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Some Fixed Point Theorems using Weak Compatibility OWC in Fuzzy Metric Space

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Abstract

In this paper, a fixed point theorem for six self mappings is presented by using the concept of weak compatible maps also presents some common fixed point theorems for occasionally weakly compatible mapping in fuzzy metric space.

Keywords: Common fixed point, Fuzzy metric space, occasionally weakly compatible mappings.

1. INTRODUCTION

The fixed point theory has been studied and generalized in different spaces. Fuzzy set theory is one of uncertainty approaches where in topological structure are basic tools to develop mathematical models compatible to concrete real life situation. Fuzzy set was defined by Zadeh [27]. Kramosil and Michalek [15] introduced fuzzy metric space, George and Veermani [7] modified the notion of fuzzy metric spaces with the help of continuous t–norms. Many researchers have obtained common fixed point theorem for mapping satisfying different types of commutativity conditions. Vasuki [26] proved fixed point theorems for R–weakly commutating mapping. Pant [19, 20, 21] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam [5] have show that Rhoades [23] open problem on the existence of contractive definition which generates a fixed point but does not force the mapping to be continuous at the fixed point, posses an affirmative answer. Recent literature on fixed point in fuzzy metric space can be viewed in [1, 2, 3, 10, 17].

Jain and Singh [29] proved a fixed point theorem for six self maps in a fuzzy metric space. In this paper, a fixed point theorem for six self maps has been established using the concept of weak compatibility of pairs of self maps in fuzzy metric space. Also presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in fuzzy metric space.

For the sake of completeness, we recall some definition and known results in fuzzy metric space.

2. PRELIMINARY NOTES

Definition 2.1. A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2. A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t–norms if * is satisfying conditions

(i) * is an commutative and associative ;

(ii) * is continuous ;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Definition 2.3. A 3–tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t–norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$,

(F1) $M(x, y, t) > 0$;

(F2) $M(x, y, t) = 1$ if and only if $x = y$

(F3) $M(x, y, t) = M(y, x, t)$;

(F4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$;

(F5) $M(x, y, \bullet) : (0, \infty) \to (0, 1]$ is continuous

Then M is called a fuzzy metric on X. Then M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Example 2.1.(Induced fuzzy metric) Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b \in [0, 1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows

$$M_d(x, y, t) = \frac{t}{t + d(x + y)}$$

Then (X, M_d, *) is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Definition 2.4. Let (X, M, *) be a fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to converges to x in X if for each $\in > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \in$ for all $n \geq n_0$.

(b) a sequence $\{x_n\}$ in X is said to be Cauchy if for each $\in > 0$ and each t>0, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \in$ for all $n, m \geq n_0$.

(c) a fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5. A pair of self mapping (f, g) of a fuzzy metric space (X, M, *) is said to be

(i) weakly commuting if $M(fgx, gfx, t) \geq M(fx, gx, t)$ for all $x \in X$ and $t > 0$

(ii) R–weakly commuting if there exists some R>0 such that $M(fgx, gfx, t) \geq M(fx, gx, t/R)$ for all $x \in X$ and $t > 0$.

Definition 2.6. Two self mapping f and g of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.7. Two self maps f and g of a fuzzy metric space (X, M, *) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.8. Let X be a set, f, g self maps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.9. A pair of maps S and T is called weakly compatible pair if they commute at coincidence points i.e. if Sx = Tx for some $x \in X$ then STx = TSx.

Definition 2.10. Two self maps f and g of a set X are occasionally weakly compatible (OWC) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

Al–Thagafi and Nasur Shahzad [4] shown that occasionally weakly is weakly compatible but converse is not true.

Example 2.2. Let R be the usual metric space. Define S, T : $R \to R$ by Sx = 2x and Tx = x^2 for all $x \in R$. Then Sx = Tx for x = 0, 2 but STO = TSO and ST2 $\neq$ TS2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Proposition 2.1. Self mapping A and S of a fuzzy metric space (X, M, *) are compatible.

Proof. Suppose $A_p = S_p$ for some p in X. Consider a sequence $\{P_n\} = P$. Now $\{Ap_n\} \to A_p$ and $\{Sp_n\} \to S_p(A_p)$. As A and S are compatible we have $M(ASp_n, SAp_n, t) \to 1$ for all $t > 0$ as $n \to \infty$. Thus $ASp_n = SAp_n$ and we get that (A, S) is weakly compatible. The following is an example of pair of self maps in a fuzzy metric space which are weakly compatible but not compatible.

Example 2.3. Let (X, M, *) be a fuzzy metric space where X = [0, 2], t–norm is defined by a * b = min {a, b} for all a, b $\in$ [0, 1] and $M(x, y, t) = e^{-\frac{|x-y|}{t}}$ for all $x, y \in X$.

Define self maps A and S on X as follows

$$A_x = \begin{cases} 2-x & \text{if } 0 \leq x \leq 1 \\ 2 & \text{if } 1 \leq x \leq 2 \end{cases}$$

and

$$S_x = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 2 & \text{if } 1 \leq x \leq 2 \end{cases}$$

Taking $x_n = 1 - \frac{1}{n}$, $n = 1, 2, 3, \ldots$

Then $x_n \to x_n < 1$ and $2 - x_n > 2$ for all.

Also $Ax_n, Sx_n \to 1$ as $n \to \infty$

$M(ASx_n, SAx_n, t) = e^{-\frac{1}{2}} \neq 1$ as $n \to \infty$

Hence the pair (A, S) is not compatible. Also set of coincidence points of A and S is [1, 2]. Now for any $x \in [1, 2]$. Ax = Sx = 2 and AS(x) = A(2) = 2 = S(2) = SA(x). Thus A and S are weakly compatible but not compatible. From the above example, it is obvious that the concept of weak compatibility is more general than that of compatibility.

Proposition 2.2. In a fuzzy metric space (X, M, *) limit of a sequence is unique.

Lemma 2.1. Let (X, M, *) be a fuzzy metric space. Then for all $x, y \in X$, $M(x, y, \bullet)$ is a non–decreasing function.

Lemma 2.2. Let (X, M, *) be a fuzzy metric space. If there exists $q \in (0, 1)$ such that for all $x, y \in X$, $M(x, y, qt) \geq M(x, y, t)$, for all $t > 0$, then x = y.

Lemma 2.3. Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *). If there exists a number $q \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t)$ for all $t > 0$ and $n \in N$. Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.4. Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Lemma 2.5. The only t–norm * satisfying $r * r \geq r$ for all $r \in [0, 1]$ is the minimum t–norm, that is $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$.

3. MAIN RESULT

Theorem 3.1. Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mappings from X into itself such that the following conditions are satisfied

(a) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$;

(b) AB = BA, ST = TS, PB = BP, QT = TQ;

(c) either AB or P is continuous

(d) (P, AB) is compatible and (Q, ST) is weakly compatible,

(e) there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq \min\{M(ABx, STy, t),$$

$$M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(Qx, ABy, t)\}$$

Proof : Let $x_0 \in X$. From (a) there exists $x_1, x_2 \in X$ such that $Px_0 = STx_1$ and $Qx_1 = ABx_2$. Inductively, we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = y_{2n}$ for n = 1, 2, 3, …

Step 1 : Put $x = x_{2n}$ and $y = x_{2n+1}$ in (e), we get

$$(Px_{2n}, Qx_{2n+1}, qt) \geq \min\{M(ABx_{2n}, STx_{2n+1}, t),$$

$$M(Px_{2n}, ABx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, t),$$

$$M(Px_{2n}, STx_{2n+1}, t), M(Qx_{2n}, ABx_{2n+1}, t)\}$$

$$= \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t),$$

$$M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+1}, t),$$

$$M(y_{2n+1}, y_{2n+1}, t)\}$$

$$\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)$$

$$M(y_{2n+1}, y_{2n+2}, qt) \geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)$$

This implies,

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t)$$

Similarly, $(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t)$

Thus,

$$M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t)$$

for n = 1, 2, 3, …

$$M(y_n, y_{n+1}, t) \geq M(y_n, y_{n+1}, \frac{t}{q}) \geq$$

$$M(y_{n-2}, y_{n-1}, \frac{t}{q^2}) \ldots\ldots \geq M(y_1, y_2, \frac{t}{q^n}) \to 1$$

as $n \to \infty$

and hence $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for any $t > 0$ for each $\in > 0$ and $t > 0$, we can choose $n_0 \in N$ such that $M(y_n, y_{n+1}, t) > 1 - \in$ for all $n > n_0$. For $m, n \in N$, we suppose $m > n$. Then we have

$$M(y_n, y_m, t) \geq \min\{M(y_n, y_{n+1}, \frac{t}{m-n}),$$

$$M(y_{n+1}, y_{n+2}, \frac{t}{m-n}), \ldots, M(y_{m-1}, y_m, \frac{t}{m-n})\}$$

$$\geq \min\{(1 - \in), (1 - \in), \ldots, (1 - \in)\ (m - n) \text{ times}\}$$

$\geq (1 - \in)$ and hence $\{y_n\}$ is a Cauchy sequence in X.

Since (X, M, *) is complete $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point $z \in X$ i.e.

$$\{Qx_{2n+1}\} \to z \text{ and } \{STx_{2n+1}\} \to z \tag{1}$$

$$\{Px_{2n}\} \to z \text{ and } \{ABx_{2n}\} \to z \tag{2}$$

Case I : Suppose AB is continuous. Since AB is continuous, we have $(AB)^2 x_{2n} \to ABz$ nd $ABPx_{2n} \to ABz$. As (P, AB) is compatible pair, then $PABx_{2n} \to ABz$.

Step 2 : Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in (e), we get

$M(PABx_{2n}, Qx_{2n+1}, qt) \geq \min\{M(ABABx_{2n}, STx_{2n+1}, t), M(PABx_{2n}, ABABx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(PABx_{2n}, STx_{2n+1}, t), M(QABx_{2n}, ABx_{2n+1}, t)\}$

Taking $n \to \infty$, we get

$M(ABz, z, qt) \geq \min\{M(ABz, z, t), M(ABz, ABz, t), M(z, z, t), M(ABz, z, t), M(ABz, z, t)\}$

$= M(ABz, z, t)$

i.e. $M(ABz, z, qt) \geq M(ABz, z, t)$

Therefore, by using Lemma 2.2, we get

$$ABz = z \qquad \ldots(3)$$

Step 3 : Put $x = z$ and $y = x_{2n+1}$ in (e), we have

$M(Pz, Qx_{2n+1}, qt) \geq \min\{M(ABz, STx_{2n+1}, t),$

$M(Pz, ABz, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(Pz, STx_{2n+1}, t), M(Qz, ABx_{2n+1}, t)\}$

Taking $n \to \infty$ and using equation (1), we get $M(Pz, z, qt) \geq \min\{M(z, z, t), M(Pz, z, t), M(z, z, t), M(Pz, z, t), M(z, z, t)\} = M(Pz, z, t)$

i.e. $M(Pz, z, qt) \geq M(Pz, z, t)$

Therefore by using Lemma 2.2, we get $Pz = z$. Therefore $ABz = Pz = z$.

Step 4 : Putting $x = Bz$ and $y = x_{2n+1}$ in condition (e), we get

$M(PBz, Qx_{2n+1},qt) \geq \min\{M(ABBz,STx_{2n+1}, t), M(PBz, ABBz, t), M(Qx_{2n+1}, STx_{2n+1}, t), M(PBz, STx_{2n+1}, t), M(QBz, ABx_{2n+1}, t)\}$

As BP = PB , AB = BA, so we have P(Bz) = B(Pz) = Bz and (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.

Taking $n \to \infty$ and using (1), we get

$M(Bz, z, qt) \geq \min\{M(Bz, z, t), M(Bz, Bz, t), M(z, z, t),M(Bz, z, t) M(Bz, z, t)\} = M(Bz, z, t)$

i.e. $M(Bz, z, qt) \geq M(Bz, z, t)$

Therefore, by using Lemma 2.2, we get Bz = z and also we have ABz = Az = z.

Therefore

$$Az = Bz = Pz = z \qquad \ldots(4)$$

Step 5 : As $P(X) \subset ST(X)$, there exists $u \in X$ such that z = Pz = STu, Putting $x = x_{2n}$ and y = u in (e), we get

$M(Px_{2n}, Qu, qt) \geq \min\{M(ABx_{2n}, STu, t), M(Px_{2n}, ABx_{2n},t),M(Qu,STu,t),M(Px_{2n},STu, t), M(Qx_{2n}, ABu, t)\}$

Taking $n \to \infty$ and using (1) and (2), we get

$M(z, Qu, qt) \geq \min\{M(z, z, t), M(z, z, t), M(Qu, z, t),M(z, z, t), M(z, z, t)\}= M(Qu, z, t)$

i.e. $M(z, Qu, qt) \geq M(z, Qu, t)$

Therefore by Lemma 2.2, we get Qu = z, Hence STu = z = Qu. Since (Q, ST) is weak compatible. Therefore, we have QSTu = STQu. Thus Qz = STz.

Step 6 : Putting $x = x_{2n}$ and y = z in (e), we get

$M(Px_{2n}, Qz, qt) \geq \min\{M(ABx_{2n}, STz, t), M(Px_{2n}, ABx_{2n}, t),M(Qz,STz,t),M(Px_{2n},STz, t), M(Qx_{2n}, ABz, t)\}$

Taking $n \to \infty$ and using (2) and step 5, we get

$M(z, Qz, qt) \geq \min\{M(z, Qz, t), M(z, z, t), M(Qz, Qz, t), M(z, Qz, t), M(z, Qz, t)\} = M(z, Qz, t)$

i.e. $M(z, Qz, qt) \geq M(z, Qz, t)$

Therefore, by using lemma 2.2, we get Qz = z.

Step 7: Putting $x = x_{2n}$ and y = Tz in (e), we get

$M(Px_{2n}, QTz, qt) \geq \min\{M(ABx_{2n}, STTz, t), M(Px_{2n}, ABx_{2n},t), M(QTz, STTz, t), M(Px_{2n}, STTz, t), M(Qx_{2n}, ABTz, t)\}$

As QT = TQ and ST = TS, we have QTz = TQz = Tz and ST(Tz) = T(STz) = TQz = Tz

Taking $n \to \infty$, we get

$M(z, Tz, qt) \geq \min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t),M(z, Tz, t), M(z, Tz, t)\}$

$= M(z, Tz, t)$

i.e. $M(z, Tz, qt) \geq M(z, Tz, t)$

Therefore by using lemma 2.2, we get Tz = z, now STz = Tz = z implies Sz = z. Hence

$$Sz = Tz = Qz = z \qquad \ldots(5)$$

Combining (4) and (5), we get

Az = Bz = Pz = Qz = Tz = Sz = z

Hence, z is the common fixed point of A, B, S, T, P and Q.

Case II: Suppose P is continuous. As P is continuous, $P^2x_{2n} \to Pz$ and $P(AB)\, x_{2n} \to Pz$. As (P, AB) is compatible, we have

$(AB)\, Px_{2n} \to Pz$.

Step 8: Putting $x = Px_{2n}$ an $y = x_{2n+1}$ in condition (e), we have

$M(PPx_{2n}, Qx_{2n+1}, qt) \geq \min\{M(ABPx_{2n}, STx_{2n+1}, t) M(PPx_{2n}, ABPx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, t),M(PPx_{2n}, STx_{2n+1}, t),M(QPx_{2n}, ABx_{2n+1}, t)\}$

Taking $n \to \infty$, we get

$M(Pz, z, qt) \geq \min\{M(Pz, z, t), M(Pz, Pz, t), M(z, z, t), M(Pz, z, t), M(Pz, z, t)\}= M(Pz, z, t)$

i.e. $M(Pz, z, qt) \geq M(Pz, z, t)$

Therefore by using Lemma 2.2, we have Pz = z. Further using steps 5, 6, 7, we get

z = STz = Sz = Tz = z

Step 9: As $Q(X) \subset AB(X)$, there exists $w \in X$, such that z = Qz = ABw. Put x = w and $y = x_{2n+1}$ in (e), we have

$M(Pw, Qx_{2n+1}, qt) \geq \min\{M(ABw, STx_{2n+1}, t), M(Pw, ABw, t), M(Qx_{2n+1},STx_{2n+1},t), M(Pw, STx_{2n+1}, t), M(Qw, ABx_{2n+1}, t)\}$

Taking $n \to \infty$, we get

$M(Pw, z, qt) \geq \min\{M(z, z, t), M(Pw, z, t), M(z, z, t), M(Pw, z, t), M(z, z, t)\}= M(Pw, z, t)$

i.e. $M(Pw, z, qt) \geq M(Pw, z, t)$

Therefore, by using Lemma 2.2, we get Pw = z

Therefore, ABw = Pw = z. As (P, AB) is compatible. We have Pz = ABz. Also, from step 4, we get Bz = z. Thus Az = Bz = Pz = z and we see that z is the common fixed point of the six maps in this case also.

Uniqueness : Let u be another common fixed point of A, B, S, T, P and Q. Then Au = Bu = Pu = Su = Tu = u. Put x = z and y = u, in (e), we get

$M(Pz, Qu, qt) \geq \min\{M(ABz, STu, t), M(Pz, ABz, t), M(Qu, STu, t), M(Pz, STu, t), M(Qz, ABu, t)\}$

Taking $n \to \infty$, we get

$M(z, u, qt) \geq \min\{M(z, u, t), M(z, z, t), M(u, u, t), M(z, u, t), M(z, u, t)\} = M(z, u, t)$

i.e. $M(z, u, qt) \geq M(z, u, t)$

Therefore, by using Lemma 2.2, we get z = u. Therefore z is the unique fixed point of self maps A, B, S, T, P and Q.

Corollary 3.1. Let (X, M, *) be a complete fuzzy metric space and let A, S, P and Q be mapping from X into itself such that the following conditions are satisfied

(a) $P(X) \subset S(X)$, $Q(X) \subset A(X)$;

(b) either A or P is continuous

(c) (P, A) is compatible and (Q, S) is weakly compatible;

(d) there exists $q \in (0, 1)$, such that for every $x, y \in X$ and $t > 0$;

(e) $M(Px, Qy, qt) \geq \min\{M(Ax, Sy, t),$
$M(Px, Ax, t), M(Qy, Sy, t),$
$M(Px, Sy, t), M(Qx, Ay, t)\}$

Then A, S, P and Q have a unique fixed point in X.

Theorem 3.2. Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self–mappings of X. Let pair {A, S}, {B,T} and {P, Q} be owc. If there exists $q \in (0, 1)$ such that

$$M(Px, Qy, qt) \geq \min\{M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(ABx, Qy, t)\} \quad \ldots(1)$$

for all $x, y \in X$ and for all $t > 0$, then there exists a unique point $w \in X$ such that ABw = Pw = w and a unique point $z \in X$ such that STy = Qy = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S, T, P and Q.

Proof: Let the pair {A, S}, {B, T} and {P, Q} be owc, so there are points $x, y \in X$ such that ABx = Px and STy = Qy. We claim that ABx = STy. If not, by inequality (1)

$M(Px, Qy, qt) \geq \min\{M(ABx, STy, t),$

$M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(ABx, Qy, t)\}$

$= \min\{M(Px, Qy, t), M(Px, Px, t), M(Qy, Qy, t), M(Px, Qy, t), M(Px, Qy, t)\}$

$= M(Px, Qy, t)$

Therefore ABx = STy i.e. ABx = Px = STy = Qy. Suppose that there is a another point z such that ABz = STz then by (1), we have ABz = Pz = STy = Qy so ABx = ABz and w = ABx = STx is the unique point of coincidence of A and S. By lemma 2.4, w is the only common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz and z = Pz = Qz

Assume that $w \neq z$, we have

$M(w, z, qt) = M(Pw, Qw, qt) \geq \min\{M(ABw, STz, t), M(Pw, ABw, t), M(Qz, STz, t), M(Pw, STz, t), M(ABw, Qz, t)\}$

$= \min\{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, t), M(w, z, t)\}$

$= M(w, z, t)$

Therefore, we have z = w by Lemma 2.4 and z is a common fixed point of A, B, S, T, P and Q. The uniqueness of the fixed point holds from (1).

Theorem 3.3. Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mapping of X. Let the pair {A, S}, {B, T} and {P, Q} be owc. If there exists $q \in (0, 1)$ such that

$$M(Px, Qy, qt) \geq \phi(M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(ABx, Qy, t)) \quad \ldots(1)$$

for all $x, y \in X$ and $\phi : [0, 1]^5 \to [0, 1]$ such that $\phi(t, 1, 1, t, t) > t$ for all $0 < t < 1$, then there exist a unique common fixed point of A, B, S, T, P and Q.

Proof : let the pairs {A, S}, {B, T} and {P, Q} are owc there are points $x, y \in X$ such that ABx = Px and STy = Qy. We claim that ABx = STy. If not, by inequality (1)

$M(Px, Qy, qt) \geq \phi(M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(ABx, Qy, t)) = \phi(M(Px, Qy, t), M(Px, Px, t), M(Qy, Qy, t), M(Px, Qy, t), M(Px, Qy, t))$

$= \phi(M(Px, Qy, t), 1, 1, M(Px, Qy, t), M(Px, Qy, t)$

$> M(Px, Qy, t)$ a contradiction.

Therefore ABx = STy, i.e.

ABx = Px = STy = Qy. Suppose that there is a another point z such that ABz = STz then by (1) we have ABz = Pz = STy = Qy, so ABx = ABz and w = ABx = STx is the unique point of coincidence of A and T. By Lemma 2.4, w is a unique common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz and z = Pz = Qz. Thus z is a common fixed point of A, B, S, T, P and Q. The uniqueness of the fixed point holds from (1).

Theorem 3.4. Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mappings of X. Let the pair {A, S}, {B, T} and {P, Q} be owc. If there exists $q \in (0, 1)$ such that

$$M(Px, Qy, qt) \geq \phi(\min\{M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t), M(ABx, Qy, t)\}) \quad \ldots(1)$$

for all $x, y \in X$ and $\phi : [0, 1] \to [0, 1]$ such that $\phi(t) > 1$ for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S, T, P and Q.

Proof : The proof follows from Theorem 3.2.

Theorem 3.5. Let (X, M, *) be a complete fuzzy metric space and let A, B, S, T, P and Q be self mappings of X. Let the pair {A, S}, {B, T} and {P, Q} are owc. If there exists a point $q \in (0, 1)$ for all $(x, y) \in X$ and $t > 0$.

$M(Px, Qy, qt) \geq M(ABx, STy,t)*M(Px, ABx, t) *M(Qy, STy, t)*M(Px, STy,t)* M(ABx, Qy, t)$

Then, there exists a unique common fixed point of A, B, S, T, P and Q.

Proof : Let the pair {A, S}, {B, T} and {P, Q} are owc, there are points $x, y \in X$ such that $ABx = Px$ and $STy = Qy$.

We claim that $ABx = STy$, by inequality (1)

$M(Px, Qy, qt) \geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t)*M(Px, STy, t), M(ABx, Qy, t)$

$= M(Px, Qy, t) * M(Px, Px, t) * M(Qy, Qy, t)$

$* M(Px, Qy, t) * M(Px, Qy, t)$

$\geq M(Px, Qy, t) * 1 * 1 * M(Px, Qy, t) * M(Px, Qy, t)$

$\geq M(Px, Qy, t)$

Thus, we have $ABx = STy$, i.e. $ABx = Px = STy = Qy$, suppose that there is a another point z such that $ABz = STz$ then by (1) we have $ABz = Pz = STy = Qy$ so $ABx = ABz$ and $w = ABx = STx$ is the unique point of coincidence of A and S. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$ and $z = Pz = Qz$. Thus z is a common fixed point of A, B, S, T, P and Q.

Corollary 3.2. Let (X, M, *) be a complete fuzzy metric space and A, B, S, T, P and Q be self–mappings of X. Let the pair {A, S}, {B, T} and {P, Q} and owc. If there exists a point $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$.

$M(Px, Qy, qt) \geq M(ABx, STy,t)*M(Px,ABx, t) * M(Qy, STy, t)* (M(Px, STy, 2t) * M(ABx, Qy, t)$

Then, there exists a unique common fixed point of A, B, S, T, P and Q.

Proof : We have

$M(Px, Qy, qt) \geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t)*M(Px, STy, 2t) * M(ABx, Qy, t)$

$\geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t)* M(STy,ABx,t) *M(ABx,Px,t)* M(ABx,Qy, t)$

$\geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t)* M(ABx, Qy, t)$

And therefore, from above theorem A, B, S, T, P and Q have a common fixed point.

REFERENCES

[1] C. T. Aage, J. N. Salunke, “Common Fixed Point Theorems in Fuzzy Metric Spaces”, 56(2), 2009, pp. 155–164, International Journal of Pure and Applied Mathematics.

[2] C. T. Aage, J. N. Salunke, “Some Fixed Point Theorems in Fuzzy Metric Space”, 56(3) 2009, pp. 311–320, International Journal of Pure and Applied Mathematics.

[3] C. T. Aage, J. N. Salunke, “On Fixed Point Theorems in Fuzzy Metric Spaces Using A Control Function”, Submitted.

[4] A. Al–Thagafi and Naseer Shahzad, *Generalized I–Nonexpansive Selfmaps and Invariant Approximation*, English Series May, 2008, Vol. 24, No. 5, pp. 867876, Acta Mathematica Sinica.

[5] P. Balasubramaniam, S. Muralisankar, R.P. Pant, “Common fixed points of four mappings in a fuzzy metric space”, 10(2) (2002), 379–384, J. Fuzzy Math. .

[6] Y. J. Cho, H. K. Pathak. S.M. Kang, J. S. Jung, “Common fixed points of compatible maps of type (A) on fuzzy metric spaces”, 93 (1998), 99–111 Fuzzy Sets and Systems.

[7] A. George, P. Veeramani, “On some results in fuzzy metric spaces”, 64 (1994), 395–399, Fuzzy Sets and Systems.

[8] M. Grabiec, “Fixed points in fuzzy metricspace”, 27(1988), 385–389 Fuzzy Sets and Systems.

[9] O. Hadzic, “Common fixed point theorems for families of mapping in complete metric space”, 29 (1984), 127–134, Math. Japan.

[10] Mohd. Imdad and Javid Ali, “Some common fixed point theorems in fuzzy metric space”,11(20060, 153–163,Mathematical Communications.

[11] G. Jungck, “Compatible mappings and common fixed points (2)” (1988), 285–288, Int. J. Math. Math. Sci.

[12] G. Jungck and B. E. Rhoades, “Fixed Point for Set Valued functions without Continuity”, 29(3) (1998), pp. 771–779, Indian J. Pure Appl. Math..

[13] G. Jungck and B. E. Rhoades, “Fixed Point Theorems for Occasionally Weakly compatible Mappings”, Volume 7, No. 2, 2006, 287–296 Fixed Point Theory.

[14] G. Jungck and B. E. Rhoades, “Fixed Point Theorems for Occasionally Weakly compatible Mappings”, Erratum, Volume 9, No. 1, 2008, 393–384, Fixed Point Theory.

[15] O. Kramosil and J. Michalek, “Fuzzy metric and statistical metric space”, 11(1975), 326–334, Kybernetika.

[16] S. Kutukcu, “A fixed point theorem for contraction type mappings in Menger spaces”, 4(6) (2007), 371–373, Am. J. Appl. Sci..

[17] Servet Kutukcu, Sushil Sharma and Hanifi Tokgoz, "A Fixed Point Theorem in Fuzzy metric Spaces", Vol. 1, 2007, no. 8, 861–872, Int. Journal of Math. Analysis.

[18] S. N. Mishra, "Common fixed points of compatible mappings in PM–spaces", 36(1991), 283–289, Math. Japan.

[19] R. P. Pant, "Common fixed points of four mappings", 90(1998), 281–286, Bull. Cal. Math. Soc. .

[20] R. P. Pant, "Common fixed point theorems for contractive maps", 226(1998), 251–258, J. Math. Anal. Appl.

[21] R.P. Pant, K. Jha, "A remark on common fixed points of four mappings in a fuzzy metric space", 12(2) (2004), 433–437, J. Fuzzy Math. .

[22] H. K. Pathak and Prachi Singh, "Common Fixed Point Theorem for Weakly Compatible Mapping", 2, 2007, no. 57, 2831–2839. International Mathematical Forum.

[23] B. E. Rhoades, "Contractive definitions and continuity", 72(1988), 233–245, Contemporary Math..

[24] B. Schweizer and A. Sklar, "Statistical metric spaces", 10(1960), 313–334. Pacific J. Math.

[25] Seong Hoon Cho, "On common fixed point in fuzzy metric space", 1, 2006, 10, 471–479. Int. Math. Forum.

[26] R. Vasuki, Common fixed points for R–weakly commuting maps in fuzzy metric spaces, 30 (1999), 419–423. Indian J. Pure Appl. Math.

[27] L. A. Zadeh, Fuzzy sets, 8(1965), 338–353. Inform and Control.

[28] S. H, Cho, on common fixed point theorem in fuzzy metric spaces, Vol. 20 (2006), No. 1–2, 523–533. J. Appl. Math. and computing.

[29] A. Jain and B. Singh, A fixed point theorem using weak compatible in fuzzy metric space, Vol. 5 No. I(2005), 297–306. Varahmihir J. Math. Sci.