International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 23 (2018) pp. 16701-16702

Distributive Law on Soft Set Theory

R. Rajarajeswa[1], **K. Rajendran**[2, *]

[1]*Head (Retd), Department of Mathematics, Sri Parasakthi College for Women, Courtallam 627 802, Tamilnadu, India.*

[2] *Research Scholar, Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627 012, Tamilnadu, India.*

Abstract In this paper, the authors prove that the distributive law of intersection over union is true in soft set theory.
Keywords: Soft sets, union and intersection of soft sets, distributive law
Mathematics Subject Classification: 03E20

1. Introduction

After the introduction of the concept of soft set theory by Molodstov[2], many researchers developed the concepts and applied in various fields. One such paper on soft set theory is by P.K. Maji, R. Biswas and A.R. Roy[1]. In [1], the proposition 2.5 result (iii) is one of the distributive law which is stated to be true without proof. Already we disproved the result (iii) i.e., the distributive law of union over intersection over soft sets. In this paper we prove the result (iv) i.e., the distributive law of intersection over union is true.

2. Soft Set Theory

Definition 2.1 *Let U be an universal set and E be a set of parameters. $P(U)$ be the power set of U and $A \subset E$. Let $F : A \to P(U)$ be a mapping, then the pair (F, A) is called a soft set over U.*

In other words, a soft set over U is a parametrized family of subsets of U. For $e \in A$, the set $F(e)$ may be considered as the set of e-elements of the soft set (F, A) or the set of e-approximate elements of the soft set (F, A).
So, a soft set (F,A) can be written as $(F, A) = \{F(e) : e \in A\}$.

Definition 2.2 *Let (F, A), (G, B) be two soft sets over U. We say that (F, A) is a soft subset of (G, B) if*

(i) $A \subset B$ and

(ii) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We then write $(F, A)\overline{\subset}(G, B)$.

Definition 2.3 *Two soft sets (F, A) and (G, B) over U are said to be soft equal if $(F, A)\overline{\subset}(G, B)$ and $(G, B)\overline{\subset}(F, A)$ and it is written as $(F, A) = (G, B)$.*

Definition 2.4 *The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C) where $C = A \cup B$ and $\forall e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\overline{\cup}(G, B) = (H, A \cup B)$.

Definition 2.5 *The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C) where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cup G(e)$. We write $(F, A)\overline{\cap}(G, B) = (K, A \cap B)$.*

Consider the example. Let $X = \{h_1, h_2, \cdots, h_6\}$ be the universal set, where $h_1, h_2, \cdots, h_6$ are houses of different nature which a person is planning to purchase and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the parameter set, where e_1 is the parameter expensive, e_2 is the parameter beautiful, e_3 is the parameter wooden, e_4 is the parameter cheap, e_5 is the parameter in the green surroundings.

*Corresponding autho

Let $E_1 = \{e_1, e_3, e_5\}$, $E_2 = \{e_1, e_2, e_3\}$ and $E_3 = \{e_1, e_4\}$.
Let $(F, E_1) = \{F(e_1), F(e_3), F(e_5)\} = \{\{h_2, h_4\}, \{h_1\}, \{h_3, h_4, h_5\}\}$,
$(G, E_2) = \{G(e_1), G(e_2), G(e_3)\} = \{\{h_1, h_3\}, \{h_3\}, \{h_1, h_4\}\}$ and $(H, E_3) = \{H(e_1), H(e_4)\} = \{\{h_2, h_5\}, \{h_1, h_4\}\}$ be the soft subsets.

The distributive law of intersection over union is

$$(F, E_1)\bar{\cap}((G, E_2)\bar{\cup}(H, E_3)) = ((F, E_1)\bar{\cap}(G, E_2))\bar{\cup}((F, E_1)\bar{\cap}(H, E_3)).$$

LHS = $(F, E_1)\bar{\cap}((G, E_2)\bar{\cup}(H, E_3))$
Let $(G, E_2)\bar{\cup}(H, E_3) = (K, E_2 \cup E_3)$, where $\forall\, e \in E_2 \cup E_3$,

$$K(e) = \begin{cases} G(e) & \text{if } e \in E_2 - E_3 \\ H(e) & \text{if } e \in E_3 - E_2 \\ G(e) \cup H(e) & \text{if } e \in E_2 \cap E_3 \end{cases}$$

But $E_2 \cup E_3 = \{e_1, e_2, e_3, e_4\}$, $E_2 - E_3 = \{e_2, e_3\}$, $E_3 - E_2 = \{e_4\}$, $E_2 \cap E_3 = \{e_1\}$.
Therefore, $K(e_2) = G(e_2) = \{h_3\}$, $K(e_3) = G(e_3) = \{h_1, h_4\}$, $K(e_4) = H(e_4) = \{h_1, h_4\}$, $K(e_1) = G(e_1) \cup H(e_1) = \{h_1, h_3\} \cup \{h_2, h_5\} = \{h_1, h_2, h_3, h_5\}$
Therefore LHS = $(F, E_1)\bar{\cap}(K, E_2 \cup E_3) = (L, E_1 \cap (E_2 \cup E_3))$, where $\forall\, e \in E_1 \cap (E_2 \cup E_3)$, $L(e) = F(e) \cup K(e)$ $\forall$ $e \in E_1 \cap (E_2 \cup E_3)$.
But $E_1 \cap (E_2 \cup E_3) = \{e_1, e_3, e_5\} \cap \{e_1, e_2, e_3, e_4\} = \{e_1, e_3\}$,
$L(e_1) = F(e_1) \cup K(e_1) = \{h_2, h_4\} \cup \{h_1, h_2, h_3, h_5\} = \{h_1, h_2, h_3, h_4, h_5\}$
$L(e_3) = F(e_3) \cup K(e_3) = \{h_1\} \cup \{h_1, h_4\} = \{h_1, h_4\}$
RHS = $((F, E_1)\bar{\cap}(G, E_2))\bar{\cup}((F, E_1)\bar{\cap}(H, E_3))$.
Let $(F, E_1)\bar{\cap}(G, E_2) = (M, E_1 \cap E_2)$ where $\forall\, e \in E_1 \cap E_2$, $M(e) = F(e) \cup G(e)$
But $E_1 \cap E_2 = \{e_1, e_3\}$
Therefore $M(e_1) = F(e_1) \cup G(e_1) = \{h_2, h_4\} \cup \{h_1, h_3\} = \{h_1, h_2, h_3, h_4\}$

$M(e_3) = F(e_3) \cup G(e_3) = \{h_1\} \cup \{h_1, h_4\} = \{h_1, h_4\}$
Let $((F, E_1)\bar{\cap}(H, E_3)) = (N, E_1 \cap E_3)$, where $\forall\, e \in E_1 \cap E_3$, $N(e) = F(e) \cup H(e)$.
But $E_1 \cap E_3 = \{e_1\}$,
Therefore, $N(e_1) = F(e_1) \cup H(e_1) = \{h_2, h_4\} \cup \{h_2, h_5\} = \{h_2, h_4, h_5\}$.
Therefore, RHS = $(M, E_1 \cap E_2)\bar{\cup}(N, E_1 \cap E_3) = (P, (E_1 \cap E_2) \cup (E_1 \cap E_3))$, where $\forall\, e \in (E_1 \cap E_2) \cup (E_1 \cap E_3)$,

$$P(e) = \begin{cases} M(e) & \text{if } e \in E_1 \cap E_2 - E_1 \cap E_3 \\ N(e) & \text{if } e \in E_1 \cap E_3 - E_1 \cap E_2 \\ M(e) \cup N(e) & \text{if } e \in (E_1 \cap E_2) \cap (E_1 \cap E_3) \end{cases}$$

But $E_1 \cap E_2 = \{e_1, e_3\}$, $E_1 \cap E_3 = \{e_1\}$
Therefore $E_1 \cap E_2 - E_1 \cap E_3 = \{e_3\}$, $E_1 \cap E_3 - E_1 \cap E_2 = \phi$
$(E_1 \cap E_2) \cap (E_1 \cap E_3) = \{e_1\}$.
Therefore $P(e_3) = M(e_3) = \{h_1, h_4\}$
$P(e_1) = M(e_1) \cup N(e_1) = \{h_1, h_2, h_3, h_4\} \cup \{h_2, h_4, h_5\} = \{h_1, h_2, h_3, h_4, h_5\}$
Therefore, $L(e_1) = P(e_1)$, $L(e_3) = P(e_3)$ Therefore, $L = P$. Hence $LHS = RHS$. So the distributive law of intersection over union for soft sets is true.

References

[1] P.K. Maji, R. Biswas and A.R. Roy, *Soft set theory*, Compu. Math. Appl., 45, (2003), 555-562.

[2] D. Molodstov, *Soft set theory - first results*, Compu. Math. Appl., 37, (4/5), (1999), 9-31.

[3] L.A. Zadeh, *Fuzzy sets, Information and Control*, 8, (1965), 338-358.