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Power Losses Formula for Optimal Power Flow Problem in Power System

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Abstract

Many of optimization techniques that have been used to solve the Optimal Power Flow (OPF) problem in power system, are used a B-coefficient losses formula, to calculate the power losses in power system, where the B-coefficient are kept constants in the optimization process, in the derivation of these coefficients, the ratio between the reactive and active generation power are assumed to be constant, this assumption is discussed in this paper, where the results shows that this assumption is valid on a narrow region. Another assumption has been proposed in this paper, where these coefficients are derived based on assuming constant generation reactive power, this assumption has better results than the previous one for two power systems, one of them is IEEE-26 buses system.

INTRODUCTION

A reduction of the generation power's cost, is one of the main concerns in power system, where the Optimal Power Flow (OPF) is used in power system, to determine the generation power from each plant in order to get lowest cost, subjected to many constraints such as; achieving the demand power and losses in the transmission lines, the generation power within the generator limits and the voltage and reactive power within stable limits.

Power losses is calculated in different ways during the optimization process, it can be evaluated from the load flow program[1], it can be written in term of the load currents[2], it can be written in terms of buses voltage[3-4], or it can be written in terms of the generation power [5-9], and they are called in this case B-coefficient matrices.

Many of deterministic and heuristic optimization methods that have been used to solve the OPF problem are using the B coefficient losses formula to determine the losses on the transmission lines[5-9], this formula can be evaluated from the system configuration once, where the losses formula is written in terms of the generation power, and during the optimization process, you need only to substitute the value of the generation power to calculate the power losses in power system.

In the derivation of losses formula, the ratio between P and Q in the generation plant is assumed to be constant , where this assumption needs to be discussed more, for different reasons, such as:

- In most of power systems, the electrical companies forced the industrial load, to keep their power factors almost constant and near to one, which means that when these loads need extra power it will generate its extra reactive power by themselves in order to keep their load factors close to one.
- When the power generation is increased, the range of reactive power delivered from the generator is decreased, so if the generator give reactive power near to its maximum value, then if the power supplied from this generator is increased, the limits of reactive power is decreased, which means that this generator will give less reactive power than before.
- There are several sources of the reactive power in power system, such as shunt capacitors, so the generation units are not the only sources of the reactive power, which means that the generation plants are not the only supplier of the increment of the demand reactive power

So for the above reasons, this assumption will be discussed in this paper, and a suggested modification of the losses formula is proposed based on the results.

POWER LOSSES DERIVATION

B coefficient matrices of the power system can be derived as follows[10]:

The injected complex power at each bus S_i equals to

$$S_i = V_i I_i^* = P_i - jQ_i \tag{1}$$

The total losses in power System($P_L + jQ_L$) equals to the summation of the total complex power

$$P_L + jQ_L = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} V_i I_i^* = V_{bus}^T I_{bus}^* \tag{2}$$

Where $V_{bus} = Z_{bus} I_{bus}$, substitute in equ.(2)

$$\begin{aligned} P_L + jQ_L &= (Z_{bus} I_{bus})^T I_{bus}^* \\ &= I_{bus}^T Z_{bus} I_{bus}^* \end{aligned} \tag{3}$$

Where the impedance matrix Z_{bus} is symmetrical matrix, the power losses in the power system equals the real part of equ.(3), and is given by

$$P_L = I_{bus}^T R_{bus} I_{bus}^* \tag{4}$$

R_{bus} is the real value of Z_{bus}

Let define I_D as the total demand current, and it will be equal to the summation of the individual load currents $I_D = I_{L1} + I_{L2} + I_{L3} + ... + I_{Lnd}$, where nd is the number of load buses. then each individual load currents can be written in terms of I_D as follows:

$$I_{Lk} = \ell_k I_D \tag{5}$$

ℓ_k is complex constant fraction, $\ell_k = \dfrac{I_{Lk}}{I_D}$

Assuming Bus one is the slack bus, the voltage of this bus is given as follows:

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 + ... + Z_{1n}I_{1n} \\ &= \sum_{m=1}^{ng} Z_{1m}I_{gm} + \sum_{i=1}^{nd} Z_{1i}I_{Li} \end{aligned} \tag{6}$$

where ng is the number of generation bus in power system.

Substitute (5) in (6)

$$V_1 = \sum_{m=1}^{ng} Z_{1m}I_{gm} + I_D \sum_{i=1}^{nd} Z_{1i}\ell_i \tag{7}$$

Let $T = \sum_{i=1}^{nd} Z_{1i}\ell_i$, and let I_0 is the total current flow away from bus 1, where $V_1 = -Z_{11}I_0$, then equ.7 will be

$$I_{D1} = \frac{-1}{T} Z_{11}I_0 - \frac{1}{T}\sum_{m=1}^{ng} Z_{1m}I_{gm} \tag{8}$$

And the load current is

$$I_{Li} = \ell_i I_D = \frac{-\ell_i}{T} Z_{11}I_0 - \frac{\ell_i}{T}\sum_{i=1}^{ng} Z_{1i}I_{gi} \tag{9}$$

Let $\rho_i = -\dfrac{\ell_i}{T}$

$$I_{Li} = \rho_i Z_{11}I_0 + \rho_i \sum_{m=1}^{ng} Z_{1m}I_{gm} \tag{10}$$

Now, define the bus current $I_{bus}^T = \begin{bmatrix} I_{g1} & I_{g2} & \cdots & I_{gng} & I_{L1} & I_{L2} & \cdots & I_{Lnd} \end{bmatrix}$ in terms of new current vector $I_{new}^T = \begin{bmatrix} I_{g1} & I_{g2} & \cdots & I_{gng} & I_0 \end{bmatrix}$ as follows:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gng} \\ I_{L1} \\ I_{L2} \\ \vdots \\ I_{Lnd} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ \rho_1 Z_{11} & \rho_1 Z_{12} & \cdots & \rho_1 Z_{1ng} & \rho_1 Z_{11} \\ \rho_2 Z_{11} & \rho_2 Z_{12} & \cdots & \rho_2 Z_{1ng} & \rho_2 Z_{11} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_{nd} Z_{11} & \rho_{nd} Z_{12} & \cdots & \rho_{nd} Z_{1ng} & \rho_{nd} Z_{11} \end{bmatrix} \begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gng} \\ I_0 \end{bmatrix} \tag{11}$$

$$I_{bus} = CI_{new} \tag{12}$$

Then P_L will be as follows:

$$P_L = (CI_{new})^T R_{bus} (CI_{new})^* = I_{new}^T C^T R_{bus} C^* I_{new}^* \tag{13}$$

The generation currents can be expressed in term of the generation complex power as follows:

$$I_{gi} = \frac{S_{gi}^*}{V_i^*} = \frac{P_{gi} - jQ_{gi}}{V_{gi}^*} = \frac{1 - j\dfrac{Q_{gi}}{P_{gi}}}{V_{gi}^*} P_{gi} \tag{14}$$

By assuming $\dfrac{Q_{gi}}{P_{gi}}$ is constant then

$$I_{gi} = \psi_i P_{gi} \tag{15}$$

Where, $\psi_i = \dfrac{1 - j\dfrac{Q_{gi}}{P_{gi}}}{V_{gi}^*}$

Now, write equ.(15) in matrix form:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gng} \\ I_0 \end{bmatrix} = \begin{bmatrix} \psi_1 & 0 & \cdots & 0 & 0 \\ 0 & \psi_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_{ng} & 0 \\ 0 & 0 & \cdots & 0 & I_0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gng} \\ 1 \end{bmatrix} \tag{16}$$

$$I_{new} = \Psi P_{Gnew} \tag{17}$$

Where $P_{Gnew}^T = \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gng} & 1 \end{bmatrix}$

Substitute equ.(17) in equ.(13)

$$P_L = (\Psi P_{Gnew})^T C^T R_{bus} C^* (\Psi P_{Gnew})^* = P_{Gnew}^T \Psi^T C^T R_{bus} C^* \Psi^* P_{Gnew}^* \tag{18}$$

Let $H = \Psi^T C^T R_{bus} C^* \Psi^*$, then

$$P_L = \Re[P_{Gnew}^T H P_{Gnew}^*] = P_{Gnew}^T \Re[H] P_{Gnew}^* \tag{19}$$

And

$$\Re[H]=\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1ng} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2ng} & B_{02}/2 \\ \vdots & \vdots & & \vdots & \vdots \\ B_{ng1} & B_{ng2} & \cdots & B_{ngng} & B_{ng0}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0ng}/2 & B_{00} \end{bmatrix} \tag{20}$$

Substitute (20) in (19)

$$P_L=\begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gng} & 1 \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1ng} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2ng} & B_{02}/2 \\ \vdots & \vdots & & \vdots & \vdots \\ B_{ng1} & B_{ng2} & \cdots & B_{ngng} & B_{ng0}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0ng}/2 & B_{00} \end{bmatrix}\begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gng} \\ 1 \end{bmatrix} \tag{21}$$

$$P_L=\begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gng} \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1ng} \\ B_{21} & B_{22} & \cdots & B_{2ng} \\ \vdots & \vdots & \ddots & \vdots \\ B_{ng1} & B_{ng2} & \cdots & B_{ngng} \end{bmatrix}\begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gng} \end{bmatrix} + \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gng} \end{bmatrix}\begin{bmatrix} B_{01} \\ B_{02} \\ \vdots \\ B_{0ng} \end{bmatrix} + B_{00} \tag{22}$$

POWER LOSSES COEFFICIENT

Now, the derivation of losses coefficient will be derived based on assuming the reactive powers from generation units are almost constants, when the demand power is constant, and the generation power is varying, starting from equ.(14), and now assume Q_{gi} is constant:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gng} \\ I_0 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_{ng} & 0 \\ 0 & 0 & \cdots & 0 & \lambda_0 \end{bmatrix}\begin{bmatrix} S_{g1}^* \\ S_{g2}^* \\ \vdots \\ S_{gng}^* \\ 1 \end{bmatrix} \tag{23}$$

$$I_{new} = \lambda S_{gnew}^*$$

Where; $\lambda_i = 1/V_i^*, i = 1:ng$, and $\lambda_0 = -V_1/Z_{11} = I_0$

Substitute equ.(23) in equ.(13)

$$\begin{aligned} P_L &= (\lambda S_{gnew}^*)^T C^T R_{bus} C^* (\lambda S_{gnew}^*)^* \\ &= S_{gnew}^{*T} \lambda^T C^T R_{bus} C^* \lambda^* S_{gnew} \\ &= S_{gnew}^{*T} \mathrm{E} S_{gnew} \end{aligned} \tag{24}$$

Where, $E = \lambda^T C^T R_{bus} C^* \lambda^*$

$$\begin{aligned} P_L &= (P_{gnew}^T - jQ_{gnew}^T)E(P_{gnew} + jQ_{gnew}) \\ &= P_{gnew}^T E P_{gnew} + jP_{gnew}^T E Q_{gnew} - jQ_{gnew}^T E P_{gnew} + Q_{gnew}^T E Q_{gnew} \end{aligned} \tag{25}$$

Now let $E = \begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}$, where size of E_1 is $ng \times ng$, size of E_2 is $ng \times 1$, size of E_3 is $1 \times ng$ and size of E_4 is 1×1, and $E_3 = E_2^T$

$$\begin{aligned} P_L &= \begin{bmatrix} P_G^T & 1 \end{bmatrix}\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} P_G \\ 1 \end{bmatrix} + j\begin{bmatrix} P_G^T & 1 \end{bmatrix}\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} Q_G \\ 0 \end{bmatrix} \\ &- j\begin{bmatrix} Q_G^T & 0 \end{bmatrix}\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} P_G \\ 1 \end{bmatrix} + \begin{bmatrix} Q_G^T & 0 \end{bmatrix}\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} Q_G \\ 0 \end{bmatrix} \end{aligned} \tag{26}$$

$$\begin{aligned} &P_L = P_G^T E_1 P_G + P_G^T(E_2 + jE_1 Q_G) + \\ &(E_3 - jQ_G^T E_1)P_G + E_4 + jE_3 Q_G - jQ_G^T E_2 + Q_G^T E_1 Q_G \end{aligned} \tag{27}$$

We still can write the B coefficient matrices but it will now differ than before as follows:

$$P_L=\begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gng} \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1ng} \\ B_{21} & B_{22} & \cdots & B_{2ng} \\ \vdots & \vdots & \ddots & \vdots \\ B_{ng1} & B_{ng2} & \cdots & B_{ngng} \end{bmatrix}\begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gng} \end{bmatrix} + \begin{bmatrix} B_{01} & B_{02} & \cdots & B_{0ng} \end{bmatrix}\begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gng} \end{bmatrix} + B_{00} \tag{28}$$

Or it can be written as follows:

$$P_L = P_G^T B P_G + B_0 P_G + B_{00} \tag{29}$$

Where,

$B = \Re[E_1]$, $\quad B_0 = \Re[(E_2^T + E_3 - 2jQ_G^T E_1)]$ and $B_{00} = \Re(E_4 + jE_3 Q_G - jQ_G^T E_2 + Q_G^T E_1 Q_G)$

SIMULATION RESULTS

In this section a comparison between the two coefficient formulas will be done by applying them to two power system; one of them is four bus system and the other is IEEE-26 buses system, where the load will be kept constant and the generation power will be varied, and this is the case that are done in optimal dispatch, where the value of the demand power is fixed and known and the generation power will be changed until the minimum cost is achieved. The bus and line data of the four buses system are listed in Table1.

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Table 1. Line data and bus data for Four buses system

Line data					Bus data					
From bus	To bus	R	X	B	Bus	Generation			Load	
						P	$\lvert V \rvert$	δ	P	Q
1	2	0.00704	0.0372	0.0775						
1	3	0.01008	0.0504	0.1025	1		1	0		
2	3	0.00704	0.0372	0.0775	2	3.18	1	-		
2	4	0.01272	0.0636	0.1275	3	-	-	-	2.20	1.3634
					4	-	-	-	2.S0	1.7352

The traditional B coefficient (based on the Q_{gi}/P_{gi} is constant) are as follows:

$$B = \begin{bmatrix} 8.3832 & -0.0494 \\ -0.0494 & 5.9636 \end{bmatrix} \times 10^{-3},$$

$$B_0 = [0.7502 \quad 0.3899] \times 10^{-3} \quad \text{and}$$

$$B_{00} = 9.0122 \times 10^{-5}$$

The modified B coefficient matrix (assume constant Q_{gi}) are as follows:

$$B' = \begin{bmatrix} 4.2822 & -0.0314 \\ -0.0314 & 5.0809 \end{bmatrix} \times 10^{-3},$$

$$B'_0 = [0.2789 \quad -0.8581] \times 10^{-4} \text{and } B'_{00} = 0.0267$$

Now, the load is kept constant, and generation power of bus 2 will be changed, then power losses in the transmission line using the load flow will be determine. At the same time the load losses will be calculated using B coefficient and modified B coefficient matrices, by multiplying them with the generation power using equ. 29. The results are shown in Fig.1, where it is clear from this figure that the modified B coefficient matrices has better estimation of the power losses than the traditional one. Fig.2 shows the reactive power generated from bus 2, where it is almost constant.

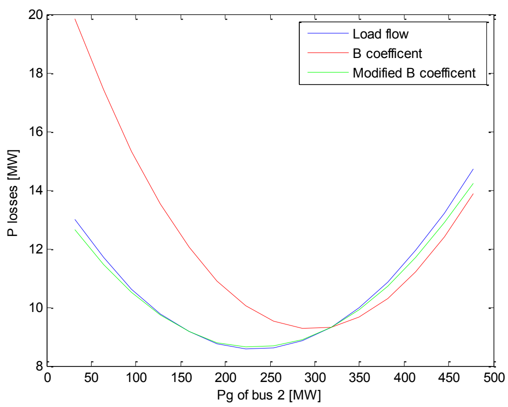

Figure 1. Power losses in the transmission line of four buses system.

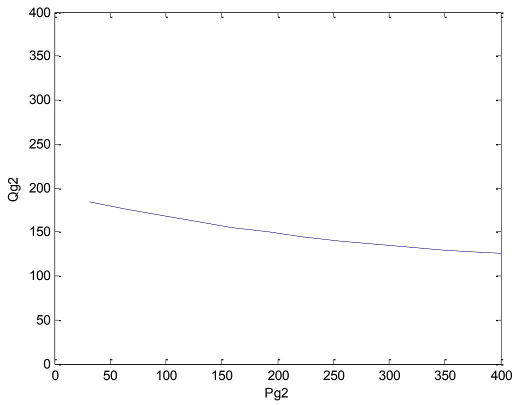

Figure 2. Generation reactive power versus the generation active power at bus 2 for four buses system.

The same work is also done for IEEE-26 bus system, where it has 6 generation buses, the B coefficient and the modified B coefficient was calculated once, then they kept constant, the losses power was calculated using these values and compared with the power losses obtained from the load flow, Fig.3 – Fig.7 show the power losses when each generation bus power changed individually, where the results show again that the modified B coefficient matrices has better power losses values. Fig. 8 shows the generation reactive power from each generation bus, when the real power from the sixth generation bus (bus number 26 in the system), where they almost constant. Fig.9 shows the contour plot of power losses when both of generation power at bus 2 and bus 3 are changed together, the results shows that power losses using modified B coefficient is closer to that calculated using load flow compared to the power losses using traditional B coefficient.

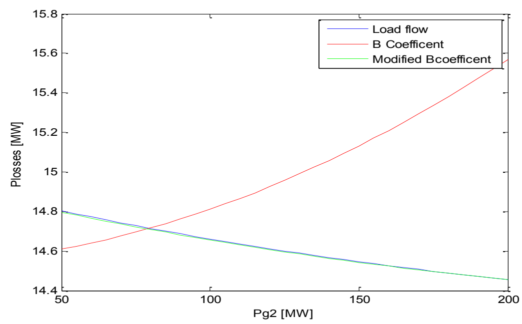

Figure 3. Power losses in the transmission line of IEEE-26 buses system, when generation power in bus2 is changed.

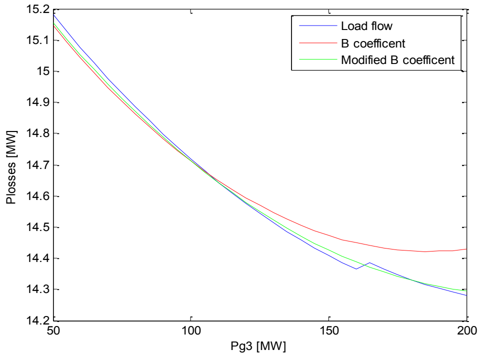

Figure 4. Power losses in the transmission line of IEEE-26 buses system, when generation power in bus3 is changed.

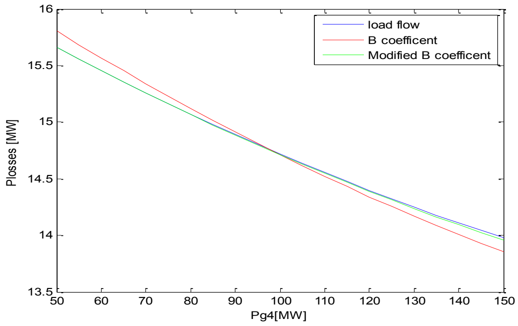

Figure 5. Power losses in the transmission line of IEEE-26 buses system, when generation power in bus4 is changed.

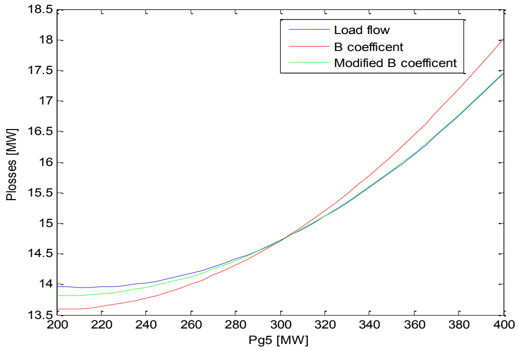

Figure 6. Power losses in the transmission line of IEEE-26 buses system, when generation power in bus5 is changed.

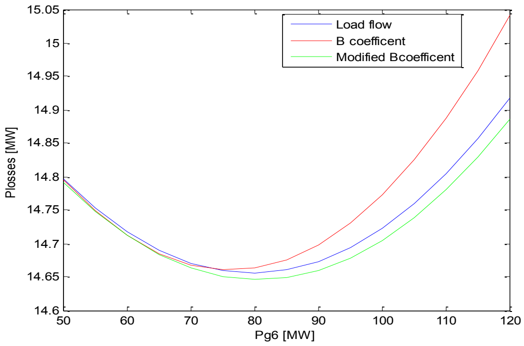

Figure 7. Power losses in the transmission line of IEEE-26 buses system, when generation power in bus26 is changed.

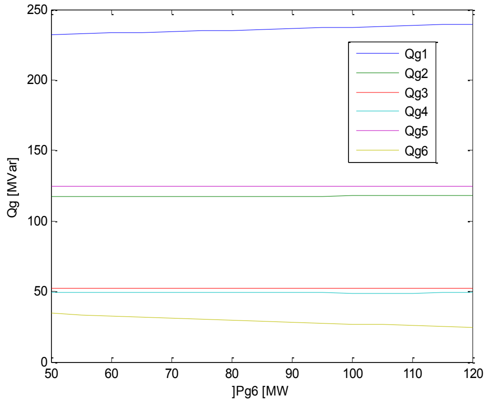

Figure 8. Reactive power from the generation buses when active power at bus 26 is changed for IEEE 26 buses system

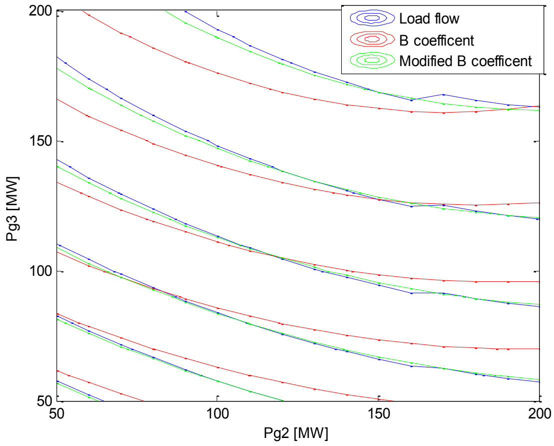

Figure 9. Contour plot for losses power for IEEE 26 buses system, when generation power at bus 2 and bus 3 are changed.

CONCLUSIONS

The results show that the power losses formula based on Assuming constant relation between generation active and reactive power is not accurate for all the generation power's range, where based on the load flow result, assuming constant generation reactive power in power losses formula gives more accurate results in two power systems, one of them is IEE-26 buses system.

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