International Journal of Applied Engineering Research, ISSN 0973-4562 Volume 13, Number 23 (2018) pp. 16532-16537

SD - PRIME CORDIAL LABELING IN DUPLICATE GRAPHS OF PATH AND STAR RELATED GRAPHS

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Abstract: In this paper, we prove that the extended duplicate graphs of path graph, comb graph, twig graph, star graph, bistar graph and double star graph admit SD - Prime cordial labeling.

AMS Subject classification: 05C78.
Keywords: Graph labeling, duplicate graph, SD - Prime cordial labeling.

INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 2000 papers [2]. The concept of duplicate graph was introduced by Sampath kumar and he proved many results on it [5]. Thirusangu, Ulaganathan and Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [7]. Thirusangu, Ulaganathan and Vijaya kumar have proved that the duplicate graph of Ladder graph L_m, $m \geq 2$, is cordial, total cordial and prime cordial [6]. Vijaya kumar, Ulaganathan and Thirusangu, proved the existence of 3-Equitable and 3-Cordial Labeling in Duplicate Graph of Some Graphs [9]. Thulukkanam, Vijaya Kumar and Thirusangu have proved the existence of V-cordial labeling in some duplicate graphs [8]. The concept of SD-Prime cordial labeling was introduced by Gee-Choon Lau et.al and proved the existence of the same in some standard graphs [3]. Lourdusamy and Patrick have studied the existence of SD-Prime cordial labeling in some more graphs [4].

PRELIMINARIES

In this section, we give the basic notions relevant to this paper.

Definition 1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling).

Definition 2. Let $G(V, E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is

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defined as : The edge uv is in E if and only if both uv' and $u'v$ are edges in E_1 [5].

Definition 3. The Star graph is a complete bipartite graph $K_{1,m}$, where m represents the number of vertices and S_m has $(m - 1)$ edges.

Definition 4. A bistar $B_{m,m}$ is a graph obtained from K_2 by joining m pendant edges to each end of K_2. The edge K_2 is called the central edge of $B_{m,m}$ and the vertices of K_2 are called the central vertices of $B_{m,m}$.

Definition 5. The Double star $DS_{m,m}$ is a tree $K_{1,m,m}$ obtained from the star $K_{1,m}$ by adding a new pendent edge of the existing m pendant vertices. It has $2m + 1$ vertices and $2m$ edges.

Definition 6. A walk is a sequence of vertices $v_1, v_2, \ldots, v_k$ such that $\forall\ i \in 1, 2, \ldots, k - 1$, v_i is adjacent to v_{i+1}.

Definition 7. A path is a walk where $v_i \neq v_j$, for all $i \neq j$. In other words, a path is a walk that visits each vertex at most once.

Definition 8. Let P_n be a path graph with n vertices. The comb graph is defined as $P_n \odot K_1$. It has $2n$ vertices and $2n - 1$ edges.

Definition 9. A graph $G(V, E)$ obtained from a path by joining exactly two pendent edges to each internal vertices of the path is called a Twig graph, denoted by T_m. A Twig T_m with m internal vertices has $3m + 2$ vertices and $3m + 1$ edges.

Definition 10. The extended duplicate graph of star denoted by $EDG(S_m)$, is obtained from the duplicate graph of star by joining v_1 and v'_1.

Definition 11. The extended duplicate graph of bistar denoted by $EDG(B_{m,m})$, is obtained from the duplicate graph of bistar by joining v_1 and v'_1.

Definition 12. The extended duplicate graph of double star denoted by $EDG(DS_{m,m})$, is obtained from the duplicate graph of double star by joining v_1 and v'_1.

Definition 13. The extended duplicate graph of path denoted by $EDG(P_m)$, is obtained from the duplicate graph of star by joining v_1 and v'_1.

Definition 14. The extended duplicate graph of comb denoted by $EDG(CB_m)$, is obtained from the duplicate graph of bistar by joining v_1 and v'_1.

Definition 15. The extended duplicate graph of twig denoted by $EDG(T_m)$, is obtained from the duplicate graph of double star by joining v_1 and v'_1.

Remark 16. The extended duplicate graph of star denoted by $EDG(S_m)$, has $2m$ vertices and $2m - 1$ edges.

Remark 17. The extended duplicate graph of bistar denoted by $EDG(B_{m,m})$ has $4m + 4$ vertices and $4m + 3$ edges.

Remark 18. The extended duplicate graph of double star denoted by $EDG(DS_{m,m})$ has $4m + 2$ vertices and $4m + 1$ edges.

Remark 19. The extended duplicate graph of path graph is denoted by $EDG(P_m)$ has $2m + 2$ vertices and $2m + 1$ edges.

Remark 20. The extended duplicate graph of twig is denoted by $EDG(T_m)$ has $6m + 4$ vertices and $6m + 3$ edges.

Remark 21. The extended duplicate graph of comb is denoted by $EDG(CB_m)$ has $4m$ vertices and $4m - 1$ edges.

MAIN RESULTS

Definition 22. A bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ induces an edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge uv in G, $f'(uv) = 1$ if $gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise, where S and D respectively represent the sum and difference of the labels at the end vertices of the edge uv. The labeling f is called SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that G is SD-prime cordial if it admits SD-prime cordial labeling.

Theorem 23. The extended duplicate graph of path $EDG(P_m)$, $m \geq 4$, admits SD-prime cordial labeling.

Algorithm-SDPP

$V \leftarrow \{v_1, v_2, \ldots, v_{m+1}, v'_1, v'_2, \ldots, v'_{m+1}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+1}, e'_1, e'_2, \ldots, e'_m\}$

Case (i): $m \equiv 0 (\bmod 4)$
Fix: $v_1 \leftarrow \frac{3m}{2} + 1, v'_1 \leftarrow 2m + 2$
For $1 \le k \le \frac{m}{4}$
$v_{2k} \leftarrow 4k - 3, v_{2k+1} \leftarrow 4k, v_{\frac{m}{2}+2k} \leftarrow m + 2k - 1$
$v_{\frac{m}{2}+2k+1} \leftarrow \frac{3m}{2} + 2k + 1$
$v'_{2k} \leftarrow 4k - 2, v'_{2k+1} \leftarrow 4k - 1, v'_{\frac{m}{2}+2k} \leftarrow \frac{3m}{2} + 2k$,
$v'_{\frac{m}{2}+2k+1} \leftarrow m + 2k$
Case (ii): $m \equiv 1 (\bmod 4)$
Fix: $v_1 \leftarrow 2m + 1, v'_1 \leftarrow 2m + 2$
For $1 \le k \le \frac{m+1}{2}$
$v_{2k} \leftarrow 4k - 3, v'_{2k} \leftarrow 4k - 2$
For $1 \le k \le \frac{m-1}{4}$
$v_{2k+1} \leftarrow 4k, v_{\frac{m-1}{2}+2k+1} \leftarrow m + 4k - 2$
$v'_{2k+1} \leftarrow 4k - 1, v'_{\frac{m-1}{2}+2k+1} \leftarrow m + 4k - 1$
Case (iii): $m \equiv 2 (\bmod 4)$
Fix: $v_1 \leftarrow m, v'_1 \leftarrow 2m + 2$
For $1 \le k \le \frac{m-2}{4}$
$v_{2k+1} \leftarrow 4k, v'_{2k} \leftarrow 4k - 2$
For $1 \le k \le \frac{m+2}{4}$
$v_{2k} \leftarrow 4k - 3, v_{\frac{m}{2}+2k} \leftarrow \frac{3m}{2} + 2k$
$v'_{2k+1} \leftarrow 4k - 1, v'_{\frac{m}{2}+2k-1} \leftarrow \frac{3m}{2} + 2k - 1$
Case (iv): $m \equiv 3 (\bmod 4)$
Fix: $v_1 \leftarrow 2m + 2, v'_1 \leftarrow 2m + 1$
For $1 \le k \le \frac{m+1}{4}$
$v_{2k} \leftarrow 4k - 3, v_{2k+1} \leftarrow 4k, v_{\frac{m+1}{2}+2k} \leftarrow \frac{3}{2}(m + 1) + 2k - 2$
$v'_{2k} \leftarrow 4k - 2, v'_{2k+1} \leftarrow 4k - 1, v'_{\frac{m+1}{2}+2k} \leftarrow m + 2k$
For $1 \le k \le \frac{m-3}{4}$
$v_{\frac{m+3}{2}+2k} \leftarrow m + 2k + 1, v'_{\frac{m+3}{2}+2k} \leftarrow \frac{3}{2}(m + 1) + 2k - 1$.

Proof. **Case (i):** $m \equiv 0 (\bmod 4)$

Using the algorithm SDPP, $2m+2$ vertices receive labels $1, 2, 3, \ldots, 2m + 2$. Using the induced function f^* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } gcd(S, D) = 1 \\ 0 & \text{otherwise} \end{cases}$$

the m edges namely $e_2, e'_2, e_3, e'_3, e_4, e'_4, e_5, e'_5, \ldots, e_{m-6}, e'_{m-6}, e_{m-5}, e'_{m-5}$ receive label 0 and the $m + 1$ edges namely $e_1, e'_1, e_8, e'_8, e_9, e'_9, \ldots, e_{m-1}, e'_{m-1}, e_m, e'_m, e_{m+1}, e'_{m+1}, e_{m+2}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (ii): $m \equiv 1 (\bmod 4)$

Using the algorithm SDPP, $2m + 2$ vertices receive labels $1, 2, 3, \ldots, 2m + 2$. Using the induced function f^* defined in Case (i), the m edges namely $e_1, e_2, e'_2, e_3, e'_3, e_4, e'_4, \ldots, e_{m-7}, e'_{m-7}, e_{m-6}, e'_{m-6}$ receive label 0 and the $m+1$ edges namely $e'_1, e_8, e'_8, e_9, e'_9, \ldots, e_{m-1}, e'_{m-1}, e_m, e'_m, e_{m+1}, e'_{m+1}, e_{m+2}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (iii): $m \equiv 2 (\bmod 4)$

Using the algorithm SDPP, $2m + 2$ vertices receive labels $1, 2, 3, \ldots, 2m + 2$. Using the induced function f^* defined in Case (i), the m edges namely $e_1, e_2, e'_2, e_3, e'_3, e_4, e'_4, \ldots, e_{m-7}, e'_{m-7}, e_{m-6}$ receive label 0 and the $m + 1$ edges namely $e'_1, e'_8, e_9, e'_9, e_{10}, e'_{10}, e_{11}, e'_{11}, \ldots, e_{m-1}, e'_{m-1}, e_m, e'_m, e_{m+1}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (iv): $m \equiv 3 (\bmod 4)$

Using the algorithm SDPP, $2m + 2$ vertices receive labels $1, 2, 3, \ldots, 2m + 2$. Using the induced function f^* defined in Case (i), the $m + 1$ edges namely $e_1, e'_1, e_2, e'_2, e_3, e'_3, \ldots, e_{m-6}, e'_{m-6}, e_{m-5}, e'_{m-5}$ receive label 0 and the m edges namely $e'_7, e'_7, e_8, e'_8, \ldots, e_{m-1}, e'_{m-1}, e_m, e'_m, e_{m+1}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (v): When $m = 2$

The vertices v_1, v_2, v_3 are labeled with 5, 1, 4 respectively and the vertices v'_1, v'_2, v'_3 are labeled with 6, 2, 3 respectively. Using the induced function f^* defined in Case (i) the two edges e_2, e'_2 receive label 0 and the three edges e_1, e'_1 and e_3 receive label 1. Thus the number of edges with label 0 and 1 differ by one.

Case (vi): When $m = 3$

The vertices v_1, v_2, v_3, v_4 an labeled with 7,1,4,5 respectively and the vertices v'_1, v'_2, v'_3, v'_4 an labeled with 8, 2, 3, 6. Using the induced function f^* defined in Case (i), the four edges e_2, e_3, e'_2, e'_3 receive label 0 and the three edges e_1, e_4, e'_1 receive label 1. Thus the number of edges with label 0 and 1 differ by one.

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Hence the extended duplicate graph of path $EDG(P_m)$, $m \geq 4$ admits SD-prime cordial labeling. ■

Theorem 24. The extended duplicate graph of comb $EDG(CB_m)$, $m \geq 2$, admits SD-prime cordial labeling.

Algorithm-SDPCB

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{2m}, e'_1, e'_2, \ldots, e'_{2m-1}\}$

Case (i): When m is odd

Fix: $v_1 \leftarrow 1$, $v'_1 \leftarrow 2$, $v_{2m} \leftarrow 2m+1$, $v'_{2m} \leftarrow 4m$

For $1 \leq k \leq \frac{m-1}{2}$

$v_{4k-2} \leftarrow 2m+4k-2$, $v_{4k-1} \leftarrow 2m+4k-1$, $v_{4k} \leftarrow 4k+1$

$v_{4k+1} \leftarrow 4k+2$

$v'_{4k-2} \leftarrow 4k-1$, $v'_{4k-1} \leftarrow 4k$, $v'_{4k} \leftarrow 2m+4k$, $v'_{4k+1} \leftarrow 2m+4k+1$

Case (ii): When m is even

Fix: $v_1 \leftarrow 1$, $v'_1 \leftarrow 2$, $v_{2m} \leftarrow 4m$, $v'_{2m} \leftarrow 2m+1$

For $1 \leq k \leq \frac{m}{2}$

$v_{4k-2} \leftarrow 2m+4k-2$, $v_{4k-1} \leftarrow 2m+4k-1$

$v'_{4k-2} \leftarrow 4k-1$, $v'_{4k-1} \leftarrow 4k$

For $1 \leq k \leq \frac{m-2}{2}$

$v_{4k} \leftarrow 4k+1$, $v_{4k+1} \leftarrow 4k+2$

$v'_{4k} \leftarrow 2m+4k$, $v'_{4k+1} \leftarrow 2m+4k+1$

Proof. **Case (i):** When m is odd

Using the algorithm SDPCB, $4m$ vertices receive labels $1, 2, 3, \ldots, 4m$. Using the induced function f^* defined in Theorem 23, the $2m$ edges namely $e_1, e'_1, e_4, e'_4, e_6, e'_6, \ldots, e_{m+3}, e'_{m+3}, e_{m+5}, e'_{m+5}, e_{m+6}, e'_{m+6}$ receive label 0 and the $2m-1$ edges namely $e_2, e'_2, e_3, e'_3, \ldots, e_{m+2}, e'_{m+2}, e_{m+4}, e'_{m+4}, e_m$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (ii): When m is even

Using the algorithm SDPCB, $4m$ vertices receive labels $1, 2, 3, \ldots, 4m$. Using the induced function f^* defined in Theorem 23, the $2m$ edges namely $e_1, e'_1, e_4, e'_4, \ldots, e_{m+5}, e'_{m+5}$ receive label 0 and the $2m-1$ edges namely $e_2, e'_2, e_3, e'_3, \ldots, e_{m+2}, e'_{m+2}, e_{m+4}, e'_{m+4}, e_m$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Hence the extended duplicate graph of comb $EDG(CB_m)$, $m \geq 2$ admits SD-prime cordial labeling. ■

Theorem 25. The extended duplicate graph of twig graph $EDG(T_m)$, $m \geq 2$, admits SD-prime cordial labeling.

Algorithm-SDPT

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_{3m+2}, v'_1, v'_2, \ldots, v'_{3m+2}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{3m+2}, e'_1, e'_2, \ldots, e'_{3m+1}\}$

Fix: $v_1 \leftarrow 46$, $v'_1 \leftarrow 24$, $v_2 \leftarrow 1$, $v'_2 \leftarrow 23$

For $1 \leq k \leq \frac{m+1}{2}$

$v_{6k-3} \leftarrow 6k+19$, $v_{6k-2} \leftarrow 6k+20$, $v_{6k-1} \leftarrow 6k+21$

$v'_{6k-31} \leftarrow 6k-4$, $v'_{6k-2} \leftarrow 6k-3$, $v'_{6k-1} \leftarrow 6k-2$

For $1 \leq k \leq \frac{m-1}{2}$

$v_{6k} \leftarrow 3m+k+3$, $v_{6k+1} \leftarrow 3m+k+4$, $v_{6k+2} \leftarrow 3m+k+5$

$v'_{6k} \leftarrow 3m+k+6$, $v'_{6k+1} \leftarrow 3m+k+7$, $v'_{6k+2} \leftarrow 3m+k+8$.

Proof. **Case (i):** When m is odd

Using the algorithm SDPT, $6m+4$ vertices receive labels $1, 2, 3, \ldots, 6m+4$. Using the induced function f^* defined in Theorem 23, the $3m+1$ edges namely $e_1, e'_2, e_3, e'_4, \ldots, e_{3m}, e'_{3m}, e'_{3m+1}, e_{3m+2}$ receive label 0 and the $3m+2$ edges namely $e_2, e'_1, e_4, e'_3, e_5, e'_5, \ldots, e_{3m-1}, e'_{3m-1}, e_{3m+1}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (ii): When m is even

Using the algorithm SDPT, $6m+4$ vertices receive labels $1, 2, 3, \ldots, 6m+4$. Using the induced function f^* defined in Theorem 23, the $3m+1$ edges namely $e_1, e'_1, e'_2, e_3, e'_4, \ldots, e_{3m}, e'_{3m}, e_{3m+1}$ receive label 0 and the $3m+2$ edges namely $e_2, e'_3, e_4, e_5, e'_5, \ldots, e_{3m-1}, e'_{3m-1}, e'_{3m+1}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Hence the extended duplicate graph of twig $EDG(T_m)$, $m \geq 2$ admits SD-prime cordial labeling. ■

Theorem 26. The extended duplicate graph of star $EDG(S_m)$, $m \geq 3$, admits SD-prime cordial labeling.

Algorithm-SDPS

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_m, v'_1, v'_2, \ldots, v'_m\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_m, e'_1, e'_2, \ldots, e'_{m-1}\}$

Fix: $v_1 \leftarrow 1$

For $1 \leq k \leq m-1$

$v_{k+1} \leftarrow m+k+1$

For $1 \leq k \leq m$

$v'_k \leftarrow k+1$

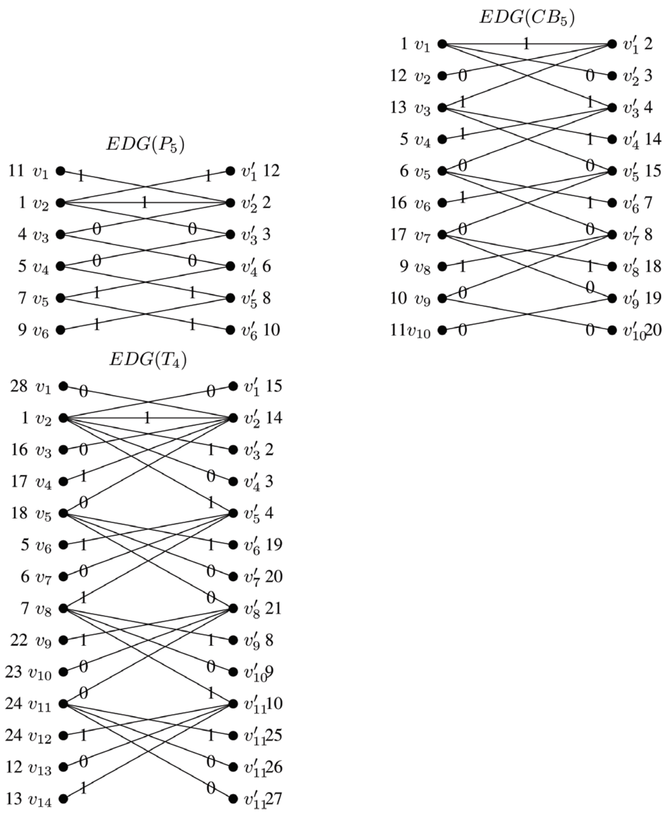

FIGURE 1. Example of SD-prime cordial labeling in $EDG(P_5)$, $EDG(CB_5)$ and $EDG(T_4)$

Proof. **Case (i):** When m is odd

Using the algorithm SDPS, $2m$ vertices receive labels $1, 2, 3, \ldots, 2m$. Using the induced function f^* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if } gcd(S, D) = 1 \\ 0 & \text{otherwise} \end{cases}$$

The m edges namely $e_1, e'_2, e_3, e'_4, \ldots, e_{m-2}, e'_{m-1}$ receive label 0 and the $m - 1$ edges namely $e_2, e'_1, e_4, e'_3, e_6, e'_5, \ldots, e'_{m-2}, e_{m-1}, e_m$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (ii): When m is even

Using the algorithm SDPS, $2m$ vertices receive labels $1, 2, 3, \ldots, 2m$. Using the induced function f^* defined in Case (i) above, the m edges namely $e_1, e'_1, e_3, e'_3, e_5, e'_5, \ldots, e_{m-1}, e'_{m-1}$ receive label 0 and the $m - 1$ edges namely $e_2, e'_2, e_4, e'_4, \ldots, e_{m-2}, e'_{m-2}, e_m$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Hence the extended duplicate graph of star $EDG(S_m)$, $m \geq 3$ admits SD-prime cordial labeling. ■

Theorem 27. The extended duplicate graph of bistar $EDG(BS_{m,m})$, $m \geq 2$, admits SD-prime cordial labeling.

Algorithm-SDPBS

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_{m+2}, v'_1, v'_2, \ldots, v'_{m+2}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{2m+2}, e'_1, e'_2, \ldots, e'_{2m+1}\}$

Case (i): When m is odd

Fix: $v_1 \leftarrow 1$

For $1 \leq k \leq 2m + 2$

$v'_k \leftarrow k + 1$

For $1 \leq k \leq 2m + 1$

$v_{k+1} \leftarrow 2m + k + 3$

Case (ii): When m is even

Fix: $v_1 \leftarrow 1$

For $1 \leq k \leq m + 2$

$v'_k \leftarrow k + 1$

For $1 \leq k \leq m + 1$

$v_{k+1} \leftarrow 2m + k + 3$

For $1 \leq k \leq m$

$v_{m+k+2} \leftarrow m + k + 3$

$v'_{m+k+2} \leftarrow 3m + k + 4$

Proof. **Case (i):** When m is odd

Using the algorithm SDPBS, $4m + 4$ vertices receive labels $1, 2, 3, \ldots, 4m + 4$. Using the induced function f^* defined in Theorem 26, the $2m + 2$ edges namely $e_1, e'_1, e_3, e'_3, \ldots, e_{2m-1}, e'_{2m-1}, e_{2m+1}, e'_{2m+1}$ receive label 0 and the $2m + 1$ edges namely $e_2, e'_2, e_4, e'_4, \ldots, e_{2m-2}, e'_{2m-2}, e_{2m}, e'_{2m}, e_{2m+2}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Case (ii): When m is even

Using the algorithm SDPBS, $4m + 4$ vertices receive labels $1, 2, 3, \ldots, 4m + 4$. Using the induced function f^* defined in Theorem 26, the $2m + 2$ edges namely $e_1, e'_1, e_3, e'_3, \ldots, e_{2m-1}, e'_{2m-1}, e_{2m+1}, e'_{2m+1}$ receive label 0 and the $2m + 1$ edges namely $e_2, e'_2, e_4, e'_4, \ldots, e_{2m-2}, e'_{2m-2}, e_{2m}, e'_{2m}, e_{2m+2}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Hence the extended duplicate graph of bistar $EDG(B_{m,m})$, $m \geq 2$ admits SD-prime cordial labeling.

■

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Theorem 28. The extended duplicate graph of double star $EDG(DS_{m,m})$, $m \geq 2$, admits SD-prime cordial labeling.

Algorithm-SDPDS

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_{2m+1}, v'_1, v'_2, \ldots, v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots, e_{2m+1}, e'_1, e'_2, \ldots, e'_{2m}\}$

Fix: $v_1 \leftarrow 1$, $v'_1 \leftarrow 2$

For $1 \leq k \leq 2m$

$v'_{k+1} \leftarrow 2k + 1$

For $1 \leq k \leq m$

$v_{m+k+1} \leftarrow 2k + 2$

$v_{k+1} \leftarrow 2m + 2k + 2.$

Proof. Using the algorithm SDPDS, $4m + 2$ vertices receive labels $1, 2, 3, \ldots, 4m + 2$. Using the induced function f^* defined in Theorem 26, the $2m$ edges namely $e_1, e'_1, e_2, e'_2, e_3, e'_3, \ldots, e_{m-1}, e'_{m-1}, e_m, e'_m$ receive label 0 and the $2m + 1$ edges namely $e_{m+1}, e'_{m+1}, e_{m+2}, e'_{m+2}, \ldots, e_{2m}, e'_{2m}, e_{2m+1}$ receive label 1. Thus the number of edges with label 0 and 1 differ atmost by 1.

Hence the extended duplicate graph of double star $EDG(DS_{m,m})$, $m \geq 2$ admits SD-prime cordial labeling. ∎

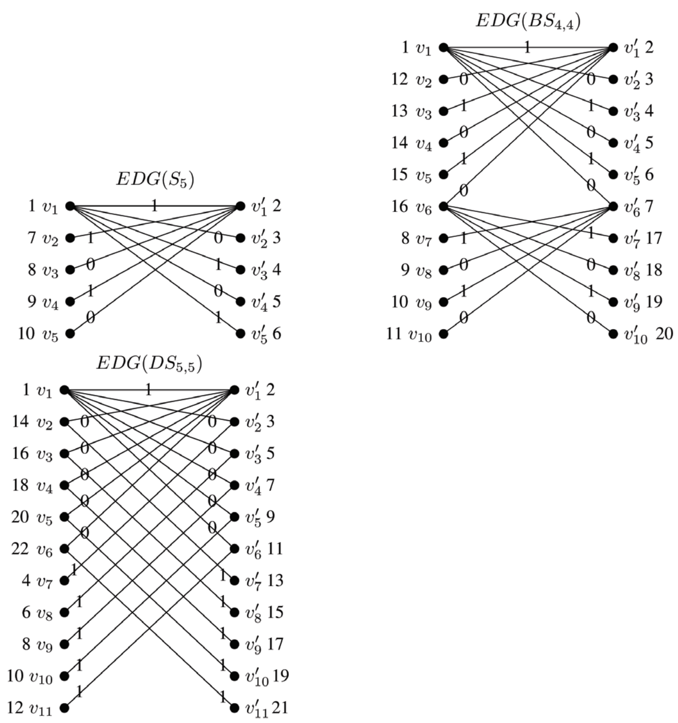

FIGURE 2. Example of SD-prime cordial labeling in $EDG(S_5)$, $EDG(BS_{4,4})$ and $EDG(DS_{5,5})$

CONCLUSION

We have proved that the extended duplicate graphs of path graph, comb graph, twig graph, star graph, bistar graph and double star graph admit SD - Prime cordial labeling.

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