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Linear and Non Linear Analysis of Double Diffusive Convection in a Vertically Oscillating Couple Stress Fluid with Cross Diffusion Effects

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Abstract

The double diffusive convection in the presence of cross diffusion in a vertically oscillating (gravity modulation) couple stress fluid is studied. The condition for onset of convection is obtained using the linear theory analysis and effect of parameter on heat and mass transfer is investigated using non linear analysis. Linear analysis is based on normal mode technique and perturbation method. Using the Venezian method the expression for correction Rayliegh number is obtained as a function of couple stress parameter, solutal Rayliegh number, frequency and amplitude of the modulation. The effect of Dufour and Soret parameters on heat and mass transfer is investigated by deriving generalized Lorenz model. It is found that Dufour parameter and negative values of Soret parameter stabilizes the system where as positive Soret parameter destabilizes the system.

Keywords: Double diffusive convection, Couple stress fluid, gravity modulation, cross diffusion.

INTRODUCTION

Convection in the presence of two or more components with different molecular diffusivities give rise to new observations compare to classical thermal problems in which heat has single diffusive components. Variations in solute concentration can also give rise to Buoyancy force. Such problems arises in nature and particularly in ocean. The convection that arises because of density difference due to destabilizing nature of temperature field and concentration field is called double diffusive convection. Motivated by the theoretical and practical applications of double diffusive convection many authors (Turner[1], Malashetty et al.[2], Siddheshwar et al.[3], Bhadauria et al.[4], Pranesh and Arun Kumar [5], Pranesh and Sameena [6]) investigated the double diffusive convection under different situations.

When both heat and mass transfer present in a fluid motion, fluxes and driving potential relations are more complicated. In the presence of cross diffusion effects the flux of each property is significantly affected by the gradient property of other. The flux of heat due to concentration gradient is called Dufour effect and the flux of solute due to temperature gradient is called Soret effect. Many studies are available in literature concerning the Dufour and Soret effect.

The effect of Dufour and Soret in a thermal convection in binary fluid has been investigated by Knobloch[7]. Rudraiah and Malashetty [8] investigated the effects of Dufour and Soret effect in a fluid saturated porous medium. Rudraiah and Siddheshwar [9] extended the previous work to include the weak non-linear analysis. Gaikwad et al.[10] performed linear and non-linear analysis to study the effects of Soret analytically in a horizontal layer of fluid in a saturated an isotropic porous layer. Malashetty and Biradar [11] studied the effect of Soret and Dufour effect in a Newtonian fluid saturated porous medium analytically using linear and non-linear analysis. Ravi et al.[12] studied the effects of second diffusing component and cross on primary and secondary thermo convective instabilities in couple stress fluid.

Many researchers investigated the effect of periodic modulation on the stability of a fluid heated from below. These periodic modulations can either stabilizes or destabilizes the system depending on amplitude and frequency of the modulation. Gravity modulation is one of the periodic modulations which has important application in space experiments. Since gravity acts on the entire fluid volume, it can be well understood by vertically oscillating horizontal fluid layer.

Gershuni and Zhukhovitskii [13] were first to study the effect of gravity modulation in a fluid layer. The effect of vertical vibration on the onset of double diffusive convection using Floquet theory was investigated by Murrary et al.[14]. Clever et al.[15] investigated the vertically oscillating fluid layer heated from below. Siddheshwar and Pranesh [16] investigated the effect of gravity modulation on the onset of convection in a conducting fluid with internal angular momentum. The effect of internal heat source and gravity modulation on onset of convection and heat transfer using linear and non-linear analysis is studied by Bhadauria et al.[17].

In recent times many industrial and natural flows are modeled using non-Newtonian fluids. In particularly researchers are shown interest in couple stress fluid due to presence of micron sized particles, which are modeled by Stoke's couple stress equation derived from Eringen's micropolar continum. Siddheshwar and Pranesh [18] were the first to study the single component thermal convection in couple stress fluid. Later many authors (Sunil et al.[19], Malashetty et al.[20], Shivakumara [21], Malashetty and Premila Kollur [22], Pranesh and Sangeetha [23], Sameena and Pranesh [24], Ravi et al.[25]) extended the above work in a couple stress fluid with or without porous medium under different situation.

All the works mentioned above on double diffusive convection in a couple stress fluid with cross diffusion, authors have considered constant gravity. Thus, the main objective of the paper is to study the effects of Dufour and Soret effect in a couple stress fluid under time dependent gravity or body force.

MATHEMATICAL FORMULATION:

Consider a Boussinesquain couple stress fluid layer occupying a space between parallel plates of infinite horizontal extend and separated by a small distance 'd'. Lower plate is considered to be hotter than upper plate. A Cartesian coordinate system is taken with origin in the lower boundary and z-axis in the time dependent sinosioudial gravity field. Let ΔT and ΔS be the temperature and concentration difference between the lower and upper surface. Further the boundary surfaces of the layer are assumed to be stress-free, isothermal and isohaline.

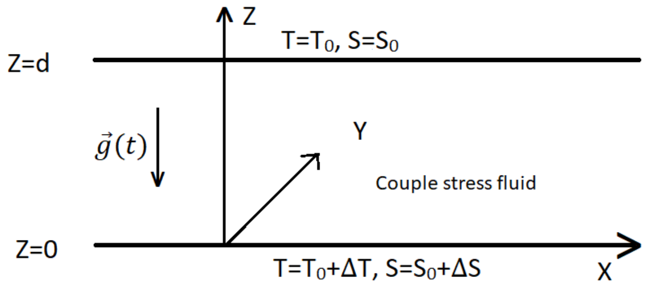


Figure 1: Schematic diagram of the physical configuration of the problem

The governing equations used to solve this problem are:

Continuity Equation:

$$\nabla \cdot \vec{q} = 0 \quad (1)$$

Conservation of linear momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g_0 (1 + \varepsilon \cos \omega t)\hat{k} + \mu \nabla^2 \vec{q} - \mu' \nabla^4 \vec{q} \quad (2)$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = K_T \nabla^2 T + K_{TS} \nabla^2 S \quad (3)$$

Conservation of Solutal Concentration:

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = K_S \nabla^2 S + K_{ST} \nabla^2 T \quad (4)$$

Equation of State:

$$\rho = \rho_0 [1 - \alpha_t (T - T_0) + \alpha_s (S - S_0)] \quad (5)$$

where, $\vec{q}$ is the velocity, ρ_0 is the density of the fluid at $T = T_0$, p is the pressure, ρ is the density, α_t is the coefficient of thermal expansion, α_s is the coefficient of solutal expansion, K_T is the thermal diffusivity, K_S is the solutal diffusivity, K_{TS} is the Dufour effect, K_{ST} is the Soret effect, g_0 is mean acceleration due to gravity, ε is the small amplitude of gravity modulation and ω is the frequency of modulation.

Basic state

The conduction state is dicribed by

$$\vec{q}_b = (0,0,0),\ \rho = \rho_b(z), P = P_b(z),\ T = T_b(z),\ S = S_b(z). \quad (6)$$

Substituting equation (6) into equations (1)-(5) we obtain,

$$\frac{d\rho_b}{dz} + \rho_b g_0 = 0 \quad (7)$$

$$K_T \frac{d^2 T_b}{dz^2} + K_{TS} \frac{d^2 S_b}{dz^2} = 0 \quad (8)$$

$$K_s \frac{d^2 S_b}{dz^2} + K_{ST} \frac{d^2 T_b}{dz^2} = 0 \quad (9)$$

Solving equation (8) and (9) using the boundary conditions $T_b = T_0 + \Delta T,\ S_b = S_0 + \Delta S$ at z = 0,

$T_b = T_0,\ S_b = S_0$ at z = d we get

$$T = T_0 + \Delta(1 - \frac{z}{d}) \quad (10)$$

$$S = S_0 + \Delta(1 - \frac{z}{d}) \quad (11)$$

$$\rho_b = \rho_0[1 - \alpha_t(T_b - T_0) + \alpha_S(S_b - S_0)] \quad (12)$$

The conduction state is disturbed by introducing the following infinite small perturbation.

$$\vec{q} = \overrightarrow{q_b} + \vec{q}\ , \rho = \rho_b(z) + \rho'\ , p = p_b(z) + p',\ T = T_b(z) + T', \quad S = S_b(z) + S' \quad (13)$$

The prime indicates that the quantities are perturbed and subscript 'b' denotes the basic state value.

Following the general method of stability analysis and non dimensionalizing by taking the characteristic length as d, characteristic time as d^2/χ, characteristic velocity χ/d ,characteristic temperature and concentration as ΔT and ΔS respectively, and characteristic pressure as $\mu\chi/d^2$, we get the following dimensionless equations.

$$\frac{1}{P_r}\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\overrightarrow{q}\right] = -\nabla p - R_a(1 + \cos \omega t)T\hat{k} + R_s(1 + \cos \omega t)\varphi_s \hat{k} + \nabla^2 \vec{q} + C\nabla^4 \vec{q} \quad (14)$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T - W = \nabla^2 T + D_u \nabla^2 T \quad (15)$$

$$\frac{\partial \varphi_s}{\partial t} + (\vec{q}.\nabla)T - W = \tau \nabla^2 \varphi_s + S_r \nabla^2 T \quad (16)$$

The non dimensional parameters Pr, Ra, Rs, Du, Sr and C are defined as below,

$P_r = \frac{\mu}{\rho_0 \chi}$ (Prandtl Number)

$R = \frac{\rho_0 g_0 \alpha_t \Delta T d^3}{\mu \chi}$ (Rayleigh Number)

$R_s = \frac{\rho_0 g_0 \alpha_t \Delta S d^3}{\mu \chi}$ (Solutal Rayleigh Number)

$C = \frac{\mu'}{\mu d^2}$ (Couple stress parameter)

$\tau = \frac{K_S}{\chi}$ (Ratio of diffusivity)

$D_u = \frac{K_{TS}\,\Delta S}{\chi \Delta T}$ (Dufour parameter)

$S_r = \frac{K_{TS}\Delta T}{\chi \Delta S}$ (Soret parameter)

We eliminate pressure term in the equation (14) by operating curl and Since two dimensional flow is considered in our study, we introduce the stream function ψ such that $u = \frac{\partial\psi}{\partial x}$ and $W = -\frac{\partial\psi}{\partial z}$

$$\frac{1}{P_r}\left[\frac{\partial}{\partial t}(\nabla^2\psi) + (\vec{q}.\nabla)\nabla^2\psi\right] = (-\mathrm{R}\frac{\partial \mathrm{T}}{\partial \mathrm{x}} + R_s\frac{\partial\varphi_s}{\partial x})(1 + \epsilon\cos\omega t) + \nabla^4\psi + C\nabla^6\psi \quad (17)$$

$$\frac{\partial T}{\partial t} - J(\psi, \mathrm{T}) + \frac{\partial\psi}{\partial x} = \nabla^2 T + D_u\nabla^2\varphi_s \quad (18)$$

$$\frac{\partial\varphi_s}{\partial t} - J(\psi.\varphi_s) + \frac{\partial\psi}{\partial x} = \tau\nabla^2\varphi_s + S_r\nabla^2 T \quad (19)$$

where, J is the Jacobian.

Linear Stability Analysis

$$\frac{1}{P_r}\left[\frac{\partial}{\partial t}(\nabla^2\psi)\right] = (-\mathrm{R}\frac{\partial \mathrm{T}}{\partial \mathrm{x}} + R_s\frac{\partial\varphi_s}{\partial x})(1 + \epsilon\cos\omega t) + \nabla^4\psi + C\nabla^6\psi \quad (20)$$

$$\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial x} = \nabla^2 T + D_u\nabla^2\varphi_s \quad (21)$$

$$\frac{\partial\varphi_s}{\partial t} + \frac{\partial\psi}{\partial x} = \tau\nabla^2\varphi_s + S_r\nabla^2 T \quad (22)$$

Eliminating T and φ_s from the equation (20)-(22), we get

$$[X_1X_2X_3\nabla^2 + (1+\epsilon f)[X_3R_s - X_3R_a + R_sS_r\nabla^2 - R_aD_u\nabla^2]\nabla_1^2 - X_1R_rD_u\nabla^6]T = 0 \quad (23)$$

Where,

$X_1 = \frac{1}{P_r}\frac{\partial}{\partial t} - \nabla^2 + C\nabla^4$

$X_2 = \frac{\partial}{\partial t} - \nabla^2$

$X_3 = \frac{\partial}{\partial t} - \tau\nabla^2$

$\mathrm{f} = \mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$ and $\mathrm{f}' = -\mathrm{i}\omega\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$

We considered stress-free, isothermal, isohaline boundary conditions in terms of T these conditions yields:

$$T = \frac{d^2T}{dz^2} = \frac{d^4T}{dz^4} = \frac{d^6T}{dz^6} = \frac{d^8T}{dz^8} = 0 \; on \; z = 0\,, z = 1 \quad (24)$$

Method of Solution

We seek the eigen function T and eigen - value R of the equation (23) in the form

$$(R_a, T) = (R_0, T_0) + \in (R_1, T_1) + \in^2 (R_2, T_2) + \cdots ... \quad (25)$$

Substituting equation (25) in equation (23), and the coefficients of various powers of ∈ are equated on either side of the equation. We get :

$$LT_0 = 0, \quad (26)$$

$$LT_1 = \left[(X_3R_0 - X_2R_s - R_sS_r\nabla^2 + R_0D_u\nabla^2)\nabla_1{}^2\right]T_0 f, \quad (27)$$

$$LT_2 = \left[(X_3R_0 - X_2R_s - R_sS_r\nabla^2 + R_0D_u\nabla^2)\nabla_1{}^2\right]T_1 f + [(X_3R_1 + X_3R_2 + R_{01}D_u\nabla^2)\nabla^2 f]\nabla_1{}^2 T_0, \quad (28)$$

Where,

$$L = X_1X_2X_3\nabla^2 + [R_SX_2 - R_0X_3 + R_SS_r\nabla^2 - R_0D_u\nabla^2]\nabla_1{}^2 - D_uS_r\nabla^6X_1$$

Solution to the zeroth order problem

The zeroth order problem is equivalent to the Rayliegh Be'nard double diffusive convection problem of couple stress fluid in the absence of gravity modulation. The marginally stable solution of the problem is the general solution of the equation (26), i.e.

$$T_0 = \sin(\pi\alpha x)\sin(\pi z) \quad (29)$$

Substituting equation (29) in equation (26) we get,

$$R_o = \frac{\left[K^6 + \eta K^8\right]D_uS_r - \left(1 + \eta K^2\right)\tau K^6}{\left(D_u - \tau\right)\left(\pi^2 a^2\right)} + \frac{R_s\left(S_r - 1\right)}{\left(D_u - \tau\right)}, \quad (30)$$

(23)

Solution to the first order problem

Equation(27) for T1 takes the form,

$$LT_1 = \left[(X_3R_0 - X_2R_s - R_sS_r\nabla^2 + R_0D_u\nabla^2)\nabla_1{}^2\right](e^{-i\Omega t})(-\pi^2a^2)T_0 \quad (31)$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null space of the operator L. This implies that the time independent part of the RHS of the equation (31) must be orthogonal to $\sin(\pi z)$. Since f varies sinusoidal with time the only steady term on RHS of the equation will vanishes, so that R1 = 0. It follows that all the odd coefficients i.e, R1 = R3 = R5 = ... = 0 in equation (25).

Using equation (27), we find that

$$LT_1 = L[(e^{-i\Omega t})\sin(\pi\alpha x)\sin(\pi Z)],$$

$$T_1 = \frac{-\pi^2 a^2}{L[\Omega]}[P_1 + iP_2](e^{-i\Omega t})T_0 \quad (32)$$

$$LT_1 = Y_1 + iY_2$$

Where,

$$Y_1 = \frac{1}{p_r}\Omega^2\tau K^4 + \frac{1}{p_r}\Omega^2 K^4 + \Omega^2 K^4 - \tau K^8 + \varsigma\Omega K^6 - \varsigma\Omega K^{10} - \pi^2 a^2 K^2 R_s + \tau K^2\pi^2 a^2 R_0 + R_s S_r K^2\pi^2 a^2 - D_u R_0 K^2\pi^2 a^2 + D_u S_r K^8 + \varsigma D_u S_r K^{10},$$

$$Y_2 = \frac{1}{P_r}\Omega\tau K^6 - \frac{1}{P_r}\Omega^3 K^2 + \tau\Omega K^6 + \Omega K^6 + \varsigma\Omega\tau K^8 + \Omega\varsigma K^8 + \Omega\pi^2 a^2 R_s - \Omega\pi^2 a^2 R_0$$

$$T_1 = \frac{-\pi^2 a^2}{[L[\Omega]]^2}[P_3 + iP_4](e^{-i\Omega t})T_0 \quad (33)$$

Where $P_3=Y_1P_1+Y_2P_2$ and $P_4=Y_1P_2-Y_2P_1$,

Equation (28) implies

$$LT_2 = Re[P_1 + iP_2](e^{-i\Omega t})(\pi^2 a^2)T_1 + (\tau K^2 - 1)R_2(-\pi^2 a^2)T_0$$

Where

$$P_1 = \tau K^2 R_0 - K^2 R_s + R_s S_r K^2 - D_u R_0 K^2 \text{ and } P_2 = \Omega(R_s - R_0)$$

On applying orthogonality condition and applying time average we get

$$R_2 = \frac{-2}{\tau K^2 - 1}\int_0^1 Re[P_1 + iP_2](e^{-i\Omega t})T_1 \sin(\pi z)dz$$

On solving we get

$$R_{2c} = \frac{\pi^2 a^2}{2(\tau K - 1^2)}\left[\frac{({P_1}^2 - {P_2}^2)Y_1 + 2P_1P_2Y_2}{{Y_1}^2 + {Y_2}^2}\right] \quad (34)$$

R_{2c} in equation(34) is the correction to the critical Rayliegh number R_{0c}.

Non- Linear Analysis:

As Linear analysis is sufficient only to obtain the condition for onset of convection, to study the rate of Heat and Mass transfer, the non-linear analysis of the problem is required. The non-linear study is done by Fourier representation.

The finite amplitude analysis is carried out here via Fourier series representation of stream function Ψ, temperature distribution θ and solutal distribution φs. The first effect of non- linearity to distort the temperature field through the interaction of ψ and θ. The distortion of the temperature field will correspond to a change in the horizontal mean, i.e., a component of the form sin (2πz) will be generated. Thus the truncated system which describes the finite amplitude free convection is given by

$$\psi = A(t)\sin(\pi\alpha x)\sin(\pi z) \quad (35)$$

$$\theta = B(t)\cos(\pi\alpha x)\sin(\pi z) + C(t)\sin(2\pi z), \quad (36)$$

$$\emptyset_s = E(t)\cos(\pi\alpha x)\sin(\pi z) + F(t)\sin(2\pi z), \quad (37)$$

Substituting (35)-(37) in equation (17)-(19) and equating the coefficients of like terms we obtain the non linear differential equations as follows

$$\dot{A} = \frac{P_r}{K^2}[(-K^4 - \varsigma K^6)A(t) - g_m R_0\pi\alpha B(t) + g_m R_s\pi\alpha E(t)], \quad (38)$$

$$\dot{B} = -A(t)\pi^2\alpha C(t\pi\alpha A(t) - K^2 B(t) - D_u K^2 E(t), \quad (39)$$

$$\dot{C} = \frac{\pi^2\alpha}{2}A(t)B(t) - 4\pi^2 C(t) - 4\pi^2 D_u F(t), \quad (40)$$

$$\dot{E} = -\pi^2\alpha A(t)E(t) - \pi\alpha A(t) - \tau K^2 E(t) - K^2 B(t)S_r, \quad (41)$$

$$\dot{F} = \frac{\pi^2\alpha}{2}A(t)E(t) - 4\tau\pi^2 F(t) - 4\pi^2 F(t) - 4\pi^2 S_r C(t), \quad (42)$$

Where, over dot denotes time derivative.

The generalized Lorenz model (38)-(42) is uniformly bounded in time and possesses many properties of the full problem. This set of non-linear ordinary differential equations possesses an important symmetry for it is invariant under the transformation,

$$(A, B, C, E, F) \to (-A, -B, -C, -E, -F), \quad (43)$$

Also the phase-space volume contracts at uniform rate given by:

$$\frac{\partial\dot{A}}{\partial A} = \frac{\partial\dot{B}}{\partial B} = \frac{\partial\dot{C}}{\partial C} = \frac{\partial\dot{E}}{\partial E} = \frac{\partial\dot{F}}{\partial F} = -[P_r\varsigma K^4 + 4\pi^2 + K^2 + \tau K^2 + 4\tau\pi^2], \quad (44)$$

Which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a fixed point, a limit cycle or perhaps, a strange attractor.

Heat and Mass Transfer at Lower Boundary

Heat and mass transport in a double diffusive system depends on the imposed temperature and concentration differences on the diffusion coefficient. In this chapter we mainly focus on the influence of double diffusion on heat and mass transport which are quantified in terms of Nusselt number (Nu) and Sherwood number (Sh). The heat transport can be quantified by a

Nusselt number Nu and is defined as,

$$Nu = \frac{\text{Heat and transport by(conduction + convection)}}{\text{Heat and transport by conduction}}$$

$$Nu = \frac{\left[\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}(1-z-T)_z dx\right]_{z=0}}{\left[\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}(1-z)_z dx\right]_{z=0}} \quad (45)$$

Where subscript in the integrand denotes the derivative with respect to z.

Substituting equation (40) into the equation (45) and completing differentiation and integration, we get the following expression for Nusselt number:

$$Nu = (1 - 2*\pi*C(t) + D_u*\frac{R_s}{R_0}*\left(1-2*\pi*F(t)\right), \quad (46)$$

The mass transport is quantified by Sherwood number, Sh and is defined as

$$Sh = \frac{\text{Convection mass transfer coefficient t}}{\text{Diffusive mass transfer coefficient t}}$$

$$Sh = \frac{\left[\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}(1-z-C)_z dx\right]_{z=0}}{\left[\frac{k}{2\pi}\int_0^{\frac{2\pi}{k}}(1-z)_z dx\right]_{z=0}} \quad (47)$$

where subscript in the integrand denotes the derivative with respect to z.

Substituting equation (40) into the equation (47) and completing differentiation and integration, we get the following expression for Sherwood number:

$$Sh = (1 - 2*\pi*F(t) + S_r*\tau*\frac{R_0}{R_s}*\left(1-2*\pi*C(t)\right) \quad (48)$$

The amplitudes C (t) and F (t) are determined from the dynamics of the Lorenz system (38)-(42) by solving the system numerically.

RESULTS AND DISCUSSIONS

The external control of convection is important in the thermal instability problems. In this problem the vertical oscillation of the fluid layer (gravity modulation) along with cross diffusion in a double diffusive system is considered to study its effect in thermal instability and also on heat and mass transfer. The expression for critical Rayleigh number and correction Rayleigh number in linear analysis is obtained using regular perturbation method. The expression for Nusselt number and Sherwood number which quantifies heat and mass transfer respectively obtained using non-linear analysis with the help of truncated representation of Fourier series which results in fifth order generalized Lorenz model. The result obtained are depicted in figure(2)-(16).

Figure (2)-(6) are the plot of critical Rayleigh number$R_c(R_c = R_{0c} + R_{2c})$, where R_{0c} is Rayliegh number given in eqn(30) and R_{2c} is the correction Rayliegh number given in eqn(34)) versus the frequency of gravity modulation γ for different values of Dufour parameter Du, Soret coefficient Sr, couple stress parameter *C*, solutal Rayleigh number R_s and ratio of diffusivity τ for linear case. From these figures we observe that for small frequency γ, R_c decreases and for moderate value of γ, R_c increases. Thus the system becomes destabilizes and stabilizes for small and moderate values of γ respectively. Also for very large values of γ the curve remains straight line vertically, which shows that for very large value of frequency the modulation disappears.

From figure(2) we observe that increase in Du increases R, Du represents the diffusion of concentration gradient with respect to thermal gradient. When Du increases, both heat and concentration diffuses each other which increases the temperature difference interms increases the buoyoncy force. Hence onset of convection is delayed. Thus, Du stabilizes the system.

Figure(3) shows the variation of R_c with γ for different values of Sr. From figure we found that positive increase in Sr decreases R_c and negative increase in Sr increases the R_c. Sr represents the ratio of thermal diffusive coefficient to concentration diffusive coefficient, hence reverse effect is observed when Sr increases positively and negatively.

Figure (4) shows the variation of couple stress parameter on R_c for different values of γ , we observed that increase in *C* increases R_c. This is due to increase in couple stress, a fluid becomes more rigid which requires the higher heating for the convection to take place thus the couple stress parameter stabilizes the system.

The variation of R_s with γ is shown in the figure(5). The increase in R_s indicates the increase in concentration of the solute. Since the solutes are added from from below, these settles at the bottom without disturbing the system, hence more heating is required for the convection to takes place. Therefore onset of convection is delayed hence R_s stabilizes the system.

Figure(6) shows variation of R_c with frequency for different values of ratio of diffusivity τ . The increase in τ indicates the solute diffusivity is less than the heat diffusivity which advances the onset of convection, hence increase in τ destabilizes the system.

We now discuss the results obtained in the case of non-linear analysis. The study of non-linear analysis is important as the linear analysis will not give effects of parameter on heat and mass transfer. Fig(7(a)) to Fig(11(a)) are the plots of Nusselt no. Nu versus time t for different values of D_u, C, S_r , R_s and τ respectively. The following observation are made from the figures:

1) Increase in D_u parameter increases concentration gradient and hence decreases heat transfer.
2) The increase in S_r parameter increases the temperature gradient which increases the heat transfer.
3) Increase in *C* decreases the Nusselt number indicating the reduced in heat transfer.
4) The Increase in τ decreases the nussult number and hence reduces heat transfer.
5) The increase in R_s increases nussult number and enhances heat transfer.

Figure (7(b)) to Figure (11(b)) are the plots of Sherwood number (Sh) versus t for different values of D_u, C, S_r , R_s and τ respectively. From these figures we observe that the effect of parameters on mass transfer is opposite to the effects on rate of heat transfer. From these figures we also observe that heat transfer is less compared to rate of mass transfer.

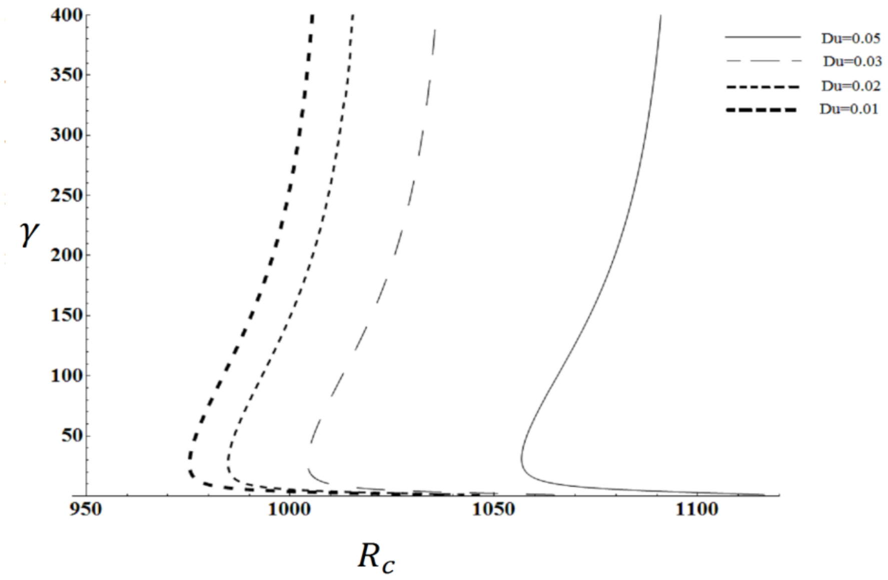


Figure 2: Plot of critical Rayliegh number R_c versus frequency of gravity modulation γ for different values of Dufour parameter Du

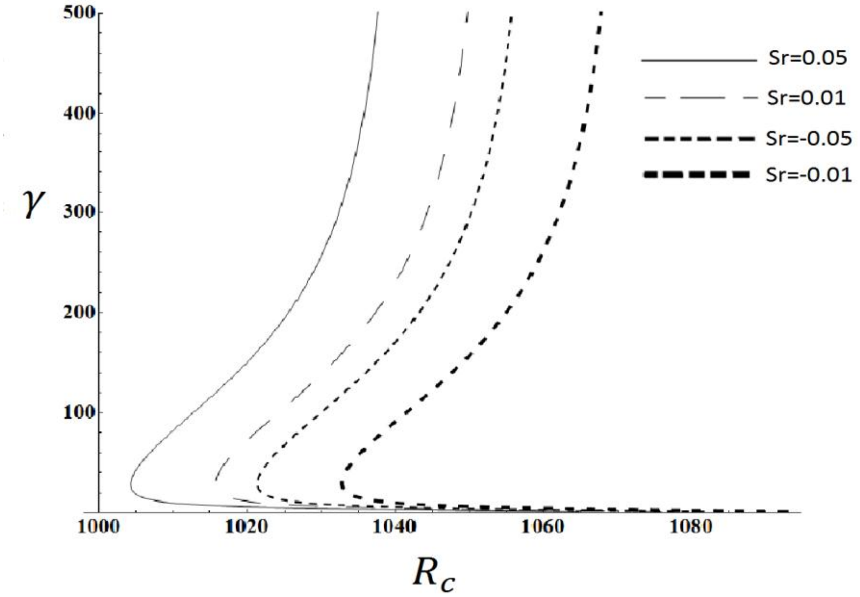


Figure 3: Plot of R_c versus γ for different values of soret parameter S_r

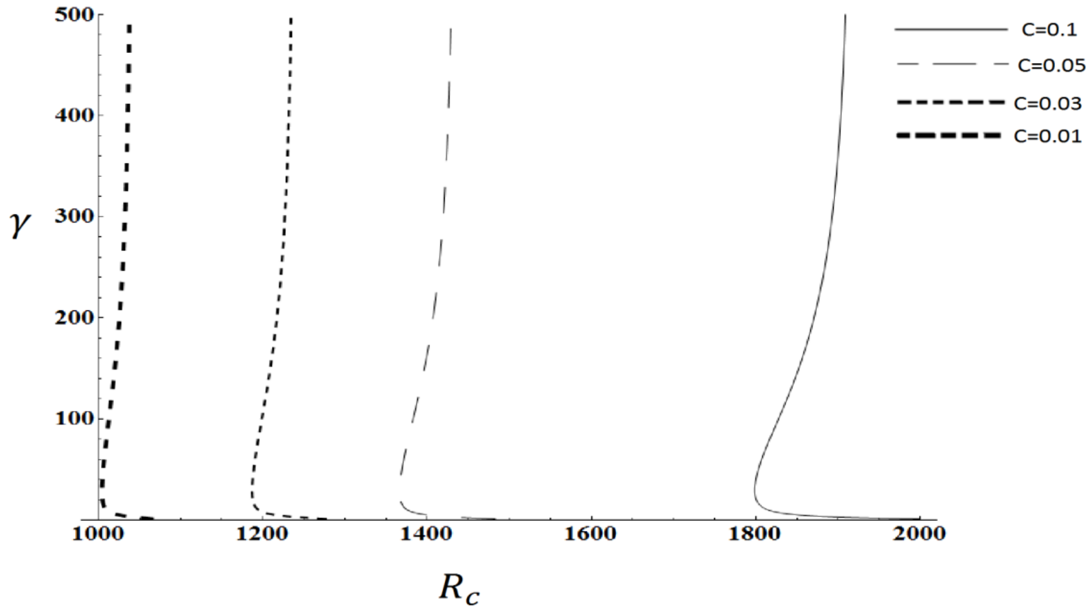


Figure 4: Plot of R_c versus γ for different values of couple stress parameter C

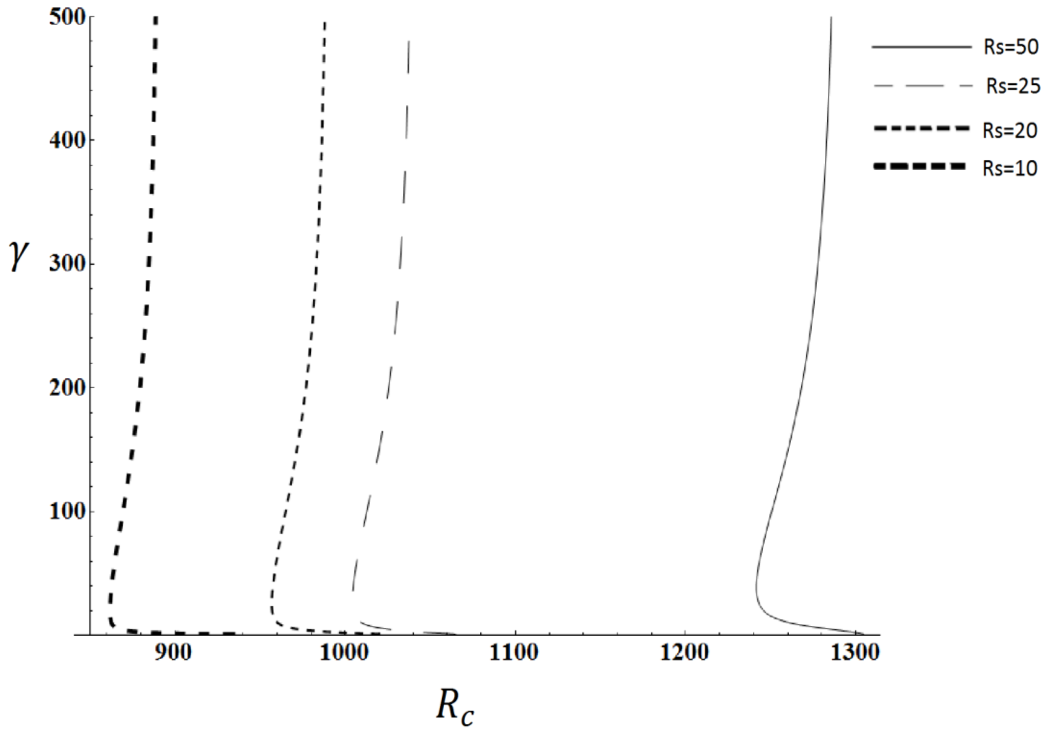


Figure 5: Plot of R_c versus γ for different values of Solutal Rayliegh number R_S

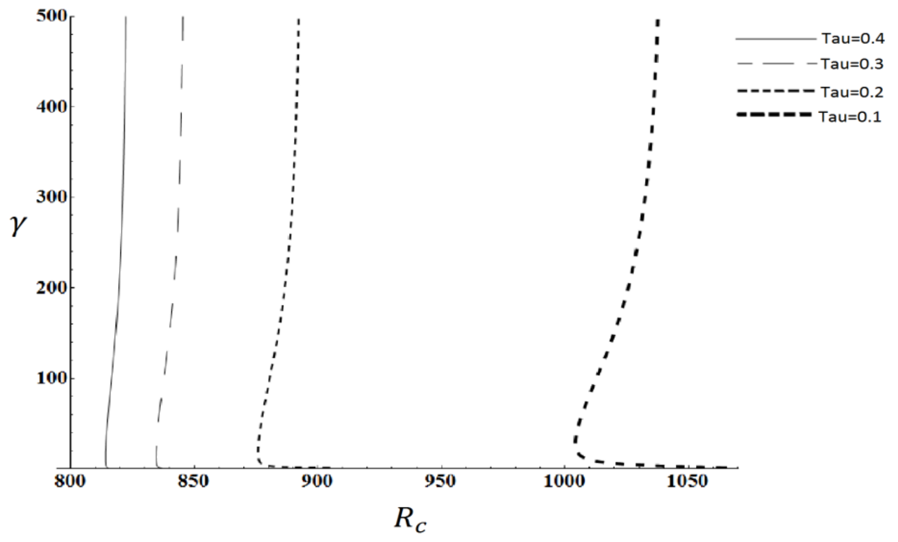
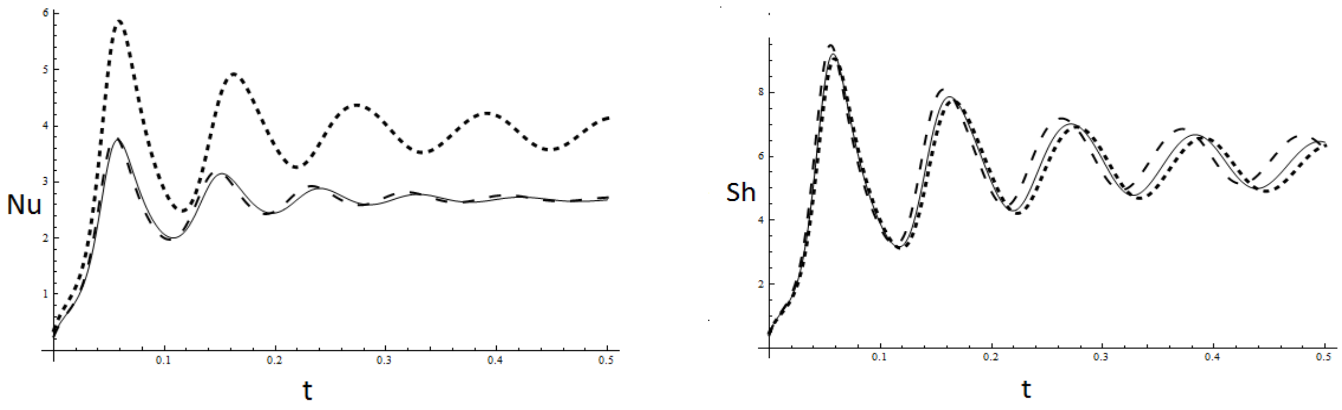
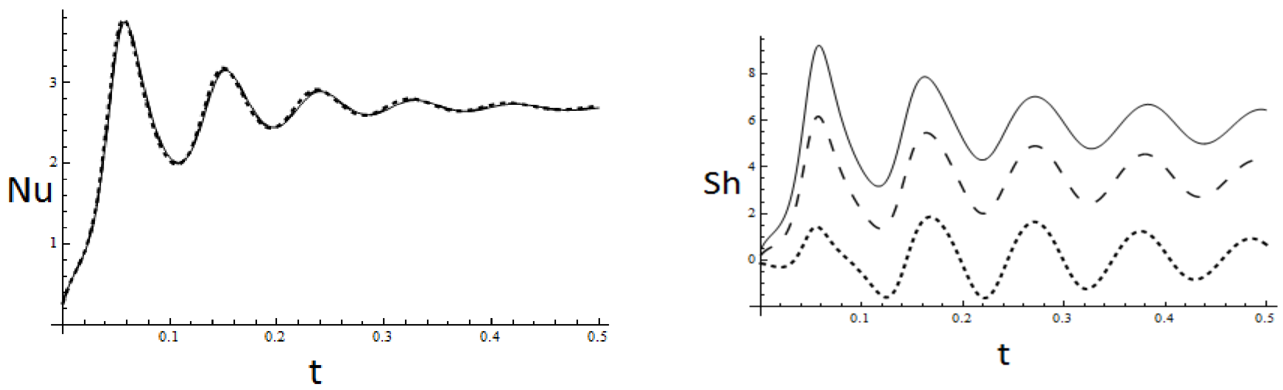


Figure 6: Plot of R_c versus γ for different values of ratio of diffusivity τ



(a) (b)

Figure 7: Plot of (a)Nu, (b)Sh versus time t for different values of Dufour parameter Du [———— Du=0.005, -- -- -- Du=0.01, - - - Du=0.002]



(a) (b)

Figure 8: Plot of (a)Nu, (b)Sh versus time t for different values of Soret parameter Sr [———— Sr=-0.05, -- -- -- Sr=0.01, - - - Sr=0.05]

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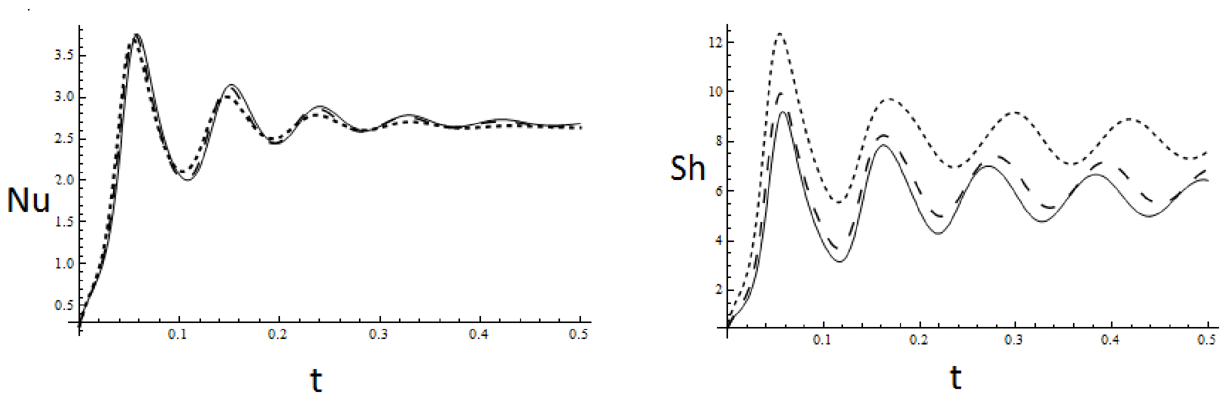
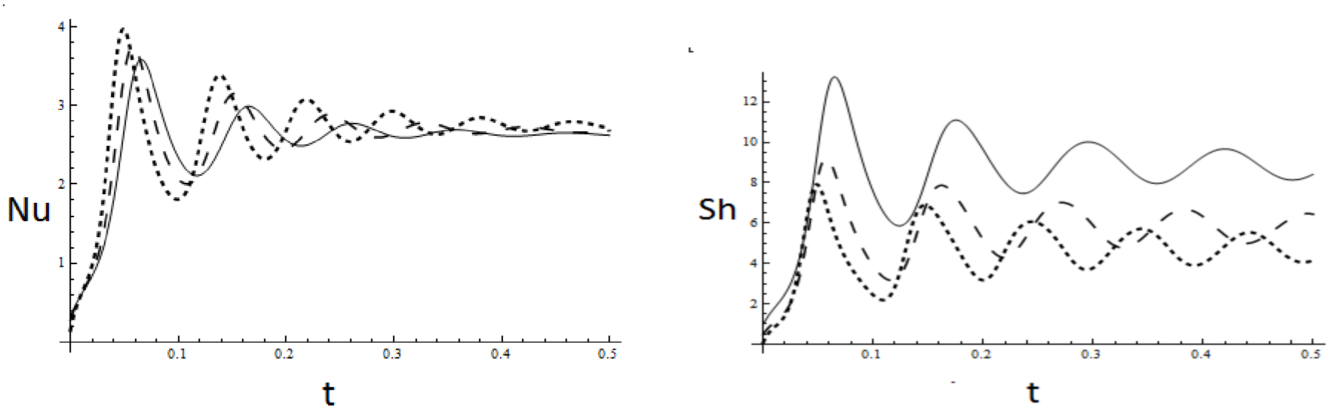


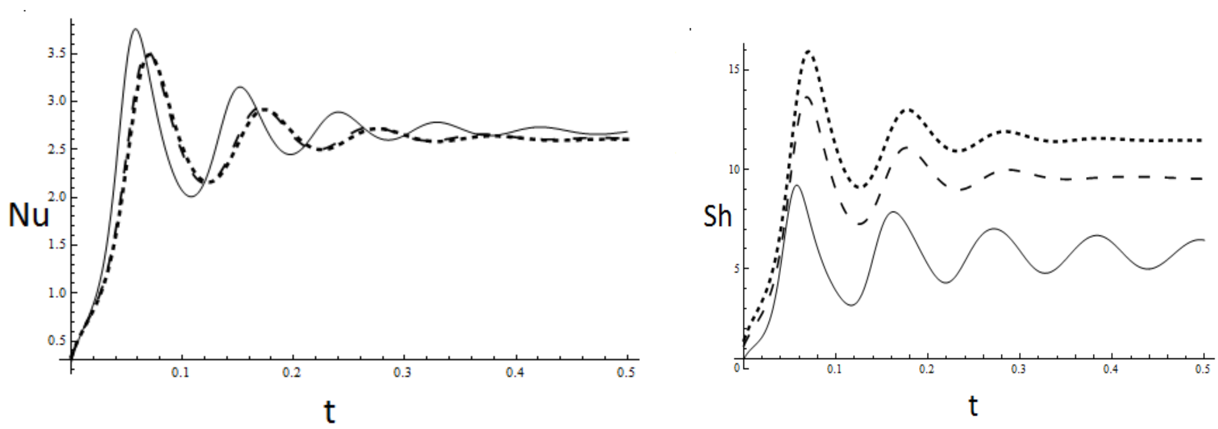
(a) (b)

Figure 9: Plot of (a)Nu, (b)Sh versus time t for different values of Couple stress parameter C[________ C=0.01, -- -- -- C=0.03, - - - C=0.1]



(a) (b)

Figure 10: Plot of (a)Nu, (b)Sh versus time t for different values of Dufour parameter R_s[________ R_s=10, -- -- -- R_s=25, - - - R_s=50]



(a) (b)

Figure 11: Plot of (a)Nu, (b)Sh versus time t for different values of Ratio of diffusivity τ[________ τ=0.1, -- -- -- τ=0.3, - - - τ=0.4]

CONCLUSIONS

From the study, following conclusions are drawn:

1) Small and moderate values of frequency γ destabilizes and stabilizes respectively and for large values modulation disappears.
2) Du stabilizes the system.
3) Positive values of S_r destabilizes whereas negative values of Sr stabilizes.
4) C and R_s stabilizes the system.
5) D_u, C and τ decreases the heat transfer and S_r and R_s increases the heat transfer.
6) Parameters have reverse effect on Mass transfer.
7) Heat transfer is less compared to mass transfer.

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REFERENCES

[1] H.E.Huppert and J.S. Turner, (1981), Journal of Fluid Mechanics, "Double diffusive convection," Vol.106, pp. 299-329.

[2] Malashetty M.S. et al., (2013), Applied Mathematical Modelling, "Soret effect on double diffusive convection in a Darcy porous medium saturated with a couple stress fluid", Vol.37, pp-172-186.

[3] P. G Siddheshwar et al., Boundary Value Problems, "On double-diffusive convection and cross diffusion effects on a horizontal wavy surface in a porous medium", pp. 25-36.

[4] B.S Bhadauria, (2012). , Transport in porous media , "Double-diffusive convection in a saturated anisotropic porous layer with internal heat source", Vol. 92, pp. 299-320,.

[5] Arun Kumar, S Pranesh, (2018), Recent Trends in Fluid Mechanics, "Study of Heat Transfer in a Time Dependent Vertically Oscillating Micropolar Liquid", Vol. 5, pp. 63-74, (2018).

[6] Sameena Tarannum, S Pranesh, (2016), Int J Eng Res, "Effect of gravity modulation on the onset of Rayleigh-Bénard Convection in a weak electrically conducting couple stress fluid with saturated porous layer", Vol. 5, pp.914-928.

[7] E. Knobloch,(1980), *The Physics of Fluid*, "Convection in binary fluids", Vol. 23.

[8] N Rudraiah and M.S. Malashetty, (1986), Journal of Heat Transfer, "The Influence of Coupled Molecular Diffusion on Double-Diffusive Convection in a Porous Medium", Vol. 108, pp. 872-876.

[9] N Rudraiah, Pradeep G Siddheshwar, (1998), Heat and mass transfer, "A weak nonlinear stability analysis of double diffusive Convection with cross-diffusion in a fluid-saturated porous medium", Vol.33, pp. 287-293.

[10] S.N. Gaikwad, (2009), Transport in Porous Media,"Linear and Non-linear Double Diffusive Convection in a Fluid-Saturated Anisotropic Porous Layer with Cross-Diffusion Effects", Vol.80, pp. 537-560.

[11] M.S. Malashetty and Bharathi S Biradar, (2012), Physics of Fluids, "The onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects", Vol. 23.

[12] R. Ravi, et al., (2017), Applied Mathematics and Mechanics,"Effects of second diffusing component and cross diffusion on primary and secondary thermoconvective instabilities in couple stress liquids", Vol.38, pp. 1579-1600, (2017).

[13] Gershuni, G.Z. and Zhukhovitskii, E.M, (1963), Journal of Applied Mathematics and Mechanics, "On the convectional instability of a two-component mixture in a gravity field", Vol. 27, pp-441-452.

[14] B.T. Murray et al., (1991), Journal of crystal growth, "The effect of gravity modulation on solutal convection during directional solidification", Vol.110, pp. 713-723.

[15] R. clever et.al., (1993), Journal of Fluid Mechanics, "Two dimensional oscillatory convectionin agravitationally modulated fluid layer," Vol. 253, pp. 663-680, (1993).

[16] Pradeep G Siddheshwar, S Pranesh, (1999), "Effect of temperature/gravity modulation on the onset of magneto- convection in weak electrically conducting fluids with internal angular momentum", Journal of Magnetism and Magnetic materials, Vol. 192, pp. 159-176.

[17] B.S. Bhadauria et al., (2013), International Journal of Heat and Mass Transfer, "Linear and non linear double-diffusive convection in a saturated anisotropic porous layer with Soret effect and internal heat source", vol. 59, pp-103-111.

[18] P.G. Siddheshwar, S Pranesh, (2004), International journal of non-linear mechanics, "An analytical study of linear and non-linear convection in Boussinesq Stokes suspensions", Vol.39, pp. 165-172.

[19] Sunil et.al.,(2004), Journal of Porous media, "Effect of suspended particles on Couple stress fluid heated and soluted from below in porus medium." vol. 7, No.1, pp. 9-18.

[20] Malashetty M. S., et al., (2010), Fluid Dynamics Research, "Double-diffusive convection in a Darcy porous medium saturated with a couple stress fluid", Vol. 42.

[21] I.S. Shivakumara, (2010), Archive of Applied Mechanics, "Onset of convection in a couple-stress fluid- saturated porous medium: effects of non - uniform temperature gradients", Vol. 80, pp. 949-957.

[22] M.S. Malashetty and Premila Kollur, (2011), Transport in Porous Media , "The Onset of Double Diffusive Convection in A Couple Stress Fluid Saturated Anisotropic Porous Layer", V0l. 86, pp. 435-459.

[23] S. Pranesh and Sangeetha George, (2014), International Journal of Applied Mathematics and Computation, "International Journal of Applied Mathematics and Computation", Vol. 5.

[24] Sameena Tarannum and S Pranesh, (2016), International Journal of Engineering Research and Technology, "Effect of Gravity Modulation on the Onset of Rayleigh-Bénard Convection in a Weak Electrically Conducting Couple Stress Fluid with Saturated Porous Layer", Vol. 5, pp. 914-928.

[25] R. Ravi, et al.,(2017), Applied Mathematics and Mechanics, "Effects of second diffusing component and cross diffusion on primary and secondary thermoconvective instabilities in couple stress liquids", Vol.38, pp. 1579-1600.