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Explicit Traveling Wave Solutions for Nonlinear Differential Difference Equations in Mathematical Physics

Khaled A Gepreel [1,2]

[1] *Mathematics Department, Faculty of Science, Taif University, Saudi Arabia.*
[2] *Mathematics Department, Faculty of Science, Zagazig University, Egypt.*

Taher A. Nofal [1,3]

[1] *Mathematics Department, Faculty of Science, Taif University, Saudi Arabia.*
[3] *Mathematics Department, Faculty of Science, Minia University, Egypt.*

Abstract

In this research, we will develop the generalized Kudryashov method from solving the nonlinear partial differential equations (NPDE's) to solve the nonlinear differential difference equations (NDDE's) in mathematical physics. We use the generalized Kudryashov method to construct the traveling wave solutions for some nonlinear differential difference equations via the lattice equation, the discrete nonlinear Klein Gordon equation, the discrete nonlinear Schrodinger equation with a saturable nonlinearity and the quintic discrete nonlinear Schrodinger equation. The proposed method is more effective and powerful to obtain many rational traveling wave solutions for nonlinear differential difference equations. This method can be used for solving more complicated system of nonlinear difference differential equations which application are extended to all branches of fields.

Keywords: Traveling wave solutions, Generalized Kudryashov method, Lattice equation, Discrete Klein Gordon equation, Discrete nonlinear Schrodinger equation with a saturable nonlinearity, Quintic discrete nonlinear Schrodinger equation..

INTRODUCTION

Nonlinear differential difference equations are describing a great nature phenomenon in different branches of sciences such as: particle vibrations in lattices, currents in electrical networks, pulses in biological chains and so on. Also it has many application in queuing problems, modern physics and discretization in solid state and quantum physics. In 1960 Fermi, Pasta, and Ulam [1] have been the focus of many nonlinear studies to the DDEs. On the other hand, there are many analytical methods are successfully extended to solve the nonlinear DDEs by researchers [2–17]. However, no method obeys the strength and the flexibility for finding all solutions to all types of nonlinear DDEs. Zhang etal. [18] and Aslan [19] used the (G'/G)-expansion method to some physically important nonlinear DDEs. Qiong etal. [12] constructed the Jacobi elliptic solutions for nonlinear DDEs. Zhang etal [20] and Gepreel [21,22] have used the Jacobi elliptic function method for constructing the exact solutions to nonlinear difference differential equations. In [23-25] generalized Kudryashov method has used to study the exact solutions for nonlinear evolutions equations and integral partial differential equations. The main objective of this paper, is to develop the generalized Kudryashov method [23-25] from solving the nonlinear partial differential equations to solve the nonlinear difference differential equations. We use the proposed method to calculate the traveling wave solutions for some nonlinear DDEs in mathematical physics. All the results in this research have been made and checked back with the aid of the Maple software package.

DESCRIPTION OF THE GENERALIZED KUDRYASHOV METHOD FOR NONLINEAR DDE's

In this section, we would like to explain the main idea for developing the generalized Kudryashov method from solving the nonlinear partial differential equations to solve the nonlinear difference differential equations. For a given nonlinear DDEs

$$\begin{aligned}&\Delta(u_{n+p_1}(x),\ldots,u_{n+p_k}(x),u'_{n+p_1}(x),\ldots,u'_{n+p_k}(x),\ldots,\\&u^{(r)}_{n+p_1}(x),\ldots,u^{(r)}_{n+p_k}(x),v_{n+p_1}(x),\ldots,v_{n+p_k}(x),\\&v'_{n+p_1}(x),\ldots,v'_{n+p_k}(x),\ldots,v^{(r)}_{n+p_1}(x),\ldots,v^{(r)}_{n+p_k}(x),\ldots.)=0,\end{aligned} \tag{1}$$

where $\Delta=(\Delta_1,\ldots,\Delta_g), x=(x_1,x_2,\ldots,x_m), n=(n_1,\ldots,n_Q)$ and $g,m,Q,p_1,\ldots,p_k$ are integers, $u_i^{(r)},v_i^{(r)}$ denotes the set of all r[th] order derivatives of u_i,v_i with respect x. We outline the mains steps for the generalized Kuderyshov method to NDDEs:

Step 1. We suppose the traveling wave solutions as follows:

$$u_n(x)=U(\xi_n),\quad v_n(x)=V(\xi_n),\ \ldots\ , \tag{2}$$

where

$$\xi_n=\sum_{i=1}^{Q} d_i n_i - \sum_{j=1}^{m} k_j x_j + \xi_0, \tag{3}$$

and $d_i(i=1,..,Q)$, $k_j,(j=1,...,m)$, the phase ξ_0 are constants to be determined later. The transformations (2) and (3) are reduced Eqs. (1) to the following nonlinear differential difference equations

$$\Omega(U(\xi_{n+p_1}),...,U(\xi_{n+p_k}), U'(\xi_{n+p_1}),...,U'(\xi_{n+p_k}),..., U^{(r)}(\xi_{n+p_1}),...,U^{(r)}(\xi_{n+p_k}),V(\xi_{n+p_1}),..., V(\xi_{n+p_k}), V'(\xi_{n+p_1}),...,V'(\xi_{n+p_k}),...,V^{(r)}_{n+p_1}(\xi_{n+p_1}),..., V^{(r)}_{n+p_k}(\xi_{n+p_k}),...)=0, \tag{4}$$

where $\Omega=(\Omega_1,...,\Omega_g)$.

Step 2. We suppose the traveling wave series expansion solutions of Eqs (4) in the following form:

$$U(\xi_n)=\frac{\sum_{i=0}^{N} a_i[Q(\xi_n)]^i}{\sum_{j=0}^{M} b_j[Q(\xi_n)]^j}, \quad V(\xi_n)=\frac{\sum_{l=0}^{L} c_l[Q(\xi_n)]^l}{\sum_{k=0}^{K} h_k[Q(\xi_n)]^i},... \tag{5}$$

where $a_i(i=01,2,...,N)$, $b_j(j=01,2,...,M)$, $c_l(l=01,2,...,L)$, $h_k(k=01,2,...,K)$ are arbitrary constants to be determined later , $Q(\xi_n)$ satisfies the following Bernoulli difference differential equation

$$Q'(\xi_n)=AQ^2(\xi_n)+BQ(\xi_n), \tag{6}$$

and A, B are arbitrary nonzero constants.

Step 3. The relations between $N,M,.K$ and L in the solutions formula (5) are determining by balancing the highest order derivatives of $U(\xi_n),V(\xi_n),...$with the nonlinear terms in Eqs. (4).

Step 4. The general solution of the proposed auxiliary difference differential equation (6) is given by

$$Q(\xi_n)=\frac{BCe^{B\xi_n}}{1-A\,Ce^{B\xi_n}} \tag{7}$$

where C is an arbitrary constant. The iteration relations corresponding to the general solution of Eq. (6) are given by

$$U(\xi_n\pm d)=\frac{\sum_{i=0}^{N} a_i[Q(\xi_n\pm d)]^i}{\sum_{j=0}^{M} b_j[Q(\xi_n\pm d)]^j},$$

$$V(\xi_n\pm d)=\frac{\sum_{l=0}^{L} c_l[Q(\xi_n\pm d)]^l}{\sum_{k=0}^{K} h_k[Q(\xi_n\pm d)]^i},... \tag{8}$$

where $Q(\xi_n+d)$ and $Q(\xi_n-d)$ are given by the formulas

$$Q(\xi_n+d)=\frac{Be^{Bd}Q(\xi_n)}{B+AQ(\xi_n)-Ae^{Bd}Q(\xi_n)} \tag{9}$$

$$Q(\xi_n-d)=\frac{Be^{-Bd}Q(\xi_n)}{B+AQ(\xi_n)-Ae^{-Bd}Q(\xi_n)} \tag{10}$$

Equations (9) and (10) can be written into unified form as follows

$$Q(\xi_n\pm d)=\frac{Be^{\pm Bd}Q(\xi_n)}{B+AQ(\xi_n)-Ae^{\pm Bd}Q(\xi_n)} \tag{11}$$

Step 5. Substituting Eqs. (5),(6),(8) and the given values of $K,L,...$from step.2 into Eqs.(4). Cleaning the denominator and collecting all terms with the same degree of $Q(\xi_n)$ together, the left hand side of Eq. (4) is converted into a polynomial in $Q(\xi_n)$. Setting each coefficients $Q^i(\xi_n)$, $i=0,1,2,...$ of these polynomials to be zero, we derive a set of algebraic equations for a_i,b_j, c_l,h_k.

Step 6. Solving the over determined system of nonlinear algebraic equations by using Maple or Mathematica software package. We end up with explicit expressions for a_i,b_j, c_l,d_k.

Step7. Substituting a_i,b_j, c_l,d_k into Eq.(5) along with (6) and (7), we can finally obtain the traveling wave solutions for nonlinear difference differential equations (1).

APPLICATIONS OF THE GENERALIZED KUDERYASHOV TO NDDE's

In this section, we apply the proposed generalized Kudryashov difference method to construct the traveling wave solutions for some nonlinear DDEs via the lattice equation, the discrete nonlinear Klein Gordon equation, the discrete nonlinear Schrodinger equation with a saturable nonlinearity and the quintic discrete nonlinear Schrodinger equation which are very important in the mathematical physics, modern physics and have been paid attention by many researchers in all fields of science

Generalized Kudryashov method for the nonlinear lattice DDE

In this section, we discuss the traveling wave solutions of the lattice equation which takes the following form [26-28]

$$\frac{du_n(t)}{dt} = (\alpha + \beta u_n + \gamma u_n^2)(u_{n+1} - u_{n-1}), \tag{12}$$

where α, β, γ are arbitrary constants. The lattice equation contains hybrid lattice equation, mKdV lattice equation, modified Volterra lattice equation, and Langmuir chain equation for some special values of the constants α, β, γ see [26-28]. According to the proposed methods, we seek the traveling wave solution of Eq. (12) as follows

$$u_n(t) = U(\xi_n), \qquad \xi_n = d\,n - c_1 t + \xi_0, \tag{13}$$

where d, c_1 and ξ_0 are constants. The transformation (13) permits us converting Eq. (12) into the following NDDE

$$-c_1 U'(\xi_n) = (\alpha + \beta U(\xi_n) + \gamma U^2(\xi_n)) \times [U(\xi_n + d) - U(\xi_n - d)], \tag{14}$$

where $' = d/d\xi_n$. We suppose the solution of Eq.(14) in the following form:

$$U(\xi_n) = \frac{a_0 + a_1\, Q(\xi_n) + a_2 Q^2(\xi_n) + \ldots + a_N Q^N(\xi_n)}{b_0 + b_1\, Q(\xi_n) + b_2 Q^2(\xi_n) + \ldots + b_M Q^M(\xi_n)}. \tag{15}$$

Differentiation Eq.(15) w.r.t. ξ_n and using the Bernoulli difference differential equation (6) to get:

$$U'(\xi_n) = \frac{L_0 + L_1\, Q(\xi_n) + \ldots + L_{N+M+1} Q^{N+M+1}(\xi_n)}{S_0 + S_1\, Q(\xi_n) + \ldots + S_{2M} Q^{2M}(\xi_n)} \tag{16}$$

where $L_i (i = 0,\ldots, M+N+1)$ and $S_j, (j = 0,\ldots,2M)$ are functions in $a_i (i = 0,\ldots,N)$ and $b_i (i = 0,\ldots,M)$. Considering the homogeneous balance between the highest order derivative $U'(\xi_n)$ and the nonlinear term $U^2(\xi_n)$ in (13), we get

$$N - M = 1 \tag{17}$$

Equation (17) has infinity number of solutions so that in the special case when $M = 1$, we get $N = 2$. Thus the solution of Eq. (14) has the following form:

$$U(\xi_n) = \frac{a_0 + a_1\, Q(\xi_n) + a_2 Q^2(\xi_n)}{b_0 + b_1\, Q(\xi_n)}, \tag{18}$$

where a_0, a_1, a_2, b_0 and b_1 are constants to be determined later. The iteration relations corresponding to the solution (18) is given by

$$U(\xi_n + d) = \frac{a_0 + a_1\, Q(\xi_n + d) + a_2 Q^2(\xi_n + d)}{b_0 + b_1\, Q(\xi_n + d)}, \tag{19}$$

$$U(\xi_n - d) = \frac{a_0 + a_1\, Q(\xi_n - d) + a_2 Q^2(\xi_n - d)}{b_0 + b_1\, Q(\xi_n - d)}, \tag{20}$$

where $Q(\xi_n + d)$ and $Q(\xi_n - d)$ are defined by formulas (9) and (10) respectively . With the aid of Maple, substituting Eqs.(9),(10),(18),(19) and (20) into Eq.(14) and collecting all terms with the same power in $Q^i(\xi_n)(i = 0,1,2\ldots)$. Setting the coefficients of these polynomial in $Q^i(\xi_n)(i = 0,1,2\ldots)$ to be zero yields a set of algebraic equations which have the following solutions:

Case 1.

$$c_1 = \frac{\gamma a_2^2 B^3 e^{2Bd}(1 + e^{Bd})}{b_0^2 A^4 (e^{Bd} - 1)^3},$$
$$a_0 = -\frac{\beta A^2 e^{Bd} b_0 - \gamma a_2 B^2 e^{Bd} - \beta A^2 b_0}{2A^2 \gamma (e^{Bd} - 1)},$$
$$a_1 = -\frac{1}{2BA\gamma(e^{Bd} - 1)e^{Bd}}\{e^{2Bd}(\beta A^2 b_0 - 3\gamma a_2 B^2) + e^{Bd}(\gamma a_2 B^2 - 2\beta A^2 b_0) + \beta A^2 b_0\},$$
$$b_1 = \frac{b_0 A(e^{Bd} - 1)}{B\, e^{Bd}},$$
$$\alpha = \frac{1}{4b_0^2 A^4 \gamma (e^{Bd} - 1)^4}\left\{e^{4Bd}(A^4 b_0^2 \beta^2 - \gamma^2 a_2^2 B^4) - 2e^{3Bd}(2A^4 b_0^2 \beta^2 + \gamma^2 a_2^2 B^4) + e^{2Bd}(6\beta^2 b_0^2 A^4 - \gamma^2 a_2^2 B^4) - 4\beta^2 A^4 b_0^2 e^{Bd} + \beta^2 A^4 b_0^2\right\}, \tag{21}$$

where b_0, a_2, γ, β and A, B are arbitrary constants In this case the traveling wave solution takes the form:

$$u_n = \frac{1 - A\, Ce^{B\xi_n}}{b_0(1 - A\, Ce^{B\xi_n}) + b_0 A(e^{Bd} - 1)\, e^{-Bd}\, Ce^{B\xi_n}} \times \left[-\frac{\beta A^2 e^{Bd} b_0 - \gamma a_2 B^2 e^{Bd} - \beta A^2 b_0}{2A^2 \gamma (e^{Bd} - 1)}\right.$$

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$$-\frac{BCe^{B\xi_n}}{2BA\gamma(e^{Bd}-1)e^{Bd}(1-A\,Ce^{B\xi_n})}\{e^{2Bd}(\beta A^2b_0-3\gamma a_2B^2)+e^{Bd}(\gamma a_2B^2-2\beta A^2b_0)+\beta A^2b_0\}+\frac{a_2B^2C^2e^{2B\xi_n}}{(1-A\,Ce^{B\xi_n})^2}\Bigg],\tag{22}$$

where

$$\xi_n=d\,n-\frac{t\gamma a_2^2B^3e^{2Bd}(1+e^{Bd})}{b_0^2A^4(e^{Bd}-1)^3}+\xi_0.$$

We note that, there are other cases which are omitted here for convenience.

Example 2. Generalized Kudryashov method for discrete nonlinear Klein Gordon equation

In this section, we consider the following discrete nonlinear Klein Gordon equation [29,30]:

$$\frac{d^2u_n(t)}{dt^2}=g(u_n)(u_{n+1}+u_{n-1}-2su_n)\tag{23}$$

The non-constant (in contrast to the standard models of harmonic coupling and linear dispersion [30]) function $g(u_n)$ ensures the presence of, nonlinear dispersion , which is critical for the existence of compactly supported solutions and s can take values in the interval [-1,1] . Kevrekidis et al [29] have obtained some exact compaction solutions and claim that this DDE does not have the traveling compact solution. If, we set $g(u_n)=\alpha+\beta u_n^2$ as similar in [29] and take the traveling wave transformation

$$u_n=U(\xi_n),\qquad \xi_n=dn-c_1t+\xi_0,\tag{24}$$

where d , c_1, s and ξ_0 are constants. The transformation (24) permits us converting Eq. (23) into the following form:

$$c_1^2U''(\xi_n)=(\alpha+\beta U^2(\xi_n))[U(\xi_n+d)+U(\xi_n-d)-2sU(\xi_n)],\tag{25}$$

where $'=d/d\xi_n$. Considering the homogeneous balance between the highest order derivative $U''(\xi_n)$ and the nonlinear term $U^3(\xi_n)$ in (25), we have :

$$N-M=1.\tag{26}$$

Equation (26) has infinitely solutions in the special case when $M=1$,we get $N=2$. Thus the solution of Eq. (25) has the following form:

$$U(\xi_n)=\frac{a_0+a_1\,Q(\xi_n)+a_2Q^2(\xi_n)}{b_0+b_1\,Q(\xi_n)},\tag{27}$$

where $a_0,a_1,\ a_2$ and b_0,b_1 are arbitrary constants to be determined later . The iteration relations corresponding to the solution (27) is given by (19) and (20). With the aid of Maple, substituting Eqs.(27),(19) and (20) into Eq.(25) and collecting all terms with the same power in $Q^i(\xi_n)(i=0,1,2\ldots)$. Setting the coefficients of these polynomial in $Q^i(\xi_n)(i=0,1,2\ldots)$. to be zero yields a set of algebraic equations which have the following results:

Case 2.

$$a_2=-\frac{4A^2a_0}{B^2},\qquad c_1=\frac{\pm 2a_0\sqrt{-\beta}}{b_0B},\ \beta<0,$$
$$\alpha=-\frac{a_0^2\beta[1+e^{Bd}]^2}{b_0^2(e^{Bd}-1)^2},\qquad b_1=\frac{-2b_0A}{B},\tag{28}$$
$$a_1=0,\qquad s=1,$$

where A,B,a_0,b_0,d are arbitrary constants. In this case the rational traveling wave solution for the discrete nonlinear Klein Gordon equation has the following form:

$$u_n(t)=\frac{a_0(1-A\,Ce^{B\xi_n})^2-4A^2a_0\,C^2e^{2B\xi_n}}{b_0(1-A\,Ce^{B\xi_n})^2-2Ab_0Ce^{B\xi_n}(1-A\,Ce^{B\xi_n})},\tag{29}$$

where

$$\xi_n=dn\mp\frac{2a_0\sqrt{-\beta}}{b_0B}t+\xi_0.\tag{30}$$

Case 3.

$$a_1=\frac{a_0(-Bb_1+2b_0A)}{Bb_0},$$

$$c_1=\frac{\pm 2a_0\sqrt{-\beta}}{b_0B},\ \beta<0$$
$$\alpha=-\frac{a_0^2\beta[1+e^{Bd}]^2}{b_0^2(e^{Bd}-1)^2},\quad a_2=0,\tag{31}$$
$$s=1$$

where A,B,a_0,b_0,d are arbitrary constants. In this case the rational traveling wave solution for the discrete nonlinear Klein Gordon equation have the following form:

$$u_n(t)=\frac{a_0(1-A\,Ce^{B\xi_n})+\frac{a_0}{b_0}(-Bb_1+2b_0A)Ce^{B\xi_n}}{b_0(1-A\,Ce^{B\xi_n})+b_1BCe^{B\xi_n}}\tag{32}$$

where

$$\xi_n = dn \mp \frac{2a_0\sqrt{-\beta}}{b_0 B}t + \xi_0 \ . \tag{33}$$

Example 3. The generalized Kudryshov method for discrete nonlinear Schrodinger equation with a saturable nonlinearity

In this section, we use the proposed method to discuss the exact solutions for the following nonlinear discrete nonlinear Schrodinger equation with a saturable nonlinearity [31-34]:

$$i\frac{d\psi_n(t)}{dt} + (\psi_{n+1} + \psi_{n-1} - 2\psi_n) + \frac{\eta|\psi_n|^2}{1+\mu|\psi_n|^2}\psi_n = 0, \tag{34}$$

which describes optical pulse propagations in various doped fibers, ψ_n is a complex valued wave function at sites n while μ and η are constants. The discrete nonlinear Schrodinger equation (DNSE) is one of the most fundamental nonlinear lattice model [31-34]. Its arise in nonlinear optics as a model of infinite wave guide arrays [31] and has been recently implemented to describe Bose-Einstein condensates in optical lattices. The class of DNSE model with saturable nonlinearity is also of particular interest in their own right, due to a feature first unveiled in [32]. We separate the solutions of the DNSE (34) in the following form:

$$\psi_n(t) = U(\xi_n)e^{-i(\sigma t+\rho)}, \qquad \xi_n = dn + k, \tag{35}$$

where σ, ρ, d and k are arbitrary constants. The transformation (35) reduced the discrete nonlinear Schrodinger equation (34) to the following difference equations

$$(\sigma - 2)U(\xi_n) + U(\xi_n + d) + U(\xi_n - d) + \frac{\nu U^3(\xi_n)}{1+\mu U^2(\xi_n)} = 0. \tag{36}$$

We suppose the solution of Eq.(36) takes the follows:

$$U(\xi_n) = \frac{a_0 + a_1\ Q(\xi_n) + a_2 Q^2(\xi_n)}{b_0 + b_1\ Q(\xi_n) + b_2 Q^2(\xi_n)}, \tag{37}$$

where a_0, a_1, a_2 and b_0, b_1, b_2 are arbitrary constants to be determined later . The iteration relations corresponding to the solution (37) is given by:

$$U(\xi_n + d) = \frac{a_0 + a_1\ Q(\xi_n + d) + a_2 Q^2(\xi_n + d)}{b_0 + b_1\ Q(\xi_n + d) + b_2 Q^2(\xi_n + d)}, \tag{38}$$

$$U(\xi_n - d) = \frac{a_0 + a_1\ Q(\xi_n - d) + a_2 Q^2(\xi_n - d)}{b_0 + b_1\ Q(\xi_n - d) + b_2 Q^2(\xi_n - d)}, \tag{39}$$

where $Q(\xi_n + d)$ and $Q(\xi_n - d)$ are defined by formulas (9) and (10) respectively . With the aid of Maple, substituting Eqs. (37),(38) and (39) into Eq.(36) and collecting all terms with the same power in $Q^i(\xi_n)(i = 0,1,2...)$. Setting the coefficients of these polynomial in $Q^i(\xi_n)(i = 0,1,2...)$ to be zero yields a set of algebraic equations which have the following families of solutions:

Case 4.

$$a_2 = -\frac{4a_0^2 A^2 - a_1^2 B^2}{4B^2 a_0},$$
$$b_2 = -\frac{(a_1^2 B^2 + 4a_0^2 A^2 - 4a_0 a_1 AB)b_0}{4B^2 a_0^2},$$
$$\eta = -\frac{8(1-e^{Bd})^2 e^{Bd} b_0^2}{(1+e^{Bd})^2 a_0^2}, \tag{40}$$
$$\mu = -\frac{(1-e^{Bd})^2 b_0^2}{(1+e^{Bd})^2 a_0^2},$$
$$\sigma = \frac{2(1-e^{Bd})^2}{(1+e^{Bd})^2}, \qquad b_1 = 0,$$

where b_0, a_0 and A, B are arbitrary constants. In this case the traveling wave solutions of Eq. (36) takes the forms:

$$U(\xi_n) = \frac{4B^2 a_0^3 + 4a_1 B^2 a_0^2\ Q(\xi_n) - a_0(4a_0^2 A^2 - a_1^2 B^2)Q^2(\xi_n)}{4B^2 a_0^2 b_0 - (a_1^2 B^2 + 4a_0^2 A^2 - 4a_0 a_1 AB)b_0 Q^2(\xi_n)}, \tag{41}$$

Substituting Eq. (7) into Eq. (41) we have:

$$U(\xi_n) = \frac{1}{\Gamma}\{4a_0^3(1 - A\ Ce^{B\xi_n})^2 + 4a_1\ Ba_0^2\ \ Ce^{B\xi_n}(1 - A\ Ce^{B\xi_n}) - a_0(4a_0^2 A^2 - a_1^2 B^2)C^2 e^{2B\xi_n}\}, \tag{42}$$

where

$$\Gamma = 4a_0^2 b_0(1 - A\ Ce^{B\xi_n})^2 - (a_1^2 B^2 + 4a_0^2 A^2 - 4a_0 a_1 AB)b_0\ C^2 e^{2B\xi_n}.$$

Consequently the traveling wave solutions of the nonlinear discrete nonlinear Schrodinger equation with a saturable nonlinearity takes the formula

$$\psi_n = \frac{e^{-i(\sigma t+\rho)}}{\Gamma}\{4a_0^3(1 - A\ Ce^{B\xi_n})^2 + 4a_1\ Ba_0^2\ \ Ce^{B\xi_n}(1 - A\ Ce^{B\xi_n}) - a_0(4a_0^2 A^2 - a_1^2 B^2)C^2 e^{2B\xi_n}\} \tag{43}$$

where $\xi_n = dn + k.$

Case 5.

$$a_1=-\frac{a_0A(1-e^{Bd})^2}{Be^{Bd}},\ a_2=-\frac{2a_0A^2(e^{2Bd}+1)}{B^2e^{Bd}},$$
$$b_1=-\frac{b_0A(e^{2Bd}+1)}{Be^{Bd}},\ \eta=-\frac{8(1-e^{Bd})^2e^{Bd}b_0^2}{(1+e^{Bd})^2a_0^2}, \qquad (44)$$
$$\mu=-\frac{(1-e^{Bd})^2b_0^2}{(1+e^{Bd})^2a_0^2},\quad \sigma=\frac{2(1-e^{Bd})^2}{(1+e^{Bd})^2},\ b_2=0,$$

where b_0,a_0 and A,B are arbitrary constants. In this case the traveling wave solutions of Eq. (36) takes the forms:

$$U(\xi_n)=\frac{1}{b_0B^2e^{Bd}-b_0BA(e^{2Bd}+1)Q(\xi_n)}\{a_0B^2e^{Bd} -a_0AB(1-e^{Bd})^2Q(\xi_n)-2a_0A^2(e^{2Bd}+1)Q^2(\xi_n)\}, \qquad (45)$$

Substituting Eq. (7) into Eq. (45) we have:

$$U(\xi_n)=\frac{1}{\Omega}\{-a_0ACe^{B\xi_n}(1-e^{Bd})^2(1-A\,Ce^{B\xi_n}) -2a_0A^2C^2e^{2B\xi_n}(e^{2Bd}+1)+a_0e^{Bd}(1-A\,Ce^{B\xi_n})^2\}, \qquad (46)$$

where

$$\Omega=b_0e^{Bd}(1-A\,Ce^{B\xi_n})^2 -b_0CAe^{B\xi_n}(e^{2Bd}+1)(1-A\,Ce^{B\xi_n})$$

Consequently the traveling wave solutions of the nonlinear discrete nonlinear Schrodinger equation with a saturable nonlinearity takes the formula

$$\psi_n=\frac{e^{-i(\sigma t+\rho)}}{\Omega}\{-a_0ACe^{B\xi_n}(1-e^{Bd})^2(1-A\,Ce^{B\xi_n}) -2a_0A^2C^2e^{2B\xi_n}(e^{2Bd}+1)+a_0e^{Bd}(1-A\,Ce^{B\xi_n})^2\} \qquad (47)$$

where $\xi_n=dn+k$.

Example 4. Generalized Kudryashov method for the quintic discrete nonlinear Schrodinger equation

In this section, we use generalized Kudryashov method to find the traveling wave exact solutions for the following nonlinear quintic discrete nonlinear Schrodinger equation [35-36]:

$$i\frac{d\psi_n(t)}{dt}+\alpha(\psi_{n+1}-2\psi_n+\psi_{n-1})+\beta|\psi_n|^2\psi_n +\gamma|\psi_n|^2(\psi_{n+1}+\psi_{n-1})+\delta|\psi_n|^4(\psi_{n+1}+\psi_{n-1})=0, \qquad (48)$$

where σ,ρ,d and β are arbitrary constants. Nonlinear quintic discrete nonlinear Schrodinger equation describes the propagation of discrete self-trapped beams in an array of weakly coupled nonlinear optical waveguides. Equation (48) was presented for the first time in [36], together with its localized solutions. We separate the solutions of Eq. (48) in the following form:

$$\psi_n(t)=U(\xi_n)\,e^{-iwt},\qquad \xi_n=dn+k, \qquad (49)$$

where d and k are arbitrary constants. The transformation (49) reduced the discrete nonlinear Schrodinger equation (48) to the following difference equations

$$U(\xi_n+d)+U(\xi_n-d)=\frac{(2\alpha-w)U(\xi_n)-\beta U^3(\xi_n)}{\alpha+\gamma U^2(\xi_n)+\delta U^4(\xi_n)} \qquad (50)$$

We suppose the solution of Eq.(50) takes the follows:

$$U(\xi_n)=\frac{a_0+a_1\,Q(\xi_n)+a_2Q^2(\xi_n)}{b_0+b_1\,Q(\xi_n)+b_2Q^2(\xi_n)}, \qquad (51)$$

where a_0, a_1, a_2 and b_0,b_1,b_2 are arbitrary constants to be determined later . The iteration relations corresponding to the solution (50) is given by Eqs. (38) and (39). With the aid of Maple, substituting Eqs. (51),(38) and (39) into Eq.(50) and collecting all terms with the same power in $Q^i(\xi_n)(i=0,1,2\ldots)$. Setting the coefficients of these polynomial in $Q^i(\xi_n)(i=0,1,2\ldots)$ to be zero yields a set of algebraic equations which have the following families of solutions:

Case 6.

$$a_2=-\frac{4a_0A^2}{B^2},\qquad b_1=-\frac{2b_0A}{B},$$
$$w=\frac{2\alpha(1-e^{Bd})^2}{(1+e^{Bd})^2},\qquad \delta=\frac{(1-e^{Bd})^2\beta b_0^2}{8e^{Bd}a_0^2},$$
$$\gamma=-\frac{\beta a_0^2(1+e^{Bd})^4+8\beta b_0^2e^{Bd}(1-e^{Bd})^2}{8e^{Bd}a_0^2(1+e^{Bd})^2},$$
$$a_1=b_2=0, \qquad (52)$$

where b_0,a_0 and A,B are arbitrary constants. In this case the traveling wave solutions of Eq. (50) takes the forms:

$$U(\xi_n)=\frac{a_0B^2-4a_0A^2Q^2(\xi_n)}{b_0B^2-2b_0ABQ(\xi_n)}, \qquad (53)$$

Substituting Eq. (7) into Eq. (53) we have:

$$U(\xi_n)=\frac{a_0(1-A\,Ce^{B\xi_n})^2-4a_0A^2C^2e^{2B\xi_n}}{b_0(1-A\,Ce^{B\xi_n})^2-2b_0ACe^{B\xi_n}(1-A\,Ce^{B\xi_n})}, \qquad (54)$$

Consequently the traveling wave solutions of the nonlinear quintic discrete nonlinear Schrodinger equation (48) takes the formula

$$\psi_n=\left[\frac{a_0(1-A\,Ce^{B\xi_n})^2-4a_0A^2C^2e^{2B\xi_n}}{\Xi}\right]e^{-2\alpha i t\left[\frac{1-e^{Bd}}{1+e^{Bd}}\right]^2} \qquad (55)$$

where $\xi_n=dn+\beta$ and

$$\Xi=b_0(1-A\,Ce^{B\xi_n})^2-2b_0ACe^{B\xi_n}(1-A\,Ce^{B\xi_n}).$$

Case 7.

$$\begin{aligned} &a_1=-\frac{2a_0A}{B}, \qquad b_2=-\frac{4b_0A^2}{B^2},\\ &w=\frac{2\alpha(1-e^{Bd})^2}{(1+e^{Bd})^2}, \qquad \delta=\frac{(1-e^{Bd})^2\beta b_0^2}{8e^{Bd}a_0^2},\\ &\gamma=-\frac{\beta a_0^2(1+e^{Bd})^4+8\beta b_0^2e^{Bd}(1-e^{Bd})^2}{8e^{Bd}a_0^2(1+e^{Bd})^2},\\ &a_2=b_1=0, \end{aligned} \qquad (56)$$

where b_0, a_0 and A, B are arbitrary constants. In this case the traveling wave solutions of Eq. (50) takes the forms:

$$U(\xi_n)=\frac{a_0B^2-2a_0ABQ(\xi_n)}{b_0B^2-4b_0A^2\;Q^2(\xi_n)}, \qquad (57)$$

Substituting Eq. (7) into Eq. (57) we have:

$$U(\xi_n)=\frac{a_0(1-A\,Ce^{B\xi_n})^2-2a_0ACe^{B\xi_n}(1-A\,Ce^{B\xi_n})}{b_0(1-A\,Ce^{B\xi_n})^2-4b_0A^2C^2e^{2B\xi_n}}. \qquad (58)$$

Then the traveling wave solutions of the the nonlinear quintic discrete nonlinear Schrodinger equation (48) takes the formula

$$\psi_n=\left[\frac{\Theta}{b_0(1-A\,Ce^{B\xi_n})^2-4b_0A^2C^2e^{2B\xi_n}}\right]e^{-2\alpha i t\left[\frac{1-e^{Bd}}{1+e^{Bd}}\right]^2}. \qquad (59)$$

where

$$\Theta=a_0(1-A\,Ce^{B\xi_n})^2-2a_0ACe^{B\xi_n}(1-A\,Ce^{B\xi_n})$$

SOME GRAPHICAL BEHAVIOR OF THE TRAVELING WAVE SOLUTIONS FOR NDDE'S

In this section, we plot the traveling wave solutions when the parameter are taken some special choose to illustrate the behavior of solutions. These graphs of the solutions determined the types of the traveling wave solutions. Exact solutions of the results describe the different nonlinear waves:

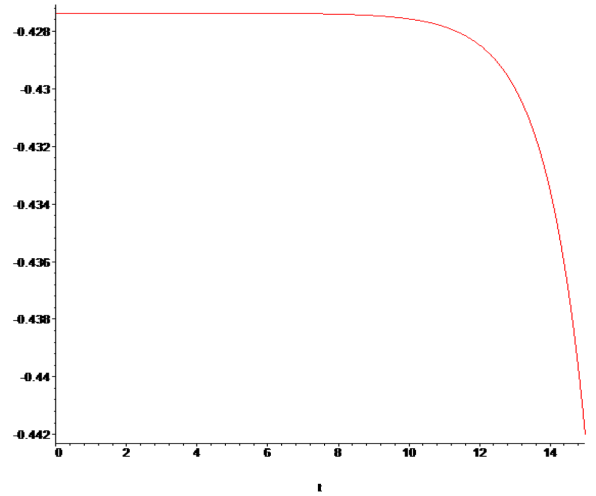

Figure 1. (a) represented the behavior of the traveling wave solution of nonlinear lattice DDE (22) the when $n=5, a2=3, B=1, b0=5, \beta=1, \gamma=7, d=1, A=2, C=5, \xi_0=9$

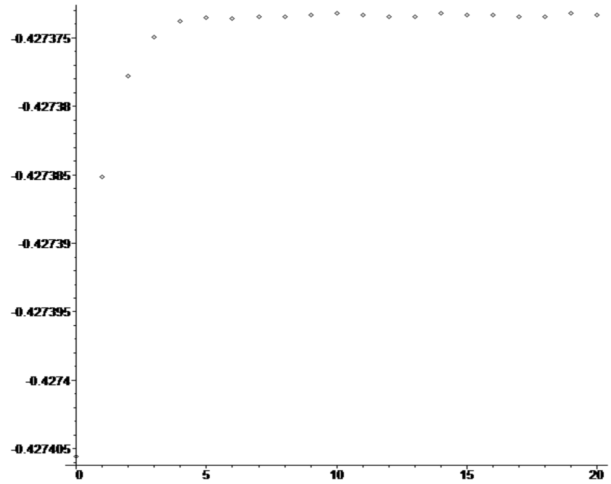

Figure 1. (b) represented the discrete traveling wave solution of nonlinear lattice DDE (22) the when t=2 and $a2=3, B=1, b0=5, \beta=1, \gamma=7, d=1, A=2, C=5, \xi_0=9$

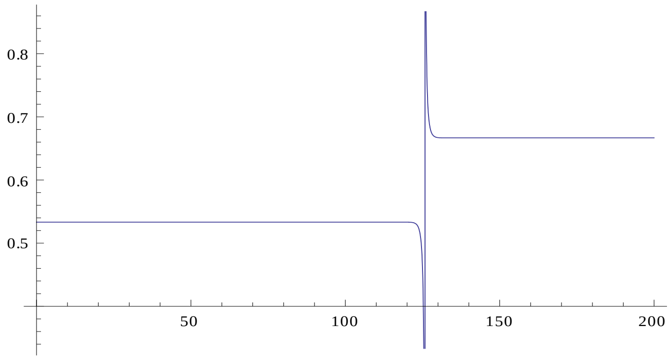

Figure 2. (a) represented the behavior of the traveling wave solution of nonlinear Klein Gordon DDE (29) the when $n=5, a_0=2, B=5, b_0=3, b_1=7, A=11, d=4, C_1=17, C=1, \xi_0=13.$

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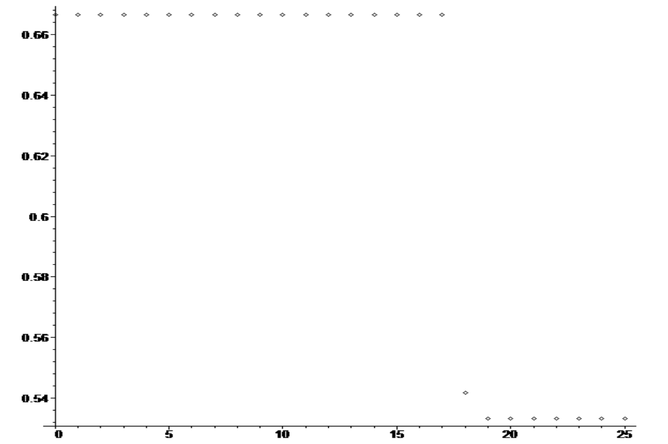

Figure 2. (b) represented the desecrate solution of nonlinear Klein Gordon DDE (29) the when

$t = 5, a_0 = 2, B = 5, b_0 = 3, b_1 = 7, A = 11, d = 4, C_1 = 17,$
$C = 1, \xi_0 = 13.$

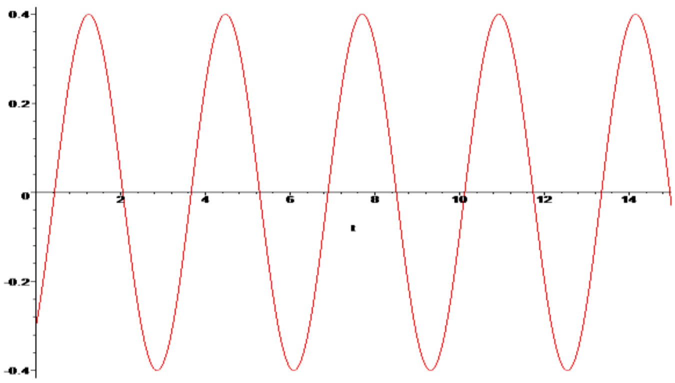

Figure 3. (a) represented real part of the traveling wave solution of nonlinear discrete Schrödinger equation (43) the when $n = 5, a_0 = 2, B = 5, b_0 = 1, a_2 = 2, A = 3, d = 1, C = 5, \rho = 7.$

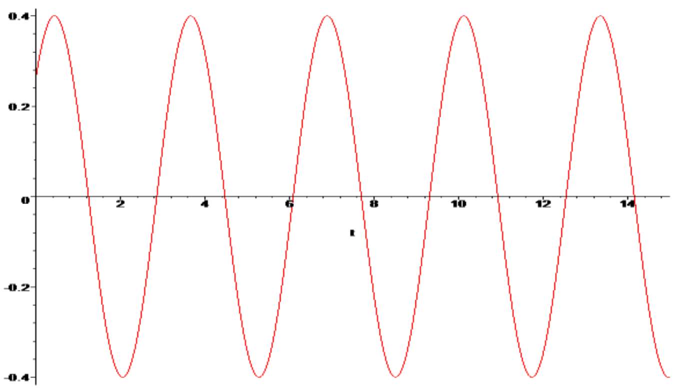

Figure 3. (b) represented imaginary part of the traveling wave solution of nonlinear discrete Schrödinger equation (43) the when $n = 5, a_0 = 2, B = 5, b_0 = 1, a_2 = 2, A = 3, d = 1, C = 5, \rho = 7.$

Figure 3.

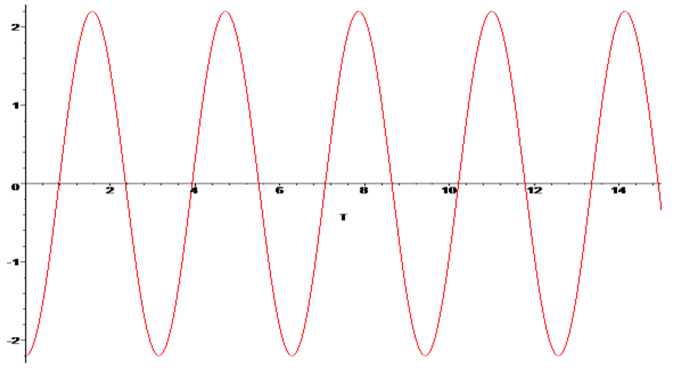

Figure 4. (a) represented real part of the traveling wave solution of nonlinear quantic Schrödinger equation (59) the when $n = 5, a_0 = 3, B = 11, b_0 = 5, A = 7, d = 13, C = 0.7, \alpha = 1.$

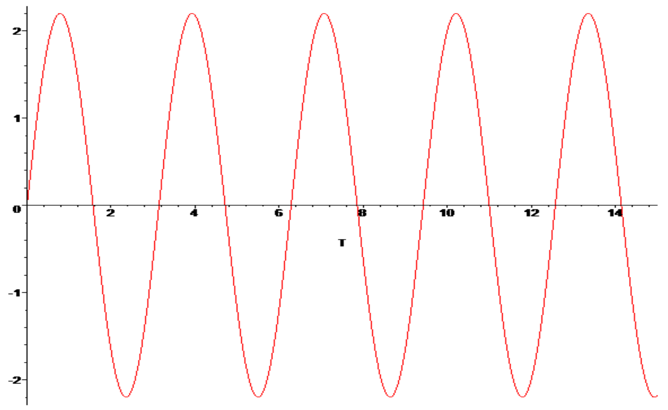

Figure 4. (b) represented imaginary part of the traveling wave solution of nonlinear quantic Schrödinger equation (59) the when $n = 5, a_0 = 3, B = 11, b_0 = 5, A = 7, d = 13, C = 0.7, \alpha = 1.$

CONCLUSION

In this paper, we developed a direct method namely generalized Kudryashov method from solving the nonlinear partial differential equations (NPDE's) to solve the nonlinear differential difference equations. We applied the improved generalized method to find the exact solutions for some physical and engineering problem the lattice equation, the discrete nonlinear Klein Gordon equation, the discrete nonlinear Schrodinger equation with a saturable nonlinearity and the quintic discrete nonlinear Schrodinger equation. The proposed method is more effective and powerful to obtain many rational traveling wave solutions for nonlinear differential difference equations. This method can be used for solving more complicated system of nonlinear difference differential equations which application are extended to all branches of fields.

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