

Effect of Entropy Generation in Viscoelastic Fluid over a Stretching Surface with Non-Uniform Heat Source/Sink and Thermal Radiation

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Abstract

The thermodynamic second law analysis is utilized to investigate the inherent irreversibility in a hydromagnetic flow over a stretching sheet with non-uniform heat source/sink and thermal radiation effects. The basic boundary layer equations including continuity, momentum and energy equations have been reduced to a non-dimensionless form via suitable similarity variables and solved analytically using Kummer's function. The effects of physical parameters such as the magnetic parameter, viscoelastic parameter are discussed in common for velocity and temperature profiles, where as parameters like thermal radiation, non-uniform heat source/sink and Prandtl number are discussed for temperature and entropy generation profiles. The wall temperature gradient is calculated and presented through table. The results indicate that the entropy generation increases with the increase in Prandtl number, viscoelastic parameter, magnetic parameter and it decreases with the radiation parameter.

Keywords: Viscoelastic fluid, MHD, Non-uniform heat source/sink, Thermal radiation.

1. INTRODUCTION

Entropy generation is a measure of the account of irreversibility associated to the real processes. The entropy generation is encountered in many of energy related applications, such as solar power collectors, geothermal energy systems and the cooling of modern electronic systems. Through entropy minimization techniques, it is possible to increase the efficiency and overall performance of all kinds of thermal systems. This becomes quite transparent when one notices that a thermal apparatus produce less entropy through irreversibility, it will destroy less available work therefore, it will present an increased efficiency; see the work of Bejan [1] for more thorough discussion on the theme of entropy generation minimization.

Entropy generation minimization method, as a thermodynamic approach, is employed to optimize the thermal engineering devices for higher energy efficiency (Mahian et al. [2]). It is important to emphasize that the second law of thermodynamics is more reliable than the first law of thermodynamic analysis, because of the limitation of the first law efficiency in the heat transfer engineering systems (Oztop and Al-salem [3]). Sahin [4] presented the effect of variable

viscosity on entropy generation rate for heated circular duct. A comparative study of entropy generation rate inside duct of different shapes and the determination of the optimum duct shape subjected to isothermal boundary condition were done by Sahin [5]. Narusawa [6] gave an analytical and numerical analysis of the second law for flow and heat transfer inside a rectangular duct. In a more recent paper, Mahmud and Fraser [7, 8] applied the second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli and Aiboud-Saouli [9]. As far as the effect of a magnetic field on the entropy generation is concerned. Mahmud et al. [10] studied in the case of mixed convection in a channel. The effects of magnetic field and viscous dissipation on entropy generation in a falling film and channel were studied by Aiboud-Saouli et al. [11]. Govindaraju et al. [12] investigated the entropy generation on the natural convection flow of an base nanofluid over a stretching sheet in the presence of magnetic field.

The study of hydromagnetic flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role (Ibrahim et al. [13], Turkyilmazoglu [14], Farooq et al. [15], Sheikholeslami et al. [16], Baag et al. [17], Sheikholeslami et al. [18], Abdul Hakeem et al. [19], Ganga et al. [20]).

The effect of thermal radiation on flow and heat transfer processes is of major importance in the design of many advanced energy conversion systems operating at high temperature. Thermal radiation with such systems occurs because of the emission by the hot walls and working fluid. Many researchers have investigated the thermal radiation effects in their studies (Abdul Hakeem et al. [211], Sheikholeslami et al. [22], Waleed Ahed Khannet al. [23], Mahabaleshwar et al. [24]).

The study of heat source/sink effects on heat transfer is very important in view of several physical problems. Aforementioned studies include only the effect of uniform heat source/sink (i.e., temperature dependent heat source/sink) on heat transfer. Eldahab and Aziz [25] have included the effect of non-uniform heat source with suction/blowing, but

confined to the case of viscous fluids only. Subhas Abel et al. [26] investigated the magneto hydrodynamic boundary layer flow and heat transfer characteristics of a non-Newtonian viscoelastic fluid over a flat sheet with a linear velocity in the presence of thermal radiation and non-uniform heat source. The study of heat transfer analysis for multiphase magnetic fluid over a stretching sheet with heat source/sink and thermal radiation are investigated by Zeeshan et al. [27].

The main originality of the present work is to analyze the entropy generation of magneto hydrodynamic flow of an incompressible viscoelastic fluid over a stretching sheet with thermal radiation and non-uniform heat source/sink parameter. The entropy generation is calculated using the entropy relation by substituting the velocity and temperature fields obtained from the momentum and energy equations.

2. FORMULATION OF THE PROBLEM

Consider the steady two-dimensional boundary layer flow of an electrically conducting, viscoelastic fluid past a stretching sheet co-incident with the plane $y = 0$; the flow being confined to $y > 0$. Two equal and opposite forces are applied along x -axis so that the surface is stretched, keeping the origin fixed. A uniform magnetic field of strength B_0 is imposed along y -axis, which produces magnetic effect in the x -direction. Under the usual boundary layer assumptions, the conservation equations of momentum and energy for the flow of viscoelastic fluid, in the usual notation, can be written as

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

$$(\rho C_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + q''' \quad (3)$$

Where $k_0 = \frac{-\alpha_1}{\rho}$ is the viscoelastic parameter, q_r is the radiative heat flux and q''' is the rate of non-uniform heat generation/absorption coefficient. The dimensionless form of q''' can be defined as

$$q''' = \left(\frac{k_{up}(x)}{xv} \right) [A^*(T_p - T_\infty)f'(\eta) + B^*(T - T_\infty)] \quad (4)$$

Where A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption. The case $A^* > 0$ and $B^* > 0$ corresponds to internal heat generation while $A^* < 0$ and $B^* < 0$ corresponds to internal absorption, T_p is the temperature of sheet and T_∞ is the constant temperature far away from the sheet.

Using Rosseland approximation for radiation, we have

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

Here, σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Further we assume that the

temperature difference within the flow is such that T^4 may be expanded in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting higher order terms we get,

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

And

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

Therefore Eq.(3) is simplified to

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} + q''' \quad (8)$$

The boundary conditions for the velocity field are of the form

$$u = u_p(x) = \lambda x \quad v = 0, \quad \text{at } y = 0$$

$$u = 0, \quad \frac{\partial u}{\partial y} = 0, \quad \text{as } y \rightarrow \infty \quad (9)$$

The equation of continuity is satisfied if we choose a dimensionless stream $\psi(x, y)$ such that.

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (10)$$

2.1 Flow Analysis

We introduce the similarity transformations

$$\eta = y \sqrt{\frac{\lambda}{\nu}}, \quad \psi(x, y) = x \sqrt{\lambda \nu} f(\eta) \quad (11)$$

Where $\psi(x, y)$ is the stream function. Momentum equation (2) becomes

$$f_\eta^2 - \eta f_{\eta\eta} = f_{\eta\eta\eta} - k_1(2f_\eta f_{\eta\eta} - \eta f_{\eta\eta\eta} - f_\eta^2) - Mn f_\eta \quad (12)$$

Where $Mn = \frac{\sigma B_0^2}{\rho \lambda}$ is the magnetic parameter and $k_1 = \frac{k_0 \lambda}{\nu}$ is the viscoelastic parameter.

The boundary conditions of Eq. (11) are given by

$$f(0) = 0, \quad f_\eta(0) = 1 \quad \text{at } \eta = 0$$

$$f_\eta(\infty) = 0, \quad f_{\eta\eta}(\infty) = 0, \quad \text{as } \eta \rightarrow \infty \quad (13)$$

Making use of the boundary conditions (13), We can obtain an analytical solution of Eq. (12)

$$f(\eta) = \frac{1 - e^{-\alpha\eta}}{\alpha} \quad \text{and} \quad f_\eta(\eta) = e^{-\alpha\eta} \quad (14)$$

$$\text{Where } \alpha = \sqrt{\frac{1 + Mn}{1 - k_1}} \quad (15)$$

2.2 Heat Transfer Analysis

The relevant boundary conditions are

$$y = 0, \quad T = T_p = A\left(\frac{x}{l}\right)^2 + T_\infty \quad y \rightarrow \infty, \quad T = T_\infty \quad (16)$$

Defining the dimensionless temperature

$$\theta = \frac{T - T_\infty}{T_p - T_\infty} \quad (17)$$

And using (14) and (17) in (8) and the boundary conditions (16) can be written as

$$\theta_{\eta\eta} + \left(\frac{3Nr}{3Nr+4}\right) \text{Pr} \left(\frac{1-e^{-\alpha\eta}}{\alpha}\right) \theta_\eta - (2\text{Pr}e^{-\alpha\eta} - B^*) \left(\frac{3Nr}{3Nr+4}\right) \text{Pr} \theta = -A^* \left(\frac{3Nr}{3Nr+4}\right) \text{Pr} e^{-\alpha\eta} \quad (18)$$

Where $\text{Pr} = \frac{\mu C_p}{k}$ is the prandtl number

$$\theta(0) = 1 \quad \text{at } \eta = 0 \quad \theta(\infty) = 0 \quad \text{as } \eta \rightarrow \infty \quad (19)$$

Introducing the new variable,

$$\xi = \frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right) e^{-\alpha\eta} \quad (20)$$

and inserting (20) in (18) we obtain

$$\xi \theta_{\xi\xi} + (1 - a_0 + \xi) \theta_\xi - \left(2 - \frac{B^*}{\alpha^2 \xi} \left(\frac{3Nr}{3Nr+4}\right)\right) \theta = -\frac{A^*}{\text{Pr}} \quad (21)$$

and (19) transform to

$$\xi = -\frac{\text{Pr}}{\alpha^2}, \quad \theta(\xi) = 1 \quad \xi \rightarrow 0, \quad \theta(0) = 0 \quad (22)$$

The solution of Eq. (21) in terms of η is written as

$$\theta(\eta) = C_1 e^{-\alpha \frac{a_0+b_0}{2} \eta} M\left[\frac{a_0+b_0}{2} - 2, b_0 + 1, -\frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right) e^{-\alpha\eta}\right] + C_2 e^{-2\alpha\eta} \quad (23)$$

Where

$$a_0 = \frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right), \quad b_0 = \sqrt{a_0^2 - \frac{4B^*}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right)}, \quad C_2 = \frac{A^* \left(\frac{3Nr}{3Nr+4}\right) \text{Pr}}{\alpha^4 \left(4 - 2a_0 + \frac{B^*}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right)\right) \left(1 - a_0 + \frac{B^*}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right)\right)},$$

$$C_1 = \frac{1 - C_2}{M\left[\frac{a_0+b_0}{2} - 1, b_0 + 1, -\frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right) e^{-\alpha\eta}\right]}, \quad M\left[\frac{a_0+b_0}{2} - 2, b_0 + 1, -\frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr+4}\right) e^{-\alpha\eta}\right] \text{ is the Kummer's function.}$$

The non-dimensional wall temperature gradient derived from Eq. (23) is read as

$$\theta'(0) = C_1 \left[-\alpha \left(\frac{a_0 + b_0}{2} \right) M \left[\frac{a_0 + b_0}{2} - 2, b_0 + 1, -\frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr + 4} \right) \right] \right] + C_1 \left[\frac{a_0 + b_0 - 4}{2(1 + b_0)} \frac{\text{Pr}}{\alpha} \left(\frac{3Nr}{3Nr + 4} \right) \right] M \left[\frac{a_0 + b_0}{2} - 2, b_0 + 1, -\frac{\text{Pr}}{\alpha^2} \left(\frac{3Nr}{3Nr + 4} \right) \right] - 2\alpha C_2 \quad (24)$$

3. ENTROPY GENERATION ANALYSIS:

According to Woods [10], the local volumetric rate of entropy generation in the presence of magnetic field can be expressed as

$$S_G = \frac{k}{T_\infty^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(1 + \frac{16\sigma^* T_\infty^3}{3k^* k} \right) \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_\infty} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_\infty} u^2 \quad (25)$$

The right hand side of the above equation consists of three parts. The first part represents the entropy generation due to heat transfer across a finite temperature difference, the second part represents the local entropy generation due to viscous dissipation and the third part represents the local entropy generation due to the effect of the magnetic field. The entropy generation number, dimensionless form of entropy generation rate N_s is defined as the ratio of the local volumetric entropy generation rate (S_G) to a characteristic entropy generation rate ($S_G)_0$. For a prescribed boundary condition the characteristic entropy generation rate is

$$(S_G)_0 = \frac{k(\Delta T)^2}{l^2 T_\infty^2} \quad (26)$$

therefore, the entropy generation number is

$$N_s = \frac{S_G}{(S_G)_0} \quad (27)$$

Using Eqs. (23), (25), (26) and (27), the entropy generation number is given by

$$N_s = \frac{4}{X^2} \theta^2 + \theta'^2 (\eta) \left(\frac{3Nr+4}{3Nr} \right) \text{Re}_1 + \frac{\text{Br}}{\Omega} f'^2 (\eta) \text{Re}_1 + \frac{\text{BrHa}^2}{\Omega} f'^2 (\eta) \quad (28)$$

where Br is the Brinkman number. Re_1 is the Reynolds number, Ω and Ha are respectively the dimensionless temperature difference and the Hartmann number. These numbers are given by the following relationships

$$\text{Br} = \frac{\mu u_p^2}{k\Delta T}, \quad \text{Re}_1 = \frac{u_l}{\nu}, \quad \Omega = \frac{\Delta T}{T_\infty}, \quad \text{Ha} = B_0 x \sqrt{\frac{\sigma}{\mu}} \quad (29)$$

4. RESULTS AND DISCUSSION:

An analysis has been carried out to study the heat transfer of a viscoelastic non-Newtonian fluid from a stretching sheet in presence of a non-uniform heat source/sink. Fig. 1. provides analysis of the variation of longitudinal and transverse velocities via Mn . It is observed that, as the magnetic parameter increases, longitudinal velocity $f'(\eta)$ as well as transverse velocity $f(\eta)$ both decreases. When magnetic parameter increases, it creates a resistible force similar to the

drag force which acts in the opposite direction of the fluid motion and slows down the fluid motion. Thus the magnetic field tends to decelerate the flow.

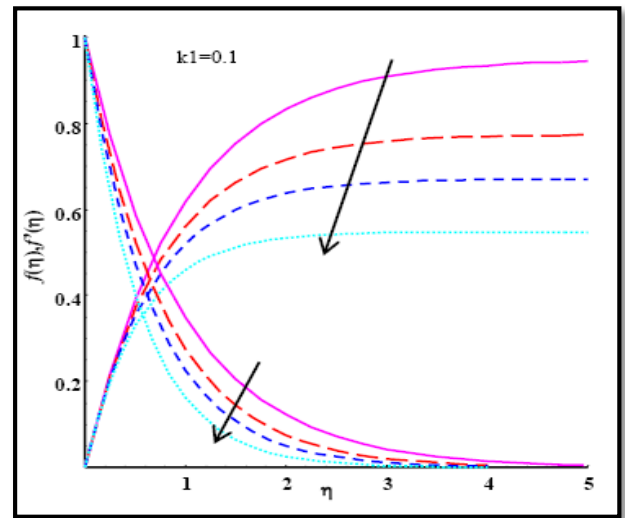


Fig.1. $f(\eta)$ and $f'(\eta)$ via variation of Mn .

Impact of k_1 on longitudinal velocity $f'(\eta)$ and transverse velocity $f(\eta)$ are considered in Fig. 2. It is clear that the steady decrease in the longitudinal velocity accompanies a rise in the viscoelastic parameter with all profiles tending asymptotically to the horizontal axis. Obviously, transverse velocity is decreased as viscoelastic parameter rises.

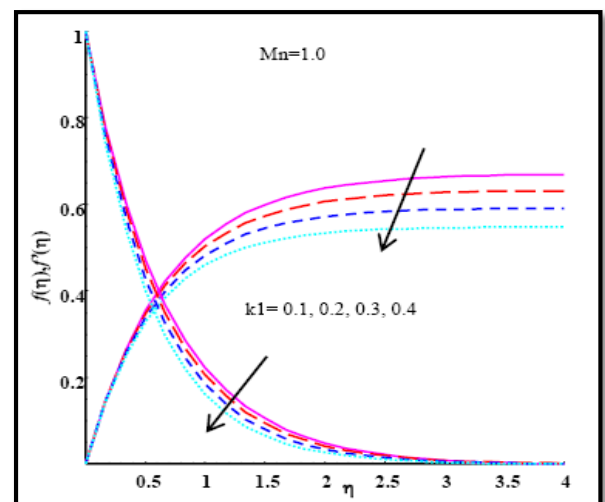


Fig.2. $f(\eta)$ and $f'(\eta)$ via variation of k_1 .

Fig. 3 and 4 demonstrates the effect of the Mn and k_l on $\theta(\eta)$. The presence of these parameters is increasing the temperature of the flow. The increasing value of magnetic parameter causes the fluid to become warmer and therefore increases its temperature (due to decreases in the heat transfer rate). Fig. 5 illustrate the effects of Nr on $\theta(\eta)$. It can be observed that the fluid temperature decreases with an increase in radiation parameter. This result qualitatively agrees with expectations, since the effect of radiation parameter decrease the rate of energy transport to the fluid thereby decreasing the temperature of the fluid.

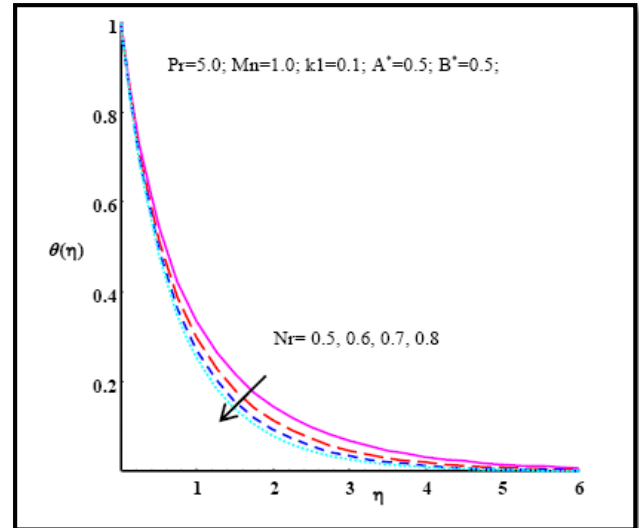


Fig. 5. $\theta(\eta)$ variation via Nr .

The effect of Pr on $\theta(\eta)$ is analyzed from Fig. 6 it is reveals that the thermal boundary layer thicknesses get decreased with increasing value of Prandtl number. In other word, the flow with large Prandtl number prevents spreading of heat in the fluid. The effect of space-dependent parameter A^* and temperature-dependent parameter B^* for heat source/sink on $\theta(\eta)$ are presented in Fig. 7 and 8 respectively. It is found that with increasing values of $A^* > 0$ and $B^* > 0$, the thermal boundary layer generates energy which causes increase in the temperature, while for the other case $A^* < 0$ and $B^* < 0$, it absorbs energy causing a fall in the temperature.

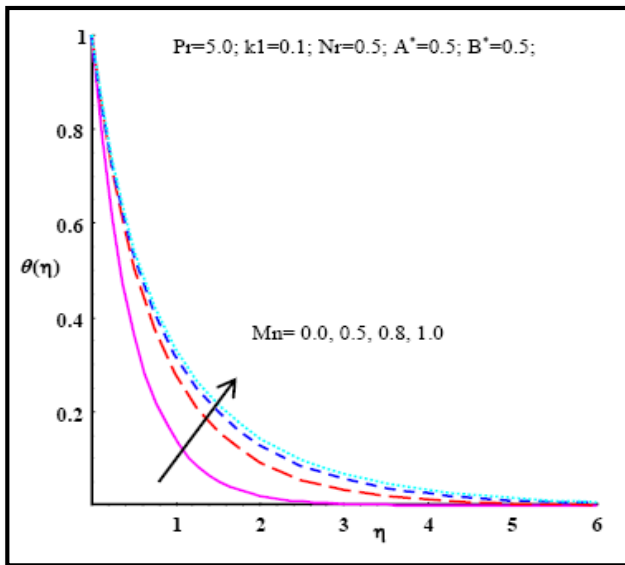


Fig. 3. $\theta(\eta)$ variation via Mn .

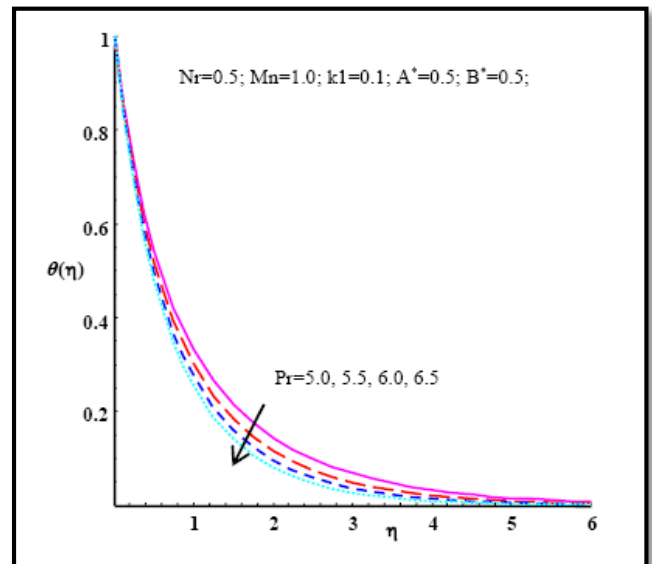


Fig. 6. $\theta(\eta)$ variation via Pr .

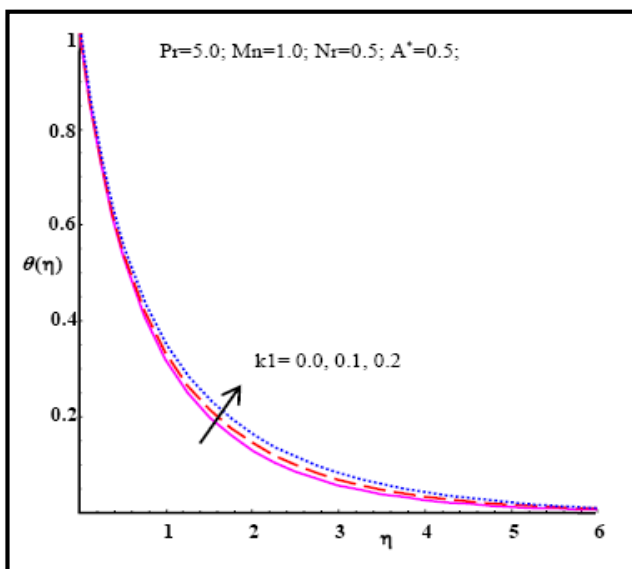


Fig. 4. $\theta(\eta)$ variation via k_l .

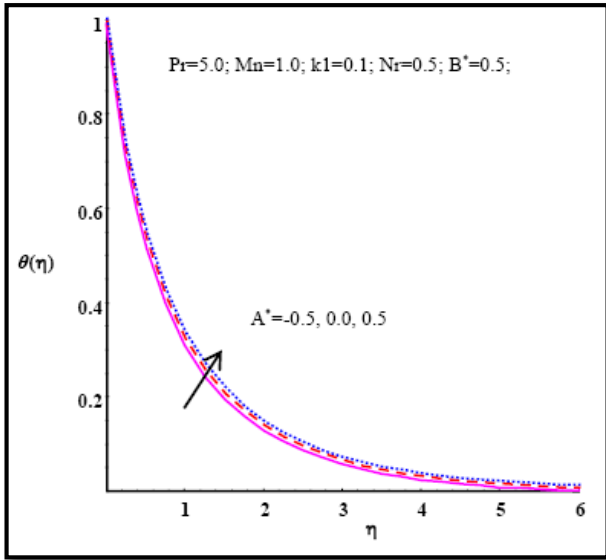


Fig. 7. $\theta(\eta)$ variation via non-uniform heat source/sink (A^*).

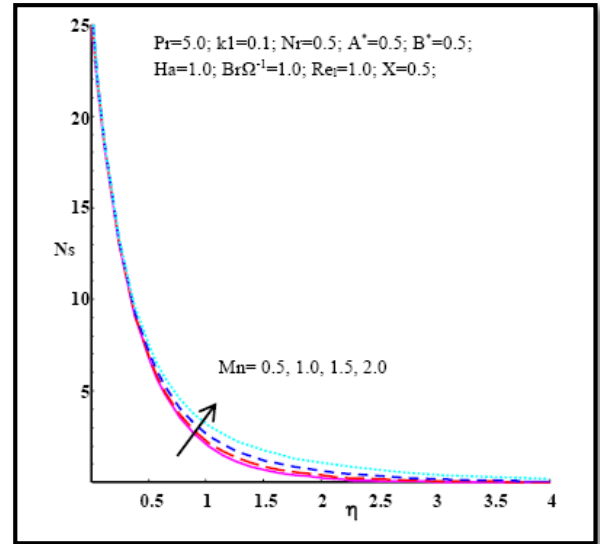


Fig. 9. N_s variation via Mn .

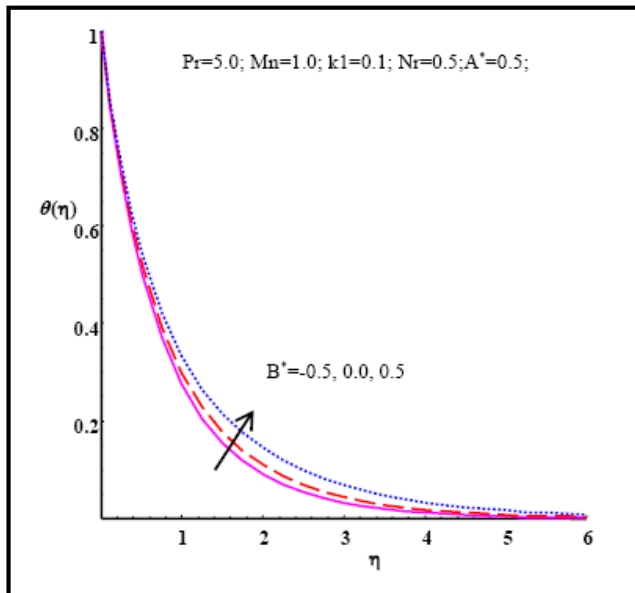


Fig. 8. $\theta(\eta)$ variation via non-uniform heat source/sink (B^*).

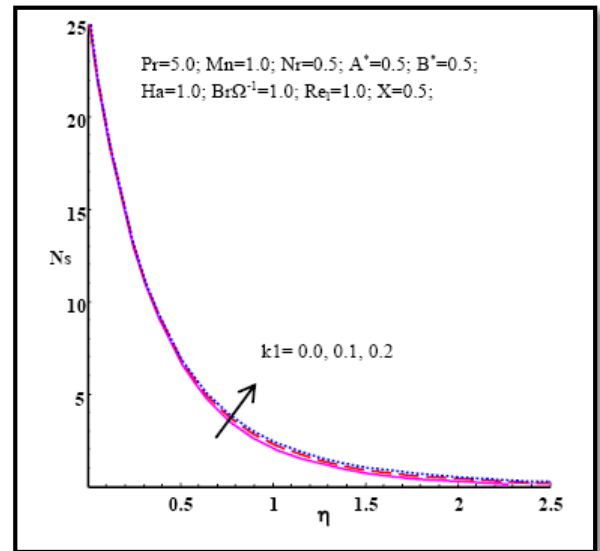


Fig. 10. N_s variation via k_1 .

Figs. 9 and 10 shows the effect of Mn and k_1 on the N_s . It is clear that both the parameters have a significant effect on entropy generation due to the decrease in the flow velocity and increase in the local fluid temperature. The generation of entropy increases when ever Mn and k_1 increases. It is clear that the presence of magnetic field and viscoelasticity creates the entropy in the field.

Fig. 11 displays the effect of Nr on N_s . This figure reveals that entropy increases considerably with the decreasing values of radiation parameter. The increase in the entropy generation is due to decrease of the absorption rate of radiation. The effect of Pr on N_s is shown in Fig.12. The generation of entropy is increases whenever the Prandtl number increases. The entropy generation increases due to three factors: (i) The increase in local velocity gradient. (ii) The increase in local temperature gradient. (iii) The decrease in temperature. As Prandtl number increases, the velocity gradient increases. This will cause an increase in the entropy generation. However, the increase in Prandtl number causes a decrease in the temperature gradient. In turn causes an increase in the entropy generation.

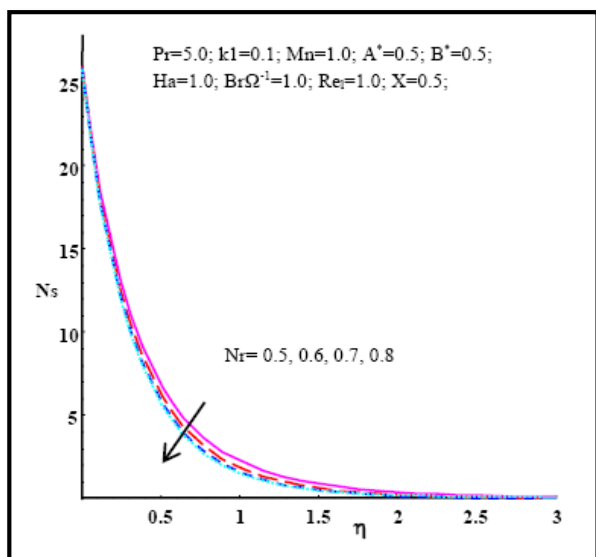


Fig. 11. N_s variation via N_r .

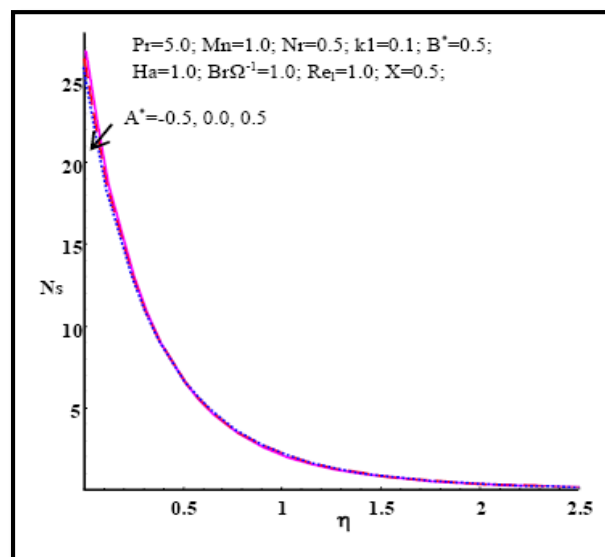


Fig. 13. N_s variation via non-uniform heat source/sink (A^*).

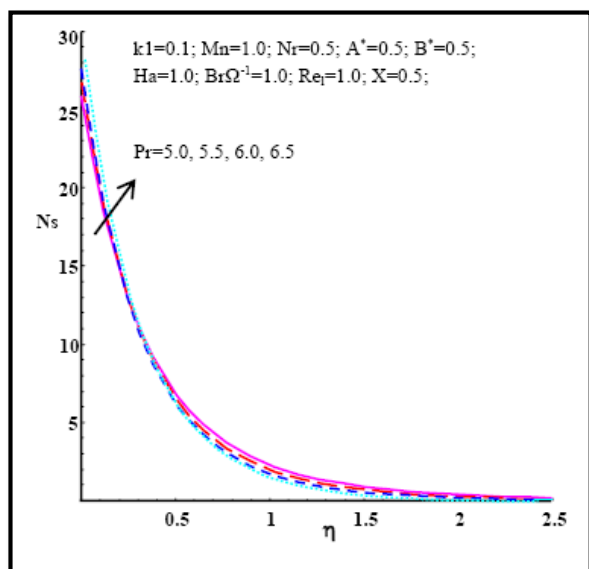


Fig. 12. N_s variation via Pr .

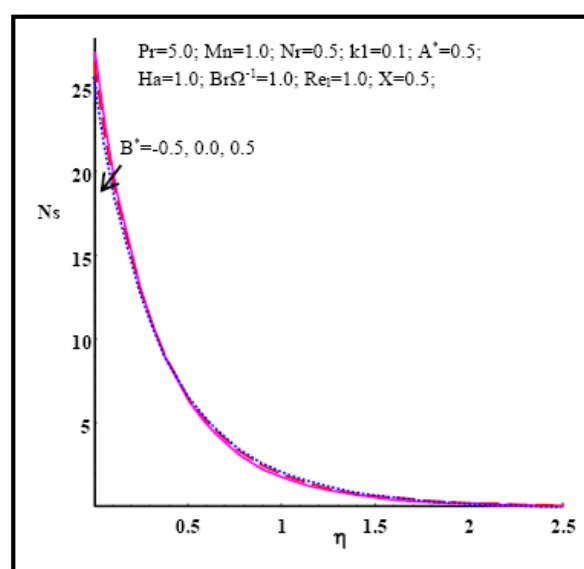


Fig. 14. N_s variation via non-uniform heat source/sink (B^*).

Figs. 13 and 14 displays the effect of non-uniform heat source/sink parameter on the entropy generation parameter. It can be seen that the entropy generation increases in the case of heat sink (i.e. $A^* < 0$ and $B^* < 0$) and decreases in the case of heat source (i.e. $A^* > 0$ and $B^* > 0$). It is also found that with increasing values of $A^* < 0$ and $B^* < 0$, the entropy generation creates more energy in that field, while for the other case $A^* > 0$ and $B^* > 0$, it absorbs energy causing a fall in the entropy generation.

The effect of wall temperature gradient for different values of Pr , k_1 , Mn , Nr , A^* and B^* are shown in Table:1 The increasing values of Pr and Nr increases the wall temperature gradient. The wall temperature gradient decreases with the increasing values of k_1 , A^* , B^* and Mn .

Table 1. Comparison of $\Theta'(0)$

Pr	K1	Mn	A^*	B^*	Nr	$\Theta'(0)$
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.5	0.1	1.0	0.5	0.5	0.5	1.43271
6.0	0.1	1.0	0.5	0.5	0.5	1.51099
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.0	0.2	1.0	0.5	0.5	0.5	1.32257

Pr	K1	Mn	A*	B*	Nr	$\Theta'(0)$
5.0	0.3	1.0	0.5	0.5	0.5	1.28944
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.0	0.1	1.5	0.5	0.5	0.5	1.29725
5.0	0.1	2.0	0.5	0.5	0.5	1.23429
5.0	0.1	1.0	-0.5	0.5	0.5	1.43813
5.0	0.1	1.0	0.0	0.5	0.5	1.39093
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.0	0.1	1.0	0.5	-0.5	0.5	1.46653
5.0	0.1	1.0	0.5	0.0	0.5	1.41215
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.0	0.1	1.0	0.5	0.5	0.5	1.34373
5.0	0.1	1.0	0.5	0.5	0.6	1.44659
5.0	0.1	1.0	0.5	0.5	0.7	1.51986

5. CONCLUSION

An analysis has been carried out to study the effects of entropy generation of MHD viscoelastic fluid flow over a stretching sheet with non-uniform heat source/sink and thermal radiation. Key points are listed below:

- 1) The velocities depend strongly on the magnetic and the viscoelastic parameters.
- 2) The temperature profiles decreases with increasing Pr , Nr and it increases with Mn , k_1 , A^* and B^* .
- 3) The entropy generation number increases with Pr , Mn , k_1 and it decreases with Nr , A^* and B^* .

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