## Economic Analysis of a System under Warranty Having Bath-Tub Curve Shaped Failure Pattern Considering Various Kinds Of Inspection And Replacement

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#### **Abstract**

The aim of the paper is to investigate economic and performance aspects of a single unit system having bath-tub curve shaped failure pattern considering various kinds of inspection and replacement. The operation of the system has been catogarised into three stages on the basis of varying failure rates viz. burn-in period, useful life period and wearout period. After burn-in stage, the available repairman periodically inspects whether fault in the system is minor or major and if the fault is minor he carries out repair/ replacement of the system online. If fault is major, he carries out inspection of the system to judge whether repair or replacement of the unit/ component is required and accordingly, he carries out offline repair/ replacement of the system. Moreover, perior to replacement, inspection of the system is also carried out to judge whether to replace the unit by an old unit of same stage or by a new one. The service engineer is assumed to be intitially available and properly install the system. Various measures of the system performance are obtained using the concepts of Markov Process and regenerative point technique. Profits of the system, both in system user and system provider points of view, are also computed. On the basis of graphical study, various conclusions regarding reliability and profit of the system are drawn.

**Keywords:** Bath-tub curve shaped, burn-in period, useful life period and wear-out period, reliability, availability, profit, Markov Process and regenerative point technique.

#### INTRODUCTION:

For reliability and performance analyses of various mechanical, electro-mechanical, electronic and telecommunication systems considering different aspects/ concepts several researchers assumed constant failure rates of the systems through out their operational life. For instance, Murari and Goyal (1983), Tuteja and Taneja (1991), Kumar et al. (2001), Tuteja et al. (2006) and Kumar and Batra (2013, 2016) etc.

However, in practical situations, many systems/ equipmens such as mobile, robot, water mains system, electric power system, turbine, compressor, generator, combustion chamber, automobiles etc. have the failure rates that varies with time. In fact, the failure rates of such systems/ equipments initially decrease during the early period or burn-in period and then the failure rates flatten out and remain nearly constant for the

useful life period. Finally, the system's failure rates become higher due to fatique and friction during the wear-out period. That is, the systems/ equipments have bath-tub failure pattern. In recent years, several researchers including Pulcini (2001), Stancliff et al. (2006), Kumar et al. (2010), Anumaka et al. (2011), Sarkar and Behera (2012), Kumar et al. (2015, 2017)etc. have considered bath-tub curve shaped failure pattern for estimation of various system parameters. The systems having bath-tub curve shaped failures are very verstile and common hence such systems are need to be analysed incorporating various practical situations.

Keeping this in view, the aim of the paper is to investigate economic and performance aspects of a single unit system having bath-tub curve shaped failure pattern working under failure free warranty policy incorporating some practical situations. The system is analysed considering various kinds of inspection and replacement. The operation of the system has been catogarised into three stages on the basis of varying failure rates viz. burn-in period, useful life period and wearout period. After burn-in stage, the available repairman periodically inspects whether fault in the system is minor or major and if the fault is minor he carries out repair/ replacement of the system online. If fault is major, he carries out further inspection of the system to judge whether repair or replacement of the unit/component has to be done. Accordingly, he carries out offline repair/ replacement of the system. Further, prior to replacement, inspection of the system is also carried out to judge whether to replace the unit by an old unit of same stage or by a new one. The service engineer properly install the system. On failure the system is first inspected to judge whether the faults is repairable or nonrepairable and in case fault is not repairable, second type (type-II) of inspection is carried out. Other assumptions of the model are:

- 1. The faults are detected immediately and properly by the service engineer/repairman.
- 2. The replacement by a new unit needs re-installation of the system.
- 3. The service engineer carries out proper installation, i.e during burn in period of the system, if he is available with the system.
- 4. The service engineer/ repairman takes some arrival time to visit the sytem for periodic perventive/ corrective maintenance.

 $g_2(t)/g_4(t)/g_7(t)$ 

 $G_2(t)/G_4(t)/G_7(t)$ 

 $h_1(t)/h_3(t)/h_6(t)$ 

 $H_1(t)/H_3(t)/H_6(t)$ 

 $h_2(t)/h_4(t) \; /h_7(t)$ 

period

period

out period

out period

out period

p.d.f of offline repair time during burn in

period / useful life period /wear-out

c.d.f. of offline repair time during burn in

period / useful life period /wear-out

p.d.f of online replacement time during

burn in period / useful life period /wear-

c.d.f of online replacement time during burn in period / useful life period /wear-

p.d.f of offline replacement time during

burn in period / useful life period /wear-

- The system is under failure free warranty after burnin period upto useful life period, i.e. as per warranty policy all repair/ replacement are done free of charges by the system provider.
- the unit works as good as new in a particular operational stage after each repair/replacement.
- distributions of the times to failure, improvement and deterioration are exponential while the other distributions are arbitrary.
- All the random variables are mutually independent.

Various measures of the system performance are obtained using the concepts of Markov Process and regenerative point technique and conclusions regarding performance and economic aspects of the system are drawn on the basis of a graphical study.

Notations:		$H_2(t)/H_4(t)/H_7(t)$	c.d.f of replacement time during burn in period / useful life period /wear-out period	
$\lambda_1/\lambda_2/\lambda_3$	rate of faults during burn in period/ useful life period/ wear-out period	$h_5(t)/h_8(t)$	p.d.f of offline replacement time by the same unit during useful life period / wear-out period	
$\eta_1/\eta_2$	rate of improvement/ deterioration of the system			
η	rate at which online inspection is done by	$H_5(t) / H_8(t)$	c.d.f of replacement time by the same unit during useful life period /wear-out period	
$p_1/p_3/p_6$	probability that online repair of the unit is carried out during burn in period/ useful	$i_1(t)/i_3(t)/i_6(t)$	p.d.f of online inspection time of the system during burn in period / useful life period /wear-out period	
$q_{1}/q_{3}/q_{6}$	probability that online replacement of the unit is carried out during burn in period/	$I_1(t)/I_3(t)/I_6(t)$	c.d.f of online inspection time of the system during burn in period /useful life period /wear-out period	
$r_1/r_3/r_6$	useful life period /wear-out period.  probability that the unit on inspection found O.K. during burn in period/ useful	$i_2(t)/i_4(t)/i_7(t)$	p.d.f of type-I inspection time of the system during burninperiod / useful life period / wear-out period	
p <sub>2</sub> /p <sub>4</sub> /p <sub>7</sub>	life period/wear-out period.  probability that repair of the unit is carried out during burn in period/ useful	$I_2(t)/I_4(t)/I_7(t)$	c.d.f of type-I inspection time on failure of the system during burn in period / useful life period /wear-out period	
$q_{2}/q_{4}/q_{7}$	life period/ wear-out period.  probability that replacement of the unit is carried out during burn in period/ useful	i <sub>5</sub> (t)/i <sub>8</sub> (t)	p.d.f of type-II inspection time of the system during useful life period /wear-out period	
$x_1/x_2$	life period/ wear-out period.  probability that the unit is replaced by the same unit after inspection during useful life period / wear-out period.  probability that the unit is replaced by the new unit after inspection during useful	$I_5(t)/I_8(t)$	c.d.f of type-II inspection time of the system during useful life period /wear-out period	
$y_1/y_2$		$k_1(t)/k_2(t)$	p.d.f of arrival time of service engineer/repairman during useful life period/wear-out period	
$g_1(t)/g_3(t)/g_6(t)$	life period /wear-out period.  p.d.f of online repair time during burn in period / useful life period /wear-out period	$K_1(t)/K_2(t)$	c.d.f. of arrival time of service engineer/repairman during useful life period /wear-out period	
$G_1(t)/G_3(t)/G_6(t)$	c.d.f. of online repair time during burn in period / useful life period /wear-out period			

States of the system:		$F_{\text{Ir}}/F_{\text{IIr}}/F_{\text{IIIr}}$	system is failed and is under offline repair	
Si	$i^{th}$ state $i = 0$ to 24		during burn in period /useful life period /wear-out period	
$O_{I}/O_{II}/O_{III}$	system is operating during burn-in period / useful life period / wear-out period	$F_{ m Irp}$	system is failed and is under offline replacement during burn in period	
$O_{Ii1}/O_{Iii3}/O_{IIIi6}$	system is operating and is under online inspection during burn in period /useful life period /wear-out period	$F_{\rm IIrp1}/F_{\rm IIIrp1}$	system is failed and is under offline replacement by the new unit during useful life period /wear-out period	
$O_{Ir}/O_{IIr}/O_{IIIr}$	system is operating and is under online repair during burn in period /useful life period / wear-out period	$F_{\rm IIrp2}/F_{\rm IIIrp2}$	system is failed and is under offline replacement by the same unit during useful life period /wear-out period.	
$O_{Irp}/O_{IIrp}/O_{IIIrp}$	system is operating and is under online replacement during burn in period /useful life period / wear-out period	The state transition diagram in fig. 1 shows various states of transitions of the system. The epochs of entry into states 0 to 24 are regeneration points and thus these are regenerative states.		
$F_{\rm li2}/F_{\rm Ili4}/F_{\rm IIli7}$	system is failed and is under inspection during burn in period /useful life period /wear-out period	The states 4, 5, 6, 11 to 15 and 20 to 24 are failed states.		

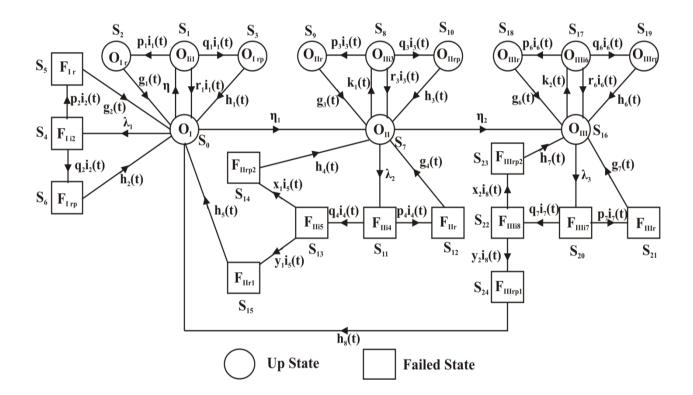


Fig. 1: State Transition Diagram

### Transition Probabilities and Mean Sojourn Times:

The state transition probability in steady state the non-zero elements,  $p_{ij}$  obtained as  $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$  and are given by

$p_{01} = \eta/D;$	$p_{10} = r_1;$	$p_{12} = p_1;$
$p_{13} = q_1;$	$p_{04} = \lambda_1 / D;$	$p_{45} = p_2;$
$p_{46} = q_2;$	$p_{07} = \eta_1 / D;$	$p_{78}=k_1^*(\lambda_2+\eta_2);$
$p_{7,11} = (1 - k_1^* (\lambda_2 + \eta_2))\lambda_2/D_0;$	$p_{8,10} = q_3;$	$p_{87} = r_3;$
$p_{7,16} = (1 - k_1^* (\lambda_2 + \eta_2)) \eta_2 / D_0;$	$p_{89} = p_3;$	$p_{11,12} = p_4;$

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$$\begin{array}{lll} p_{11,13}=q_4; & p_{13,14}=x_1; & p_{13,15}=y_1; \\ \\ p_{17,18}=p_6; & p_{17,16}=r_6; & p_{17,19}=q_6; \\ \\ p_{16,17}=k_2^*(\ \lambda_3)\ ; & p_{16,20}=1\text{-}k_2^*(\ \lambda_3) & p_{20,21}=p_7; \\ \\ p_{20,22}=q_7; & p_{22,23}=x_2; & p_{22,24}=y_2; \end{array}$$

where

$$D=\lambda_1+\eta_1+\eta$$
 and  $D_0=\lambda_2+\eta_2$ 

By these transitions probabilities it can be verified that

$$p_{20} = p_{30} = p_{50} = p_{60} = p_{97} = p_{10,7} = p_{12,7} = p_{14,7} = p_{15,0} = p_{18,16} = p_{19,16} = p_{21,16} = p_{23,16} = p_{24,0} = 1$$
 
$$p_{01} + p_{04} + p_{07} = p_{12} + p_{13} + p_{10} = p_{45} + p_{46} = p_{78} + p_{7,11} + p_{7,16} = p_{87} + p_{89} + p_{8,10} = p_{11,12} + p_{11,13} = p_{13,14} + p_{13,15} = p_{16,17} + p_{16,20} = p_{17,16} + p_{17,18} + p_{17,19} = p_{20,21} + p_{20,22} = p_{22,23} + p_{22,24} = 1$$

The mean sojourn times in regenerative states i  $(\mu_i)$  are:

$\mu_0=1/D$	$\mu_1 = -i_1^{*'}(0)$	$\mu_2 = -g_1^*(0)$
$\mu_3 = -h_1^{*'}(0)$	$\mu_4 = -i_2^{*'}(0)$	$\mu_5 = -g_2^{*'}(0)$
$\mu_6 = -h_2^{*'}(0)$	$\mu_7 = (1 - k_1^* (\lambda_2 + \eta_2))/D_0$	$\mu_8 = -i_3^{*'}(0)$
$\mu_9 = -g_3^{*'}(0)$	$\mu_{10} = -h_3^{*'}(0)$	$\mu_{11} = -i_4^{*'}(0)$
$\mu_{12} = -g_4^*(0)$	$\mu_{13} = -i_5^{*'}(0)$	$\mu_{14} = -h_4^{*'}(0)$
$\mu_{15} = -h_5^{*'}(0)$	$\mu_{16} = (1 - k_2^*(\lambda_3))/\lambda_3$	$\mu_{17} = -i_6^{*}(0)$
$\mu_{18} = -g_6^{*}(0)$	$\mu_{19} = -h_6^{*'}(0)$	$\mu_{20} = -i_7^{*'}(0)$
$\mu_{21} = -g_7^{*'}(0)$	$\mu_{22} = -i_8^{*'}(0)$	$\mu_{23} = -h_7^{*'}(0)$
$\mu_{24} = -h_8^{*'}(0)$		

where D and D<sub>0</sub> are already specified.

The unconditional mean time taken by the system to transit for any state j when it is counted from the epoch of entrance into the state i, is mathematically stated as:

$$m_{ij} = \int_{0}^{\infty} t q_{ij}(t) dt = -q_{ij}^{*'}(0)$$

Thus

$m_{01} + m_{04} + m_{07} = \mu_0;$	$m_{12} + m_{13} + m_{10} = \mu_{1}$ ;	$m_{20} = \mu_2;$	$m_{30} = \mu_3;$	$m_{45} + m_{46} = \mu_4;$
$m_{50} = \mu_5;$	$m_{60} = \mu_6;$	$m_{78}+m_{711}+m_{716}=\mu_7;$	$m_{87}+m_{89}+m_{810}=\mu_8;$	$m_{97} = \mu_9;$
$m_{10,7} = \mu_{10};$	$m_{11,12} + m_{11,13} = \mu_{11};$	$m_{12,7} = \mu_{12};$	$m_{13,14}+m_{13,15}=\mu_{13};$	$m_{14,7}=\mu_{14} \\$
$m_{15,0}=\mu_{15}$	$m_{16,17} + m_{16,20} = \mu_{16};$	$m_{17,16}+m_{17,18}+m_{17,19}=\mu_{17};$	$m_{18,16} = \mu_{18};$	$m_{19,16} = \mu_{19};$
$m_{20,21} + m_{20,22} = \mu_{20};$	$m_{21,16} = \mu_{21};$	$m_{22,23}+m_{22,24}=\mu_{22};$	$m_{23,16}=\mu_{23}$	$m_{24,0}=\mu_{24}$

#### Other Measures of System Performance:

Using the probabilistic arguments for regenerative process, recursive relations for various other measures of the system performance for different stages of operation are obtained. On solving them using Laplace/Laplace-Shieltjes transforms, we get

$$\mbox{Mean time to System Failure } (T_0) \ = \ \ \frac{N_1}{D_1} \label{eq:mean}$$

#### **Burn-in Period**

Steady-State Availability( $A_0$ ) =  $\frac{N_2}{D_2}$ 

Expected busy period of the service engineer repair time only (BR<sub>0</sub>) =  $\frac{N_3}{D_2}$ 

Expected busy period of the service engineer inspection time only (BI<sub>0</sub>) =  $\frac{N_4}{D_2}$ 

Expected number of the replacement by the service engineer:

- (i) online replacement by new unit (RPO<sub>0</sub>) =  $\frac{N_s}{D_2}$
- (ii) offline replacement by new unit (RPI<sub>0</sub>) =  $\frac{N_6}{D_2}$

Expected number of the visit by the service engineer  $(V_0) = \frac{N_7}{D_2}$ 

#### **Useful-life Period**

Steady-State Availability(A<sub>7</sub>) =  $\frac{N_8}{D_2}$ 

Expected busy period of the available repairman repair time only (BR<sub>7</sub>) =  $\frac{N_9}{D_2}$ 

Expected busy period of the available repairman inspection time only (BI<sub>7</sub>)=  $\frac{N_{10}}{D_2}$ 

Expected number of the replacement by the available repairman:

- (i) online replacement by new unit (RPO<sub>7</sub>) =  $\frac{N_{11}}{D_2}$
- (ii) offline replacement by new unit (RPI<sub>7</sub>) =  $\frac{N_{12}}{D_2}$
- (iii) offline replacement by old same operational unit (RPII<sub>7</sub>) =  $\frac{N_{13}}{D_2}$

Expected number of the visit by the available repairman  $(V_7) = \frac{N_{14}}{D_2}$ 

#### **Wear-out Period**

Steady-StateA vailability( $A_{16}$ ) =  $\frac{N_{15}}{D_2}$ 

Expected busy period of the available repairman repair time only (BR<sub>16</sub>) =  $\frac{N_{16}}{D_2}$ 

Expected busy period of the available repairman inspection time only(BI<sub>16</sub>) =  $\frac{N_{17}}{D_2}$ 

Expected number of the replacement by available repairman :

 $\frac{N_{18}}{D_2}$ 

(ii) offline replacement by new unit 
$$(RPI_{16}) =$$

 $\frac{N_{19}}{D_2}$ 

(iii) offline replacement by old same operational unit (RPII
$$_{16}$$
) =

 $\frac{N_{20}}{D_2}$ 

(iv) Expected number of the visit by the available repairman 
$$(V_{16})$$
 =

 $\frac{N_{21}}{D_2}$ 

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where

 $N_1 = (1 - p_{48}) p_{16,20} [\mu_0 + p_{01}(\mu_1 + \mu_2 p_{12} + \mu_3 p_{13})] + p_{07} p_{16,20} [\mu_7 + p_{78}(\mu_3 \mu_8 + \mu_9 p_{89} + \mu_{10} p_{8,10})] + p_{07} p_{7,16} [\mu_{16} + p_{16,17}(\mu_{17} + \mu_{18} p_{17,18} + \mu_{19} p_{17,19})]$ 

 $N_2 \!\!=\!\! p_{16,20}\; p_{20,22}\; p_{22,24}(p_{7,16} \!\!+\! p_{7,11}p_{11,13}p_{13,15}) \left[\mu_0 \!\!+\! \mu_1 p_{01} \!\!+\! \mu_2\; p_{01}p_{12} \!\!+\! \mu_3\; p_{01}p_{13}\right]$ 

 $N_3 = p_{16,20} p_{20,22} p_{22,24} (p_{7,16} + p_{7,11} p_{11,13} p_{13,15}) [\mu_2 p_{01} p_{12} + \mu_5 p_{04} p_{45}]$ 

 $N_4 \!\!=\!\! p_{16,20}\; p_{20,22}\; p_{22,24}(p_{7,16} \!\!+\! p_{7,11}p_{11,13}p_{13,15}) \left[\mu_1\; p_{01} \!\!+\! \mu_4\; p_{04}\right]$ 

 $N_5 = p_{01}p_{13}p_{16,20} p_{20,22} p_{22,24}(p_{7,16} + p_{7,11}p_{11,13}p_{13,15})$ 

 $N_6 = p_{04}p_{46}p_{16,20} p_{20,22} p_{22,24}(p_{7,16} + p_{7,11}p_{11,13}p_{13,15})$ 

 $N_7 = p_{16,20} p_{20,22} p_{22,24} (p_{01} + p_{04}) [p_{7,16} + p_{7,11} p_{11,13} p_{13,15}]$ 

 $N_8 = p_{07} p_{16,20} p_{20,22} p_{22,24} (\mu_7 + \mu_8 p_{78} + \mu_9 p_{78} p_{89} + \mu_{10} p_{78} p_{8,10})$ 

 $N_9 = p_{07}p_{16,20} p_{20,22} p_{22,24}(\mu_9 p_{78}p_{89} + \mu_{12} p_{7,11}p_{11,12})$ 

 $N_{10} = p_{07}p_{16,20} \ p_{20,22} \ p_{22,24}(\mu_8 p_{78} + \mu_{11} p_{7,11} + \mu_{13} p_{7,11} p_{11,13})$ 

 $N_{11} = p_{07}p_{78}p_{8,10}p_{16,20} p_{20,22} p_{22,24}$ 

 $N_{12} = p_{07}p_{7,11}p_{11,13} p_{13,15}p_{16,20} p_{20,22} p_{22,24}$ 

 $N_{13} = p_{07}p_{7,11}p_{11,13} p_{13,14}p_{16,20} p_{20,22} p_{22,24}$ 

 $N_{14} = p_{07}p_{16,20} \ p_{20,22} \ p_{22,24} (\ p_{78} + p_{7,11})$ 

 $N_{15} = p_{07} p_{7,16} (\mu_{16} + \mu_{17} p_{16,17} + \mu_{18} p_{16,17} p_{17,18} + \mu_{19} p_{16,17} p_{17,19})$ 

 $N_{16}=p_{07} p_{7,16} (\mu_{18} p_{16,17}p_{17,18}+\mu_{21} p_{16,20}p_{20,21})$ 

 $N_{17} = p_{07} p_{7,16} (\mu_{17} p_{16,17} + \mu_{20} p_{16,20} + \mu_{22} p_{16,20} p_{20,22})$ 

 $N_{18} = p_{07}p_{7,16}p_{11,13} * p_{16,17}p_{17,19}$ 

 $N_{19} = p_{07}p_{7,16}p_{16,20} p_{20,22} p_{22,24}$ 

 $N_{20} \!\!= p_{07}p_{7,16}p_{16,20}\;p_{20,22}\;p_{22,23}$ 

 $N_{21} = p_{07}p_{7,16} (p_{16,17} + p_{16,20})$ 

 $D_1 = p_{16,20} (1 - p_{01}) (1 - p_{78})$ 

and  $D_2=p_{16,20}$   $p_{20,22}$   $p_{22,24}$   $[(\mu_0+\mu_1p_{01})p_{7,11}p_{11,13}p_{13,15}+(\mu_2\ p_{01}p_{12}+\mu_3\ p_{01}p_{13}+\mu_5\ p_{04}p_{45}+\mu_6\ p_{04}p_{46})(\ p_{7,16}+p_{7,11}\ p_{11,13}\ p_{13,15})+\mu_4\ p_{04}p_{7,11}$   $p_{11,13}$   $p_{13,15}$   $p_{13,15}$   $p_{14}$   $p_{14}$   $p_{14}$   $p_{15}$   $p_{15,15}$   $p_{15,17}$   $p_{11,13}$   $p_{13,15}$   $p_{15,17}$   $p_{11,13}$   $p_{13,15}$   $p_{15,17}$   $p_{15,15}$   $p_{15,15}$   $p_{15,15}$   $p_{15,$ 

#### **Profit Analysis of the System**

(A) Expected Profit for System User  $(P_1)$  is given by

 $P_1 = C_0(A_0 + A_7 + A_{16}) - C_1(BI_{16}) - C_2(BR_{16}) - C_3(RPO_{16} + RPI_{16}) - C_4(RPII_{16}) - C_5(V_{16}),$ 

where

 $C_0$  = revenue per unit up time of the system

 $C_1 = cost per unit time of inspection by the available repairman$ 

 $C_2 = \cos t$  per unit time of repair by the available repairman

 $C_3 = cost per unit replacement by the new unit$ 

 $C_4 = \cos t$  per unit replacement by the old same operational unit

 $C_5 = cost per visit of the available repairman$ 

(B) Expected Profit for the System Provider  $(P_2)$  is given by

 $P_2 = (SP-CP) - C_6(BI_0 + BI_7) - C_7(BR_0 + BR_7) - C_8(RPO_0 + RPI_0 + RPO_7 + RPI_7) - C_9(RPII_7) - C_{10}(V_0 + V_7) \; , \\$  where

SP/CP = sale price/ cost price per unit of the system

 $C_6$  = cost per unit time of inspection by the service engineer

 $C_7$  = cost per unit time of repair by the service engineer

 $C_8$  = cost per unit replacement(online/new unit) by the service engineer

C<sub>9</sub> = cost per unit replacement (old same unit) by the service engineer

 $C_{10}$  = cost per visit of the service engineer

#### **Graphical Interpretations and Conclusions**

For the graphical analysis of the system at various stages of its operation following particular cases is considered:

$$\begin{split} i_{j}\left(t\right) &= \alpha_{j}e^{-\alpha_{j}t} &, \quad h_{j}\left(t\right) = \gamma_{j}e^{-\gamma_{j}t} & \text{where } j=1 \text{ to } 8 \\ g_{i}\left(t\right) &= \beta_{i}e^{-\beta_{i}t} &, \quad k_{i}\left(t\right) = \delta_{i}e^{-\delta_{i}t} & \text{where } i=1,2. \end{split}$$

Various graph plotted for mean time to system failure and profits incurred for the system w.r.t.different failure rates  $(\lambda_1, \lambda_2, \lambda_3)$ , repair rates  $(\beta_1, \beta_2, \beta_3)$  inspection rate  $(\eta)$  and improment/deterioration rate  $(\eta_1, \eta_2)$  etc.. Following interpretations and conclusions are made from the graphical analysis.

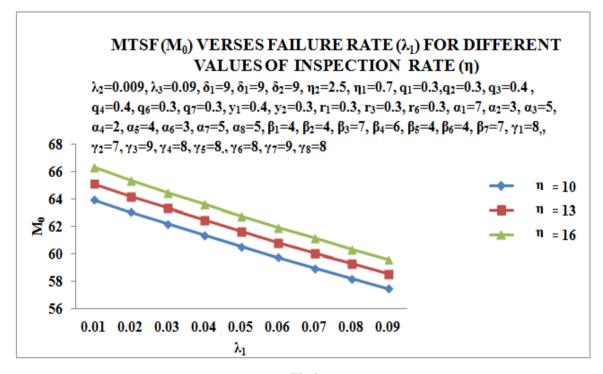


Fig.2

Fig. 2 shows the behavior of mean time to system failure  $(M_0)$  with respect to failure rate  $(\lambda_1)$  during burn-in period for different values of inspection rate  $(\eta)$ .

It can be conclude from the graph that mean time to system failure  $(M_0)$  deccreases with the increase in the values of  $\lambda_1$  when other parameters are fixed and has higher values for higher values of  $\eta$ .

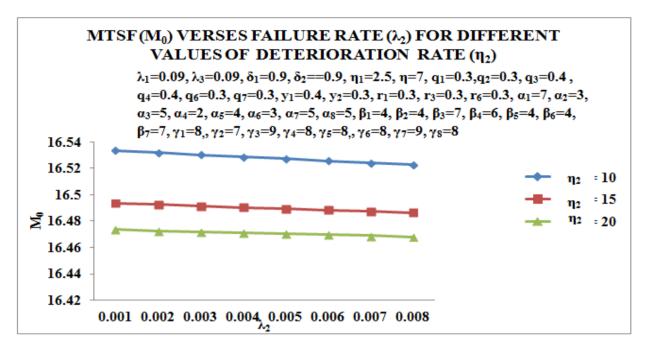


Fig.3

The pattern in fig. 3 reveals the behavior of mean time to system failure  $(M_0)$  with respect to failure rate  $(\lambda_2)$  for different values of deterioration rate  $(\eta_2)$ .

It can be conclude from the graph that mean time to system failure  $(M_0)$  decreases with the increase in the values of  $\lambda_2$  when other parameters are fixed and has lower values for higher values of  $\eta_2$ .

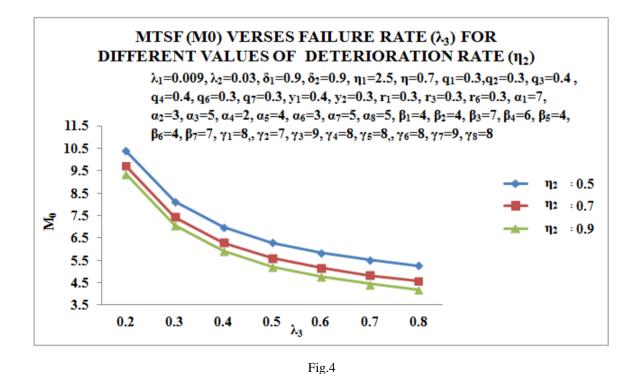


Fig. 4 presents the behavior of mean time to system failure  $(M_0)$  with respect to failure rate  $(\lambda_3)$  for different values of deterioration rate  $(\eta_2)$ .

It can be conclude from the graph that mean time to system failure  $(M_0)$  decreases with the increase in the values of  $\lambda_3$  when other parameters are fixed and has lower values for higher values of  $\eta_2$ .

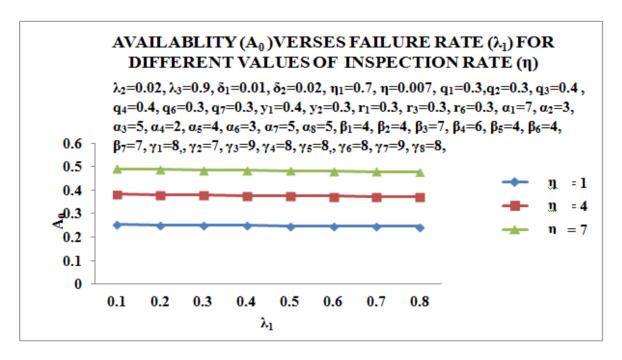


Fig 5

Fig. 5 shows the behavior of availability  $(A_0)$  with respect to failure rate  $(\lambda_1)$  for different values of inspection rate  $(\eta)$ .

It can be conclude from the graph that  $A_0$  decreases with the increase in the values of  $\lambda_1$  when other parameters are fixed and has higher values for higher values of  $\eta$ .

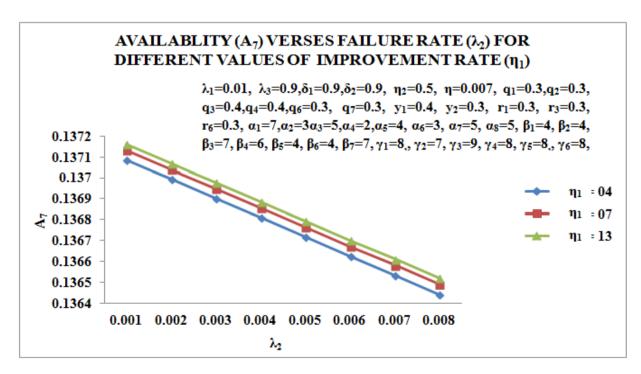


Fig. 6

Fig. 6 shows the behavior of availability (A<sub>7</sub>) with respect to failure rate ( $\lambda_2$ ) for different values of improvement rate ( $\eta_1$ ).

It can be conclude from the graph that  $A_7$  decreases with the increase in the values of  $\lambda_2$  when other parameters are fixed and has higher values for higher values of  $\eta_1$ .

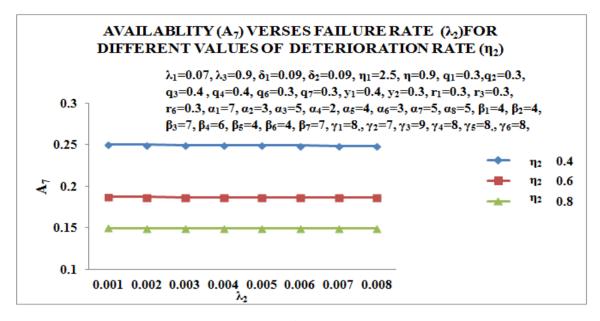


Fig. 7

Fig. 7 shows the behavior of availability  $(A_7)$  with respect to failure rate  $(\lambda_2)$  for different values of deterioration rate  $(\eta_2)$ .

It can be conclude from the graph that  $A_7$  decreases with the increase in the values of  $\lambda_2$  when other parameters are fixed and has lower values for higher values of  $\eta_2$ .

The curve in fig. 8 depicts the behaviour of profit of system user  $(P_1)$  with respect to failure rate  $(\lambda_3)$  in wear-out period for

different values of deterioration rate ( $\eta_2$ ). It is concluded from the graph that  $P_1$  decreases with the increase in the values of  $\lambda_3$  and has lower values for higher values of  $\eta_2$ . From the fig. 8, it can also be observed that for  $\eta_2$ = 5,  $P_1$  is positive or zero or negative as  $\lambda_3 <$  or = or > 0.7661 and thus in this case, the system is profitable whenever  $\lambda_3$  is less than 0.7661. Similarly for  $\eta_2$  =8 and  $\eta_2$  = 11, the system user is profitable whenever  $\lambda_3 <$  0.76068 and 0.75833 respectively.

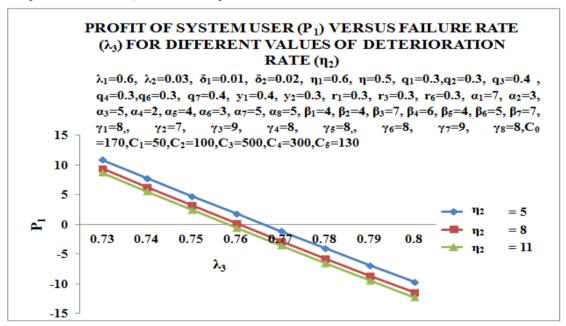


Fig. 8

Fig. 9 gives the behaviour of profit of system user  $(P_1)$  with respect to revenue per unit up time  $(C_0)$  for different values of cost per visit  $(C_5)$  of the available repairman. It can be concluded that profit of system user increases with the

increase in the values of revevue per unit up time and has lower values for higher values of cost per visit of the available repairman. From the fig. 9, it can also be observed that for  $C_5$ = Rs.100,  $P_1$  is positive or zero or negative as  $C_0 > \text{or} = \text{or}$ 

< Rs.192.052 and thus in this case, the system is profitable whenever revenue per unit up time is greater than Rs. 192.052. Similarly for  $C_5 = Rs.250$  and  $C_5 = Rs.300$ , the

system user is profitable whenever  $C_0 > Rs.264.039$  and Rs.336.026 respectively.

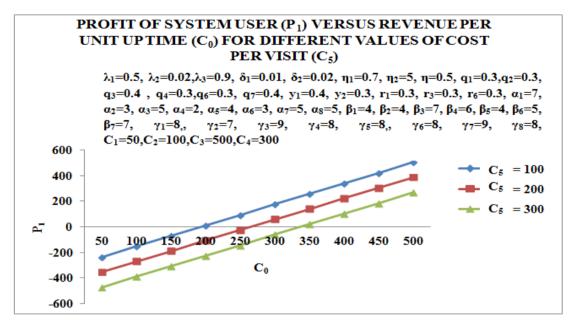


Fig. 9

The pattern in fig. 10 depicts the behaviour of profit of system provider  $(P_2)$  with respect to cost per visit  $(C_{10})$  for different values of failure rate  $(\lambda_2)$ . It is concluded from the graph that  $P_2$  decreases with the increase in the values of  $C_{10}$  and has lower values for higher values of  $\lambda_2$ . From the fig. 10, it can also be observed that for  $\lambda_2 = 0.1$ ,  $P_2$  is positive or zero or

negative as  $C_{10} < \text{or} = \text{or} > \text{Rs.}4216.763$  and thus in this case, the system is profitable whenever  $C_{10}$  should be fixed less than Rs.4216.763. Similarly for  $\lambda_2 = 0.5$  and  $\lambda_2 = 0.9$ , the system provider is profitable whenever  $C_{10} < \text{Rs.}4144.659$  and Rs.4081.479 respectively.

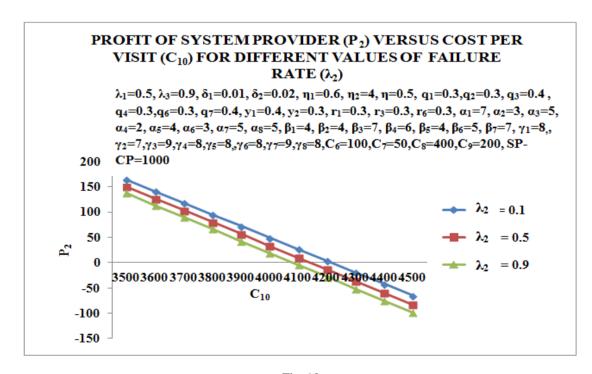


Fig. 10

Fig. 11 depicts the behaviour of profit of system provider ( $P_2$ ) with respect to profit (SP-CP) for different values of failure rate ( $\lambda_2$ ). It is concluded from the graph that  $P_2$  increase with the increase in the values of SP-CP and has lower values for higher values of  $\lambda_2$ . From the fig. 11, it can also be observed that for  $\lambda_2 = 0.4$ ,  $P_2$  is positive or zero or negative as SP-CP >

or = or < Rs.53.925 and thus in this case, the system is profitable whenever SP-CP should be fixed greater than Rs.53.925. Similarly for  $\lambda_2 = 0.6$  and  $\lambda_2 = 0.8$ , the system provider is profitable whenever SP-CP > Rs.67.114 and Rs.76.464 respectively.

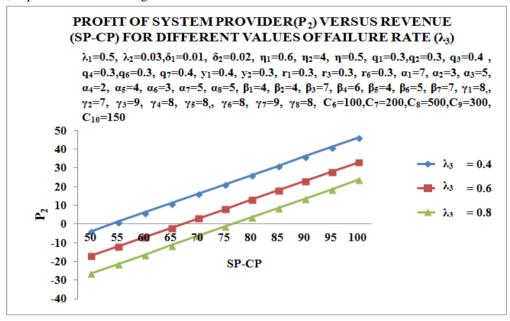


Fig. 11

The pattern in fig. 12 depicts the behaviour of profit of system provider  $(P_2)$  with respect to failure rate  $(\lambda_1)$  for different values of improvement rate  $(\eta_1)$ . It is concluded from the graph that  $P_2$  decreases with the increase in the values of  $\lambda_1$  and has higher values for higher values of  $\eta_1$ . From the fig. 12, it can also be observed that for  $\eta_1 = 0.015$ ,  $P_2$  is positive or

zero or negative as  $\lambda_1 < \text{or} = \text{or} > 0.2813$  and thus in this case, the system is profitable whenever  $\lambda_1$  should be fixed less than 0.2813. Similarly, for  $\eta_1 = 0.018$  and  $\eta_1 = 0.021$ , the system provider is profitable whenever  $\lambda_1 < 0.4739$  and 0.6665 respectively.

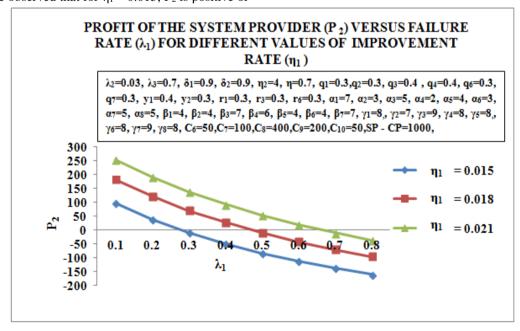


Fig. 12

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