with two types of repairman. Lim et al. (2000) introduced an idea of the analysis of system reliability with dependent repair models. Khaled and Mohamed (2005)studied about the profit analysis of a two unit cold standby system with preventive maintenance and random change in unit. Goyal et al. (2009) analyzed two-unit cold standby system working in a sugar mill with operating and rest periods. Mahmoud and Moshref (2010) studied on a two-unit cold standby system considering hardware, human error failures and preventive maintenance. Eryilmaz and Tank (2012) studied a series system with two active components and a single cold standby unit. Manocha and Taneja (2015) studied the stochastic analysis of a two-unit cold standby system with arbitrary distributions for life, repair and waiting times. Taneja et al. (2016) discussed about the comparative analysis of two reliability models with varying demand.

Rahbi et al. (2017) studied about the reliability analysis of rodding anode plant in aluminium industry. Adlakha et al. (2017) revealed about the reliability and cost-benefit analysis of a two-unit cold standby system used for communication through satellite with assembling and activation time. So, lot of work has been done by many researchers taking standby units. Though, study to optimize the number units has also been carried out by some researchers yet the optimum analysis was not carried out on the basis of profit analysis using various measures like MTSF, availability of the system, busy period and expected number of the visits of the repairman in these studies and thereafter. Thus, in the present paper, we carry out the reliability and optimum analysis for number of standby units considering the following system comprising i) one operative unit no standby unit; ii) one operative unit one standby unit; iii)one operative unit two standby units. Discussion and comparative analysis has been made for the models discussed. We have done optimum analysis comparing three cases i.e. with no cold standby units upto two as the system comparing more than two standby units with one operative unit does not seen feasible.

Various measures of system effectiveness are obtained for each of the models by making use of regenerative point technique and semi-Markov processes. The profit aspect has been taken into consideration on the basis of various measures of system effectiveness to arrive at a conclusion regarding optimum number of standby units.

NOMENCLATURE

λ	Constant rate of failure
g(t),G(t)	p.d.f. and c.d.f. of the repair rate
Op	Operative unit
Cs	Standby unit
F_r	Failed unit under repair
F_{wr}	Failed unit is waiting for the repair
F_R	Repair of the failed unit is continuing from previous state
C_0	Revenue per unit up time
C_1	Cost per unit up time for which the repairman is busy
C_2	Cost per visit of the repairman
IC	Installation cost of an additional identical unit
P_i	Profit of model i; i=1,2,3
$\Phi_i(t)$	c.d.f. of the first passage time from regenerative state i to a failed state
$q_{ij}(t)$, $Q_{ij}(t)$	Probability density function (p.d.f.), cumulative distribution function (c.d.f.) of the first passage time from regenerative state S_i to a regenerative state S_j
$p_{ij}(t)$	$\lim_{s\to 0} q^*_{ij}(s)$
$A_i(t)$	Probability that system is up and working in full capacity at the instant t given that system entered regenerative state S_i at t=0
$B_i(t)$	Probability that the repairman is busy in repairing the failed unit at instant t given that the system started from regenerative state S_i at t=0
$V_i(t)$	Expected number of visits of the repairman in (0, t]; given that the system entered regenerative state S_i at t=0

Assumptions

1. The system is operative with one operating unit and no standby unit / one standby unit / two standby units.
2. All random variables are independent.
3. Failures are assumed to follow exponential distribution, whereas the repair times have arbitrary distributions.
4. After every repair, the system becomes operative.
5. The operating unit operates with or without standby unit.
6. Only one repairman remains at a time with the system who comes immediately as and when required.
7. Units are repaired on FCFS pattern.

ANALYSIS OF THE MODELS

MODEL 1

In this model, we have considered a system wherein initially one unit is operative and no standby unit is there. Possible transitions from one state to other are given as follows:

From	S_0	S_1
To	S_1	S_0

where

S_0 is the state showing the system is operative. S_1 is the state showing the system is failed and under repair. Both these states are regenerative states.

Transition Probabilities And Mean Sojourn Times

$$q_{01}(t) = \lambda e^{-\lambda t} dt \qquad q_{10}(t) = g(t)dt$$

Mean Sojourn time (μ_i) in state, i.e., the expected time to stay in state i is

If T denotes the stay time in the regenerative state *i*, then:

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}{}^{*\prime}(0)$$

$$m_{01} = \int_0^\infty t\lambda e^{-\lambda t}dt = \frac{1}{\lambda} = \mu_0 \qquad m_{10} = \int_0^\infty t g(t)dt = -g^{*\prime}(0) = \mu_1$$

Measures of System Effectiveness

Mean Time to System Failure (MTSF)

To determine the mean time to system failure (MTSF) of the system, we regard the failed states as absorbing states. By probabilistic arguments, we obtain the following recursive relation for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t)$$

MTSF when the system starts from the state '0' is

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 \qquad D = 1$$

Availability

The availability $A_i(t)$ is seen to be satisfy

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$

$$A_1(t) = q_{10}(t) © A_0(t)$$

$$M_0(t) = e^{-\lambda t}$$

In steady-state, the availability of the system is given by

$$A_0 = \frac{N_1}{D_1}$$

where

$N_1 = \mu_0$ $\quad$ $D_1 = \mu_0 + \mu_1$

Similarly,

$N_2 = \mu_1$ $\quad$ $N_3 = 1$

Profit Analysis

At steady state, the expected total Profit (P_1) per unit time incurred to the system is given by

Profit (P_1) = $C_0AC_0^1 - C_1B_0^1 - C_2V_0^1$

where

C_0 = Revenue per unit up time.

C_1 = Cost per unit up time for which the repairman is busy

C_2 = Cost per visit of the repairman

MODEL 2

In this model, we have considered a system wherein initially, one unit is operative and one standby unit is there. Possible transitions from one state to other one given as follows:

From	S_0	S_1	S_1	S_1	S_2
To	S_1	S_0	S_1	S_2	S_1
Via	...	...	S_2	...	...

Where

S_0 = (Op, Cs)

S_1= (Fr, Op),

S_2 = (F_R, F_{wr})

States S_0 and S_1 are regenerative states and S_2 is non-regenerative state.

Transition Probabilities and Mean Sojourn Times

The transition probabilities are:

$q_{01}(t) = \lambda e^{-\lambda t}dt$ $\quad$ $q_{10}(t) = e^{-\lambda t}g(t)dt$

$q_{12}(t) = \lambda e^{-\lambda t}\bar{G}(t)dt$ $\quad$ $q_{11}^{(2)}(t) = (\lambda e^{-\lambda t} © 1)g(t)dt$

Mean Sojourn time (μ_i) in state, i.e., the expected time to stay in state i is

If T denotes the stay time in the regenerative state *i*, then:

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}^{*\prime}(0)$$

$m_{01} = \frac{1}{\lambda}$ $\quad$ $m_{10} = -g^{*\prime}(0)$

$m_{12} = \frac{1}{\lambda} + \lambda g^{*\prime}(0)$ $\quad$ $m_{11}^{(2)} = g^{*\prime}(\lambda) - g^{*\prime}(0)$

From the above values, we conclude that

$m_{01} = \mu_0$ $\quad$ $m_{10} + m_{12} = \mu_1$

$m_{10} + m_{11}^{(2)} = \mu_2$

Measures of System Effectiveness

Mean Time to System Failure (MTSF)

To determine the mean time to system failure (MTSF) of the system, we regard the failed states as absorbing states. Thus

$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t)$

$\phi_1(t) = Q_{10}(t) Ⓢ \phi_0(t) + Q_{12}(t)$

where

$N = \mu_0 + \mu_1$ $\quad$ $D = p_{12}$

Availability

The availability $A_i(t)$ is seen to be satisfy

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{11}^{(2)}(t) © A_1(t)$$

where,

$M_0(t) = e^{-\lambda t}$

$M_1(t) = e^{-\lambda t}\overline{G(t)}$

where

$N_1 = p_{10}\mu_0 + \mu_1$ $\quad$ $D_1 = p_{10}\mu_0 + \mu_2$

Similarly,

$N_2 = \mu_2$ $\quad$ $N_3 = 1$

Profit Analysis

At steady state, the expected total Profit (P_2) per unit time incurred to the system is given by

Profit (P_2) = $C_0AC_0^2 - C_1B_0^2 - C_2V_0^2 - (IC_0)$

where

C_0 = Revenue per unit up time

C_1 = Cost per unit up time for which the repairman is busy

C_2 = Cost per visit of the repairman

IC_0 = Installation cost of a unit per unit time

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MODEL 3

In this model, we have considered a system wherein initially, one unit is operative and two standby units are there. Possible transitions from one state to other one given as follows:

From	S_0	S_1	S_1	S_1	S_1	S_4	S_4
To	S_1	S_0	S_1	S_3	S_4	S_1	S_4
Via	...	...	S_2	S_2	S_2 and S_3	...	S_3

where

S_0 = (Op, Cs, Cs)

S_1= (Fr, Op, Cs)

S_2 = (F_R, Fwr, Op)

S_3 = (F_R, Fwr, Fwr)

S_4= (Op, Fr, Fwr)

States S_0, S_1 andS_4 are regenerative states and S_2 and S_3 are non regenerative states.

Transition Probabilities and Mean Sojourn Times

The transition probabilities are:

$q_{01}(t)= \lambda e^{-\lambda t}dt$ $\quad$ $q_{10}(t)= e^{-\lambda t}g(t)dt$

$q_{11}^{(2)}(t)= (\lambda e^{-\lambda t}©e^{-\lambda t})g(t)dt$ $\quad$ $q_{13}^{(2)}(t)= (\lambda e^{-\lambda t}©\lambda e^{-\lambda t})\overline{G}(t)dt$

$q_{14}^{(2,3)}(t)= (\lambda e^{-\lambda t}©\lambda e^{-\lambda t}©1)g(t)dt$ $\quad$ $q_{44}^{(3)}(t)=(\lambda e^{-\lambda t}©1)g(t)dt$

$q_{41}(t)= g(t).e^{-\lambda t}dt=e^{-\lambda t}g(t)dt$

Mean Sojourn time (μ_i) in state, i.e., the expected time to stay in state i is

If T denotes the stay time in the regenerative state *i*, then

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}^{*\prime}(0)$$

$m_{01}=\frac{1}{\lambda}$ $\quad$ $m_{10}=-g^{*\prime}(\lambda)$

$m_{11}^{(2)}=\lambda g^{*\prime\prime}(\lambda)$ $\quad$ $m_{13}^{(2)}=-\lambda^2\overline{G}^{*\prime\prime}(\lambda)$

$m_{14}^{(2,3)}= -(g^{*\prime}(0)-g^{*\prime}(\lambda)+\lambda g^{*\prime\prime}(\lambda))$ $\quad$ $m_{41}=-g^{*\prime}(\lambda)$

$m_{44}^{(3)}=g^{*\prime}(\lambda)-g^{*\prime}(0)$

From the above values, we conclude that

$m_{01}=\mu_0$ $\quad$ $m_{10}+m_{11}^{(2)}+m_{13}^{(2)}=k_1$(say)

$m_{10}+ m_{11}^{(2)}+m_{14}^{(2,3)}=\mu_3$ $\quad$ $m_{41}+m_{44}^{(3)}=\mu_3$

Measures of System Effectiveness

Mean Time to System Failure (MTSF) :

To determine the mean time to system failure (MTSF) of the system, we regard the failed states as absorbing states. Thus

$\phi_0(t) = Q_{01}(t) Ⓢ\phi_1(t)$

$\phi_1(t) = Q_{10}(t) Ⓢ\phi_0(t) + Q_{11}^{(2)}(t) Ⓢ\phi_1(t) + Q_{13}^{(2)}(t)$

MTSF when system starts from the state '0'is

where

$N= k_1+\mu_0(1-p_{11}^{(2)})$ $\quad$ $D= p_{13}^{(2)}$

Availability

The availability $A_i(t)$ is seen to be satisfy

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11}^{(2)}(t)©A_1(t) + q_{14}^{(2,3)}(t)©A_4(t)$$

$$A_4(t) = M_4(t) + q_{41}(t)©A_1(t) + q_{44}^{(3)}(t)©A_4(t)$$

where,

$$M_0(t) = e^{-\lambda t}$$

$$M_1(t) = \overline{G(t)}e^{-\lambda t} + (\lambda e^{-\lambda t} © e^{-\lambda t})\overline{G(t)}$$

$$M_4(t) = e^{-\lambda t}\overline{G(t)}$$

where

$N_1= p_{41}p_{10}\mu_0+p_{41}k_2+p_{14}^{(2,3)}\mu_4$ $\quad$ $D_1= p_{41}p_{10}\mu_0+(p_{14}^{(2,3)}+p_{41})\mu_3$

and $M_1^*(0)=k_2$(say)

Similarly,

$N_2= (p_{41}+p_{14}^{(2,3)})\mu_3$ $\quad$ $N_3= p_{41}(1-p_{11}^{(2)})+p_{14}^{(2,3)}(1+p_{41})$

Profit Analysis

At steady state, the expected total Profit (P_3) per unit time incurred to the system is given by

Profit (P_3) = $C_0AC_0^3 - C_1B_0^3 - C_2V_0^3$-2*($IC_0$)

where

C_0 = Revenue per unit up time

C_1 = Cost per unit up time for which the repairman is busy

C_2 = Cost per visit of the repairman

IC_0 = Installation cost of a unit per unit time

COMPARATIVE STUDY AMONG THE MODELS

COMPARATIVE ANALYSIS OF THE PROFITS WITH RESPECT TO REVENUE PER UNIT UP TIME

a) Model 1 will be more or less or equally beneficial than Model 2 if according as

P_1-P_2 >0 or <0 or =0

i.e. if ($C_0AC_0^1 - C_1B_0^1 - C_2V_0^1$)-($C_0AC_0^2 - C_1B_0^2 - C_2V_0^2$-($IC_0$)) >0 or <0 or =0

i.e. if $C_0(AC_0^1-AC_0^2)$- $C_1(B_0^1- B_0^2)$- $C_2(V_0^1- V_0^2)$+ IC_0 >0 or <0 or =0 i.e. if $C_0(AC_0^1-AC_0^2)$ > or < or= $C_1(B_0^1- B_0^2)+ C_2(V_0^1- V_0^2)-IC_0$

Hence, No Standby or One Standby may be used according as

$$\text{i.e. if}\begin{cases} C_0 > \text{or} < \text{or} = \dfrac{(C_1(B_0^1-B_0^2)+C_2(V_0^1-V_0^2)-IC_0)}{\left(AC_0^1-AC_0^2\right)}, \text{for} AC_0^1>AC_0^2 \\ C_0 < \text{or} > \text{or} = \dfrac{(C_1(B_0^1-B_0^2)+C_2(V_0^1-V_0^2)-IC_0)}{\left(AC_0^1-AC_0^2\right)}, \text{for} AC_0^1<AC_0^2 \end{cases}$$

b) Model 2 will be more or less or equally beneficial than Model 3 if according as

P_2-P_3 >0 or <0 or =0

i.e. if $(C_0AC_0^2 – C_1B_0^2 – C_2V_0^2-(IC_0))-(C_0AC_0^3 – C_1B_0^3 – C_2V_0^3-2*(IC_0))$ >0 or <0 or =0

i.e. if $C_0(AC_0^2 – AC_0^3)-C_1(B_0^2- B_0^3)-C_2(V_0^2- V_0^3)+ IC_0$>0 or <0 or =0

i.e. if $C_0(AC_0^2 – AC_0^3)$ > or < or=$C_1(B_0^2- B_0^3)+C_2(V_0^2- V_0^3)- IC_0$

Hence, One Standby or Two Standby may be used according as

$$\text{i.e. if}\begin{cases} C_0 > \text{or} < \text{or} = \dfrac{C_1\left(B_0^2- B_0^3\right)+C_2\left(V_0^2- V_0^3\right)- IC_0}{\left(AC_0^2 – AC_0^3\right)}, \text{for} AC_0^2>AC_0^3 \\ C_0 < \text{or} > \text{or} = \dfrac{C_1\left(B_0^2- B_0^3\right)+C_2\left(V_0^2- V_0^3\right)- IC_0}{\left(AC_0^2 – AC_0^3\right)}, \text{for} AC_0^2<AC_0^3 \end{cases}$$

c) Model 3 will be more or less or equally beneficial than Model 1 if according as

P_3-P_1>0 or <0 or =0

i.e. if $(C_0AC_0^3 – C_1B_0^3 – C_2V_0^3-2*(IC_0))-(C_0AC_0^1 – C_1B_0^1 – C_2V_0^1)$>0 or <0 or =0

i.e. if $C_0(AC_0^3- AC_0^1)-C_1(B_0^3- B_0^1)-C_2(V_0^3- V_0^1)-2*(IC_0)$>0 or <0 or =0

i.e. if $C_0(AC_0^3- AC_0^1)$ > or < or = $C_1(B_0^3- B_0^1)+C_2(V_0^3- V_0^1)+2*(IC_0)$

Hence, Two Standby or No Standby may be used according as

$$\text{i.e. if}\begin{cases} C_0 > \text{or} < \text{or} = \dfrac{C_1(B_0^3-B_0^1)+C_2(V_0^3-V_0^1)+2*\left(IC_0\right)}{(AC_0^3-AC_0^1)}, \text{for} AC_0^3>AC_0^1 \\ C_0 < \text{or} > \text{or} = \dfrac{C_1(B_0^3-B_0^1)+C_2(V_0^3-V_0^1)+2*\left(IC_0\right)}{(AC_0^3-AC_0^1)}, \text{for} AC_0^3<AC_0^1 \end{cases}$$

Let us consider the following particular case to illustrate the comparative analysis with respect to revenue per unit up time. For the special case λ=0.03, β=0.07, C_1=1000, C_2=1000, IC_0=2000.

a) P_1-P_2 >0 or < 0 or =0 if revenue per unit up time >10975.263 or < 10975.263 or = 10975.263.

b) P_3-P_1>0 or < 0 or =0 if revenue per unit up time >23141.714 or <23141.714 or =23141.714.

COMPARATIVE ANALYSIS OF THE PROFITS WITH RESPECT TO INSTALLATION COST OF A UNIT

a) Model 1 will be more or less or equally beneficial than Model 2 if according as

P_1-P_2 >0 or <0 or =0

i.e. if $(C_0AC_0^1 – C_1B_0^1 – C_2V_0^1)-(C_0AC_0^2 – C_1B_0^2 – C_2V_0^2-(IC_0))$ >0 or <0 or =0

i.e. if $C_0(AC_0^1-AC_0^2)- C_1(B_0^1- B_0^2)- C_2(V_0^1- V_0^2)+ IC_0$ >0 or <0 or =0

Hence, No Standby or One Standby may be used according as

i.e. if IC_0 > or < or= $-C_0(AC_0^1-AC_0^2)+C_1(B_0^1- B_0^2)+ C_2(V_0^1- V_0^2)$

b) Model 2 will be more or less or equally beneficial than Model 3 if according as

P_2-P_3 >0 or <0 or =0

i.e. if $(C_0AC_0^2 – C_1B_0^2 – C_2V_0^2-(IC_0))-(C_0AC_0^3 – C_1B_0^3 – C_2V_0^3-2*(IC_0))$ >0 or <0 or =0

i.e. if $C_0(AC_0^2 – AC_0^3)-C_1(B_0^2- B_0^3)-C_2(V_0^2- V_0^3)+ IC_0$>0 or <0 or =0

Hence, One Standby or Two Standby may be used according as

i.e. if IC_0 > or < or = $-C_0(AC_0^2 – AC_0^3)+C_1(B_0^2- B_0^3)+C_2(V_0^2- V_0^3)$

c) Model 3 will be more or less or equally beneficial than Model 1 if according as

P_3-P_1 >0 or <0 or =0

i.e. if $(C_0AC_0^3 – C_1B_0^3 – C_2V_0^3-2*(IC_0))-(C_0AC_0^1 – C_1B_0^1 – C_2V_0^1)$>0 or <0 or =0

i.e. if $C_0(AC_0^3- AC_0^1)-C_1(B_0^3- B_0^1)-C_2(V_0^3- V_0^1)-2*(IC_0)$>0 or <0 or =0

i.e. if $-2*(IC_0)$> or < or = $-C_0(AC_0^3- AC_0^1)+C_1(B_0^3- B_0^1)+C_2(V_0^3- V_0^1)$

Hence, Two Standby or No Standby may be used according as

$$\text{i.e.if } IC_0 < \text{ or} > \text{or} = \frac{C_0(AC_0^3-AC_0^1)-C_1(B_0^3-B_0^1)-C_2(V_0^3-V_0^1)}{2}$$

Let us consider the following particular case to illustrate the comparative analysis with respect to installation cost of a unit. For the special case λ=0.2, β=0.5, C_0=1000, C_1=900, C_2=1000.

a) P_1-P_2 >0 or < 0 or =0 if installation cost of a unit >83.146 or < 83.146 or =83.146 .

b) P_2-P_3>0 or < 0 or =0 if installation cost of a unit> 23.803 or < 23.803 or =23.803

c) P_3-P_1>0 or < 0 or =0 if installation cost of a unit >54.4618 or <54.4618 or =54.4618.

COMPARATIVE ANALYSIS OF THE PROFITS WITH RESPECT TO COST PER VISIT OF THE REPAIRMAN:

a) Model 1 will be more or less or equally beneficial than Model 2 if according as

P_1-P_2 >0 or <0 or =0

i.e. if $(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1)-(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2-(IC_0))$ >0 or <0 or =0

i.e. if $C_0(AC_0^1-AC_0^2)- C_1(B_0^1- B_0^2)- C_2(V_0^1- V_0^2)+ IC_0$ >0 or <0 or =0

i.e. if $C_2(V_0^1- V_0^2)$ < or > or = $C_0(AC_0^1-AC_0^2) -C_1(B_0^1-B_0^2)+ IC_0$

Hence, No Standby or One Standby may be used according as

$$\text{i.e. if } \begin{cases} C_2 < \text{or} > \text{or} = \dfrac{(C_0\left(AC_0^1-AC_0^2\right)-C_1(B_0^1-B_0^2)+IC_0)}{\left(V_0^1-V_0^2\right)}, \text{for } V_0^1>V_0^2 \\ C_2 > \text{or} < \text{or} = \dfrac{(C_0\left(AC_0^1-AC_0^2\right)-C_1(B_0^1-B_0^2)+IC_0)}{\left(V_0^1-V_0^2\right)}, \text{for } V_0^1<V_0^2 \end{cases}$$

b) Model 2 will be more or less or equally beneficial than Model 3 if according as

P_2-P_3 >0 or <0 or =0

i.e. if $(C_0AC_0^2 - C_1B_0^2 - C_2V_0^2-(IC_0))-(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3-2*(IC_0))$ >0 or <0 or =0

i.e. if $C_0(AC_0^2 - AC_0^3)-C_1(B_0^2- B_0^3)-C_2(V_0^2- V_0^3)+ IC_0$>0 or <0 or =0

i.e. if $C_2(V_0^2- V_0^3)$ < or > or = $C_0(AC_0^2 - AC_0^3)-C_1(B_0^2-B_0^3)+ IC_0$

Hence, One Standby or Two Standby may be used according as

$$\text{i.e. if } \begin{cases} C_2 < \text{or} > \text{or} = \dfrac{C_0\left(AC_0^2 - AC_0^3\right)-C_1\left(B_0^2- B_0^3\right)+IC_0}{\left(V_0^2- V_0^3\right)}, \text{for } V_0^2>V_0^3 \\ C_2 > \text{or} < \text{or} = \dfrac{C_0\left(AC_0^2 - AC_0^3\right)-C_1\left(B_0^2- B_0^3\right)+IC_0}{\left(V_0^2- V_0^3\right)}, \text{for } V_0^2<V_0^3 \end{cases}$$

b) Model 3 will be more or less or equally beneficial than Model 1 if according as

P_3-P_1>0 or <0 or =0

i.e. if $(C_0AC_0^3 - C_1B_0^3 - C_2V_0^3-2*(IC_0))-(C_0AC_0^1 - C_1B_0^1 - C_2V_0^1)$>0 or <0 or =0

i.e. if $C_0(AC_0^3- AC_0^1)-C_1(B_0^3- B_0^1)-C_2(V_0^3- V_0^1)-2*(IC_0)$>0 or <0 or =0

i.e. if $C_2(V_0^3- V_0^1)$ < or > or = $C_0(AC_0^3- AC_0^1)-C_1(B_0^3-B_0^1)-2*(IC_0)$

$$\text{i.e. if } \begin{cases} C_2 < \text{or} > \text{or} = \dfrac{C_0(AC_0^3-AC_0^1)-C_1(B_0^3-B_0^1)-2*(IC_0)}{(V_0^3-V_0^1)}, \text{for } V_0^3>V_0^1 \\ C_2 > \text{or} < \text{or} = \dfrac{C_0(AC_0^3-AC_0^1)-C_1(B_0^3-B_0^1)-2*(IC_0)}{(V_0^3-V_0^1)}, \text{for } V_0^3<V_0^1 \end{cases}$$

Let us consider the following particular case to illustrate the comparative analysis with respect to cost per visit of the repairman. For the special case λ=0.2, β=0.3, C_0=5000, C_1=2000, IC_0=700, we observe that

a) P_1-P_2 >0 or < 0 or =0 if cost per visit of the repairman >68.60 or < 68.60 or = 68.60 .

b) P_2-P_3>0 or < 0 or =0 if cost per visit of the repairman> 12521.39 or <12521.39 or =12521.39.

c) P_3-P_1>0 or < 0 or =0 if cost per visit of the repairman >25383.78 or <25383.78 or =25383.78.

CONCLUSION

Various measures of system effectiveness have been obtained for each of the following three situations:

i) When one unit is operative and no unit is standby initially

ii) When one unit is operative and one unit is standby initially

iii) When one unit is operative and two units are standby initially

Comparative analysis has been made to see as to how many number of standby units should be used to get optimum profit. The cut off points for the revenue per unit up time, intallation cost of a unit and cost per visit of the repairman have been obtained which reveal as to when no or one or two standby should be used in order to obtain optimum profit.

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