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Generalizations of $(\in, \in \vee q)$-Anti Intuitionistic Fuzzy Soft Subalgebras of BG-algebras

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Abstract: In this paper, we try to establish the notions of $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebras and $(\in, \in)$-anti intuitionistic fuzzy soft subalgebras of BG-algebra are introduced, and its sevral properties. The homomorphic image and inverse image are investigated in $(\in, \in \vee q)$-anti intuitionistic fuzzy soft ideals of BG-agebras. Characterizations of $(\in, \in \vee q_k)$-anti intuitionistic fuzzy soft subalgebra of BG-algebra are dicussed.

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INTRODUCTION

In 1965, Zadeh [1] introduced the concept of a fuzzy subset of a set as a method of representing uncertainty in real physical world. In 1986, Atanassov [2] introduced the notion of intuitionistic fuzzy set which generalizes of the notion of fuzzy sets. Fuzzy set give a degree of members of an element in a given set, while intuitionistic fuzzy sets give both a degree of members and non-members. In 1966, Imai and Iseki [3] introduced the two classes of abstract algebras, viz., BCK/BCI-algebras. It is known that the class of BCK-algebra is proper subclass of the class of BCI-algebras. In 2002, Neggers and Kim [4] introduced a new concept called B-algebras, which are related to several classes of algebras such as BCK/BCI-algebras. In 2008, Kim and Kim [5] introduced the concept of BG-algebra which is generalization of B-algebra. In 2005, Zarandi and Saied [6] developed intuitionistic fuzzy ideal of BG-algebras. In 2012, Senapati, Bhowmik and Pal [7] studied intuitionistic fuzzy ideals of BG-algebras. In 2004, Ahn and Lee [8] introduced the notion of fuzzy subalgebras of BG-algebras. In 1996, Bhakat and Das [9] used the relation of "belongs to" and "quasi coincident with" between fuzzy point and fuzzy set to introduce the concept of $(\in, \in \vee q)$-fuzzy subgroup and $(\in, \in \vee q)$-fuzzy subring. In 2009, Jun [10] introduced the notion of generalization of $(\in, \in \vee q)$-fuzzy subalgebras in BCK/BCI-algebras. In 2011, Basnet and Singh [11] introduced the concept of $(\in, \in \vee q)$-fuzzy ideals of BG-algebras. In 2014, Barbhuiya and Choudhury [12] introduced the notion of $(\in, \in \vee q)$-fuzzy ideals of d-algebra. In 2015, Barbhuiya [13] introduced the concept of $(\in, \in \vee q)$-intuitionistic fuzzy ideals of BG-algebras. In 2014, Barbhuiya [14] introduced the concept of doubt fuzzy ideals of BG-algebras. In 1999, Molodstov [15] introduced the notion of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled that usual theoretical approach. In 2001, Maji, Biswas and Roy [16] introduced the concept of fuzzy soft sets, as a generalization of the standard soft sets.

In 2001, Maji, Biswas and Roy [17] introduced the notion of intuitionistic fuzzy soft sets, as a generalization of the standard fuzzy soft sets. In this paper, we try to establish the the notion of ($\in, \in \vee q$)-anti intuitionistic fuzzy soft subalgebras of BG-algebra and investigate some of their usual properties. We also characterize the ($\in, \in \vee q_k$)-anti intuitionistic fuzzy soft subalgebra of BG-algebra which is a generalization of ($\in, \in \vee q$)-anti intuitionistic fuzzy soft subalgebra.

PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included.

Definition 1. A BG-algebra is a non-empty set X with a constant 0 and a binary operation * satisfying the following axioms:

$(BG-1)\ x * x = 0$,
$(BG-2)\ x * 0 = x$,
$(BG-3)\ (x * y) * (0 * y) = x$ for all $x, y \in X$.

For Short, we also call X a BG-algebra.

Definition 2. A non-empty subset S of a BG-algebra X is called subalgebra of X if it is closed under the binary operation * on S.

Definition 3. A non-empty subset A of a BG-algebra X is called a BG-subalgebra of X if for any $x, y \in X$ satisfies

$(i)\ 0 \in A$,
$(ii)\ x \in A, y \in A \Rightarrow x * y \in A$.

Definition 4. A pair (M, A) is called a soft set over X if and only if M is a mapping of A into the set of all subsets of the set X.

Definition 5. A fuzzy set A of BG-algebra X is an object of the form $A = (x, \sigma_A(x)) : x \in X$, where $\sigma_A(x) : X \to [0, 1]$ with the condition $0 \leq \sigma_A(x) \leq 1$ for all $x \in X$. The number $\sigma_A(x)$ denote the degree of members of the element x in the set A.

Definition 6. Let X is a primary universe set and let E act as a set of factors. Let $\mathscr{F}(X)$ intimate the set of all fuzzy sets. Then $(\tilde{M}, A)$ is called a fuzzy soft set over X and $A \subseteq E$, where $\tilde{M} : A \to \mathscr{F}(x)$.

Definition 7. Let $(\tilde{M}, A)$ be fuzzy soft set over X and $\delta \in A \subseteq E$. We say that $(\tilde{M}, A)$ is called an anti fuzzy soft subalgebra over BG-algebra X if the fuzzy set $\tilde{M}[\delta] := \{(x, \sigma_{\tilde{M}[\delta]}(x)) : x \in X \textit{ and } \delta \in A\}$ satisfies the following conditions for all $x, y \in X$ *and* $\delta \in A$

(AFSS1) $\sigma_{\tilde{M}[\delta]}(0) \leq \sigma_{\tilde{M}[\delta]}(x)$,
(AFSS2) $\sigma_{\tilde{M}[\delta]}(x * y) \leq max\{\sigma_{\tilde{M}[\delta]}(x), \sigma_{\tilde{M}[\delta]}(y)\}$ or $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$.

Definition 8. An intuitionistic fuzzy set A of a BG-algebra X is an object of the form $A = \{(x, \sigma_A(x), \tau_A(x)) : x \in X\}$, where $\sigma_A(x) : X \to [0, 1]$ and $\tau_A(x) : X \to [0, 1]$ with the condition $0 \leq \sigma_A(x) + \tau_A(x) \leq 1$ for all $x \in X$. The numbers $\sigma_A(x)$ and $\tau_A(x)$ denote the degree of members and non-members of the element x in the set A. For the sake of simplicity, we shall use the symbol $A = (\sigma_A, \tau_A)$ for the intuitionistic fuzzy set $A = \{(x, \sigma_A(x), \tau_A(x)) : x \in X\}$.

Definition 9. Let X is a primary universe set and let E act as a set of factors. Let $\mathscr{IF}(X)$ intimate the set of all intuitionistic fuzzy sets in X. Then $(\tilde{M}, A)$ is called an intuitionistic fuzzy soft set over X and $A \subseteq E$, where $\tilde{M} : A \to \mathscr{IF}(X)$.

Definition 10. Let $(\tilde{M}, A)$ be an intuitionistic fuzzy soft set over X and $\delta \in A \subseteq E$. We say that $(\tilde{M}, A)$ is called an anti intuitionistic fuzzy soft subalgebra over BG-algebra X if intuitionistic fuzzy set $\tilde{M}[\delta] := \{(x, \sigma_{\tilde{M}[\delta]}(x)), \tau_{\tilde{M}[\delta]}(x)) : x \in X \textit{ and } \delta \in A\}$ satisfies the following conditions for all $x, y \in X$ *and* $\delta \in A$

(AIFSS1) $\sigma_{\tilde{M}[\delta]}(0) \leq \sigma_{\tilde{M}[\delta]}(x)$ and $\tau_{\tilde{M}[\delta]}(0) \geq \tau_{\tilde{M}[\delta]}(x)$,
(AIFSS2) $\sigma_{\tilde{M}[\delta]}(x * y) \leq max\{\sigma_{\tilde{M}[\delta]}(x), \sigma_{\tilde{M}[\delta]}(y)\}$ or $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$,
(AIFSS3) $\tau_{\tilde{M}[\delta]}(x * y) \geq min\{\tau_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(y)\}$ or $\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$.

Example 11. A set $X = \{0, 1, 2\}$ has the cayley table given below:

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then X is a BG-algebra.
Define an intuitionistic fuzzy soft set $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ and $\delta \in A\}$ given by $\sigma_{\tilde{M}[\delta]}(0) = \sigma_{\tilde{M}[\delta]}(1) = 0.2, \sigma_{\tilde{M}[\delta]}(2) = 0.4$, and $\tau_{\tilde{M}[\delta]}(0) = \tau_{\tilde{M}[\delta]}(1) = 0.7, \tau_{\tilde{M}[\delta]}(2) = 0.3$. Then it is easy to verify that $\tilde{M}[\delta]$ is an AIFSS over X.

$(\in, \in \vee q)$-ANTI INTUITIONISTIC FUZZY SOFT SUBALGEBRAS OF BG-ALGEBRAS

In this section, $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebras of BG-algebras are defined with some results studied.

Definition 12. A fuzzy soft set $\sigma_{\tilde{M}[\delta]}$ of the form

$$\sigma_{\tilde{M}[\delta]}(y) = \begin{cases} t, & if\ y = x,\ t \in (0,1] \\ 0, & if\ y \neq x \end{cases}$$

is called a fuzzy soft point with support x and value t and is denoted by x_t.

Definition 13. A fuzzy soft point x_t is said to belong to a fuzzy soft set $\sigma_{\tilde{M}[\delta]}$ written as $x_t \in \sigma_{\tilde{M}[\delta]}$ or $x_t q \sigma_{\tilde{M}[\delta]}$ if $\sigma_{\tilde{M}[\delta]} \geq t$ or $\sigma_{\tilde{M}[\delta]}(x) + t > 1$. If $x_t \in \sigma_{\tilde{M}[\delta]}$ or $x_t q \sigma_{\tilde{M}[\delta]}$, then $x_t \in \vee q \sigma_{\tilde{M}[\delta]}$.

Definition 14. A fuzzy soft subset $\sigma_{\tilde{M}[\delta]}$ of a BG-algebra X is said to be an $(\in, \in \vee q)$-AFSS of X if

$x_t, y_s \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t \vee s} \in \vee q \sigma_{\tilde{M}[\delta]}$ for all $x, y \in X$ and $\delta \in A$, where $t \vee s = max\{t, s\}$.

Definition 15. A fuzzy soft point x_t is said to belong to an intuitionistic fuzzy soft set $\tilde{M}[\delta] := \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ written as $x_t \in \tilde{M}[\delta]$ or $x_t q \tilde{M}[\delta]$ if $\sigma_{\tilde{M}[\delta]}(x) \geq t$ or $\sigma_{\tilde{M}[\delta]}(x) + t > 1$ and $\tau_{\tilde{M}[\delta]}(x) \leq t$ or $\tau_{\tilde{M}[\delta]}(x) + t < 1$. If $x_t \in \tilde{M}[\delta]$ or $x_t q \tilde{M}[\delta]$, then $x_t \in \vee q \tilde{M}[\delta]$.

Definition 16. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ in a BG-algebra X is said to be an $(\in, \in \vee q)$-AIFSS of X if its satisfies the following conditions:

(i) $x_t, y_s \in \sigma_{\tilde{M}[\delta]}(x) \Rightarrow x_{t \vee s} \in \vee q \sigma_{\tilde{M}[\delta]}(x)$,

i.e., $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq s \Rightarrow \sigma_{\tilde{M}[\delta]}(x*y) \leq t \vee s$ or $\sigma_{\tilde{M}[\delta]}(x*y) + t \vee s < 1$, for all $x, y \in X$ and $\delta \in A$, where $t \vee s = max\{t, s\}$.

(ii) $x_t, y_s \in \tau_{\tilde{M}[\delta]}(x) \Rightarrow x_{t \vee s} \in \vee q \tau_{\tilde{M}[\delta]}(x)$,

i.e., $\tau_{\tilde{M}[\delta]}(x) \geq t, \tau_{\tilde{M}[\delta]}(y) \geq s \Rightarrow \tau_{\tilde{M}[\delta]}(x*y) \geq t \wedge s$ or $\tau_{\tilde{M}[\delta]}(x*y) + t \wedge s > 1$, for all $x, y \in X$ and $\delta \in A$, where $t \wedge s = min\{t, s\}$.

Theorem 17. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ in a BG-algebra X is an AIFSS of X if and only if $\tilde{M}[\delta]$ is an $(\in, \in)$-AIFSS of X.

Proof. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ be an AIFSS of X. Then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$

and

$\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$.

Let $x, y \in X$ and $\delta \in A$ such that $x_t, y_s \in \sigma_{\tilde{M}[\delta]}(x)$, where $t, s \in (0, 1)$. Then $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq s$ and $\tau_{\tilde{M}[\delta]}(x) \geq t, \tau_{\tilde{M}[\delta]}(y) \geq s$.

Now $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \leq t \vee s \Rightarrow (x * y)_{t \vee s} \in \sigma_{\tilde{M}[\delta]}$

and

$\tau_{\tilde{M}[\delta]}(x*y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \geq t \wedge s \Rightarrow (x*y)_{t \wedge s} \in \tau_{\tilde{M}[\delta]}$.

Therefore, $\tilde{M}[\delta]$ is an $(\in, \in)$-AIFSS of X.

Conversely, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ be an $(\in, \in)$-AIFSS of X. To prove that $\tilde{M}[\delta]$ is an AIFSS of X.

Let $x, y \in X$ and $t = \sigma_{\tilde{M}[\delta]}(x), s = \sigma_{\tilde{M}[\delta]}(y)$. Then

$\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq s$

$\Rightarrow x_t \in \sigma_{\tilde{M}[\delta]}, y_s \in \sigma_{\tilde{M}[\delta]}$

$\Rightarrow (x * y)_{t \vee s} \in \sigma_{\tilde{M}[\delta]}$ [since $\tilde{M}[\delta]$ be an $(\in, \in)$-AIFSS of X]

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s$

$$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y). \qquad (1)$$

Again, let $x, y \in X$ and $t = \tau_{\tilde{M}[\delta]}(x), s = \tau_{\tilde{M}[\delta]}(y)$. Then

$\tau_{\tilde{M}[\delta]}(x) \geq t, \tau_{\tilde{M}[\delta]}(y) \geq s$

$\Rightarrow x_t \in \tau_{\tilde{M}[\delta]}, y_s \in \tau_{\tilde{M}[\delta]}$

$\Rightarrow (x * y)_{t \wedge s} \in \tau_{\tilde{M}[\delta]}$ [since $\tilde{M}[\delta]$ be an $(\in, \in)$-AIFSS of X]

$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s$

$$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y). \qquad (2)$$

Hence (1) and (2) $\Rightarrow \tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ is an AIFSS of X. ■

Theorem 18. If $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ be a (q, q)-AIFSS of a BG-algebra X, then it is also an $(\in, \in)$-AIFSS of X.

Proof. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X\ and\ \delta \in A\}$ be a (q, q)-AIFSS of a BG-algebra X. Let $x, y \in X$ and $\delta \in A$ such that $x_t, y_s \in \sigma_{\tilde{M}[\delta]}$. Then

$\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq s$

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x) - \varepsilon < t$ and $\sigma_{\tilde{M}[\delta]}(y) - \varepsilon < s$, where ε is arbitrary small positive number.

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x)+1-\varepsilon-t < 1$ and $\sigma_{\tilde{M}[\alpha]}(y)+1-\varepsilon-s < 1$
$\Rightarrow (x)_{1-\varepsilon-t} q \sigma_{\tilde{M}[\delta]}$ and $(y)_{1-\varepsilon-s} q \sigma_{\tilde{M}[\delta]}$.
Since $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an ($\in, \in$)-AIFSS of X. Therefore, we have $(x * y)_{(1-\varepsilon-t)\vee(1-\varepsilon-s)} q \sigma_{\tilde{M}[\delta]}$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + (1-\varepsilon-t) \vee (1-\varepsilon-s) < 1$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + 1 - \varepsilon - (t \wedge s) < 1$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < (t \wedge s) + \varepsilon$ [Since ε is an arbitrary]
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < t \wedge s$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < t \wedge s < t \vee s$
$\Rightarrow (x * y)_{t\vee s} \in \sigma_{\tilde{M}[\delta]}$.
Therefore,

$$x_t, y_s \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t\vee s} \in \sigma_{\tilde{M}[\delta]}. \quad (3)$$

Again, let $x, y \in X$ and $\delta \in A$ such that $x_t, y_s \in \tau_{\tilde{M}[\delta]}$. Then
$\tau_{\tilde{M}[\delta]}(x) \geq t, \tau_{\tilde{M}[\delta]}(y) \geq s$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x) + \varepsilon > t$ and $\tau_{\tilde{M}[\delta]}(y) + \varepsilon > s$, where ε is an arbitrary small positive number.
$\Rightarrow \tau_{\tilde{M}[\delta]}(x)+\varepsilon-t+1 > 1$ and $\tau_{\tilde{M}[\delta]}(y)+\varepsilon-s+1 > 1$
$\Rightarrow (x)_{\varepsilon-t+1} q \tau_{\tilde{M}[\delta]}$ and $(y)_{\varepsilon-s+1} q \tau_{\tilde{M}[\delta]}$.
Since $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an ($\in, \in$)-AIFSS of X. Therefore, we have
$(x * y)_{(\varepsilon-t+1)\wedge(\varepsilon-s+1)} q \tau_{\tilde{M}[\delta]}$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + (\varepsilon - t + 1) \wedge (\varepsilon - s + 1) < 1$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + \varepsilon + 1 - (t \vee s) > 1$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) > (t \vee s) - \varepsilon$ [Since ε is an arbitrary]
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) > t \vee s$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) > t \vee s > t \wedge s$
$\Rightarrow (x * y)_{t\wedge s} \in \tau_{\tilde{M}[\delta]}$.
Therefore,

$$x_t, y_s \in \tau_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t\wedge s} \in \tau_{\tilde{M}[\delta]}. \quad (4)$$

Hence (3) and (4) $\Rightarrow \tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an ($\in, \in$)-AIFSS of X. ■

Theorem 19. An intuitionistic fuzzy subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ of a BG-algebra X is an ($\in, \in \vee q$)-AIFSS of X if and only if

(i) $\sigma_{\tilde{M}[\delta]}(x * y) \leq max\{\sigma_{\tilde{M}[\delta]}(x), \sigma_{\tilde{M}[\delta]}(y), 0.5\}$ or $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$,
(ii) $\tau_{\tilde{M}[\delta]}(x * y) \geq min\{\tau_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(y), 0.5\}$ or $\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5$ for all $x, y \in X and\ \delta \in A$.

Proof. (i) First, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an ($\in, \in \vee q$)-AIFSS of X.

Case 1. Let $\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) > 0.5$ for all $x, y \in X$ and $\delta \in A$. Then
$\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$.
If possible, let $\sigma_{\tilde{M}[\delta]}(x * y) > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$. Choose a real number t such that
$\sigma_{\tilde{M}[\delta]}(x * y) > t > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x) < t, \sigma_{\tilde{M}[\delta]}(y) < t$
$\Rightarrow x_t \in \sigma_{\tilde{M}[\delta]}, y_t \in \sigma_{\tilde{M}[\delta]}$.
But $\sigma_{\tilde{M}[\delta]}(x * y) > t$
$\Rightarrow (x * y)_t \notin \sigma_{\tilde{M}[\delta]}$ and $\sigma_{\tilde{M}[\delta]}(x * y) + t > 2t$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t > 2(\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y))$
$> 2 \times 0.5 = 1$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t > 1$,
which contradicts the fact that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an ($\in, \in \vee q$)-AIFSS of X.
Therefore, $\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$.

Case 2. Let $\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$ for all $x, y \in X$ and $\delta \in A$. Then
$\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) = 0.5$.
If possible, $\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 = 0.5$. Then
$\sigma_{\tilde{M}[\delta]}(x) \leq 0.5$ and $\sigma_{\tilde{M}[\delta]}(y) \leq 0.5$.
Therefore, $x_{o.5}, y_{o.5} \in \sigma_{\tilde{M}[\delta]}$.
But $\sigma_{\tilde{M}[\delta]}(x) > 0.5$, therefore $x_{0.5} \notin \sigma_{\tilde{M}[\delta]}$ and $\sigma_{\tilde{M}[\delta]}(x) + 0.5 > 0.5 + 0.5 = 1$,
which again contradicts that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an ($\in, \in \vee q$)-AIFSS of X.
Hence, we must have $\sigma_{\tilde{M}[\delta]}(x * y) \leq 0.5 = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$.

Converse Part:
Let

$$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5. \quad (5)$$

Let $x, y \in X$ and $\delta \in A$ such that $x_t, y_s \in \sigma_{\tilde{M}[\delta]}$. Then $\sigma_{\tilde{M}[\delta]}(x) \leq t$ and $\sigma_{\tilde{M}[\delta]}(y) \leq s$. Therefore
$\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \leq t \vee s$. By Eq. (5), $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \vee 0.5$.
Now, if $t \vee s \geq 0.5$, then $t \vee s \vee 0.5 = t \vee s$. Therefore,
$\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s$

$$\Rightarrow (x * y)_{t\vee s} \in \sigma_{\tilde{M}[\delta]}. \quad (6)$$

Again, if $t \vee s > 0.5$, then $t \vee s \vee 0.5 = 0.5$. Therefore,
$\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \vee 0.5 = 0.5$

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t \vee s < 0.5 + 0.5 = 1$

$$\Rightarrow (x * y)_{t \vee s} q \sigma_{\tilde{M}[\delta]}. \quad (7)$$

From Eqs. (6) and (7), we have

$$x_t, y_s \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t \vee s} \in \vee q \sigma_{\tilde{M}[\delta]}. \quad (8)$$

Therefore, $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X.

(ii) First, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an $(\in, \in \vee q)$-AIFSS of X.

Case 1. Let $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) < 0.5$ for all $x, y \in X$ and $\delta \in A$. Then

$\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5 = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$.

If possible, let $\tau_{\tilde{M}[\delta]}(x * y) < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$. Choose a real number t such that

$\tau_{\tilde{M}[\delta]}(x * y) < t < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$

$\Rightarrow \tau_{\tilde{M}[\delta]}(x) > t, \tau_{\tilde{M}[\delta]}(y) > t$

$\Rightarrow x_t \in \tau_{\tilde{M}[\delta]}, y_t \in \tau_{\tilde{M}[\delta]}$.

But, $\tau_{\tilde{M}[\delta]}(x * y) < t$

$\Rightarrow (x * y)_t \notin \tau_{\tilde{M}[\delta]}$ and $\tau_{\tilde{M}[\delta]}(x) + t = 2t$

$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + t < 2(\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y))$

$< 2 \times 0.5 = 1$

$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + t < 1$,

which contradicts the fact that

$\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X.

Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5$.

Case 2. Let $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5$ for all $x, y \in X$ and $\delta \in A$. Then

$\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) = 0.5$.

If possible, $\tau_{\tilde{M}[\delta]}(x * y) < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5 = 0.5$. Then

$\tau_{\tilde{M}[\delta]}(x) \geq 0.5$ and $\tau_{\tilde{M}[\delta]}(y) \geq 0.5$.

Therefore, $x_{0.5}, y_{0.5} \in \tau_{\tilde{M}[\delta]}$. But $\tau_{\tilde{M}[\delta]}(x * y) < 0.5$, therefore $(x*y)_{0.5} \notin \tau_{\tilde{M}[\delta]}$ and $\tau_{\tilde{M}[\delta]}(x*y) + 0.5 < 0.5 + 0.5 = 1$, which again contradicts that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X.

Hence, we must have $\tau_{\tilde{M}[\delta]}(x * y) \geq 0.5 = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5$.

Converse Part:

Let

$$\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5. \quad (9)$$

Let $x, y \in X$ and $\delta \in A$ such that $x_t, y_s \in \tau_{\tilde{M}[\delta]}$. Then $\tau_{\tilde{M}[\delta]}(x) \geq t$ and $\tau_{\tilde{M}[\delta]}(y) \geq s$. Therefore, $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \geq t \wedge s$. By Eq. (9), we have $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \vee 0.5$.

Now, if $t \wedge s \leq 0.5$, then $t \wedge s \wedge 0.5 = t \wedge s$.

Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s$

$$\Rightarrow (x * y)_{t \wedge s} \in \tau_{\tilde{M}[\delta]}. \quad (10)$$

Again, if $t \wedge s > 0.5$, then $t \wedge s \wedge 0.5 = 0.5$. Therefore,

$\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \wedge 0.5 = 0$,

i.e., $\tau_{\tilde{M}[\delta]}(x * y) + t \wedge s > 0.5 + 0.5 = 1$

$$\Rightarrow (x * y)_{t \wedge s} q \tau_{\tilde{M}[\delta]}. \quad (11)$$

From Eqs. (10) and (11), we have

$$x_t, y_s \in \tau_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t \wedge s} \in \vee q \tau_{\tilde{M}[\delta]}. \quad (12)$$

Therefore, $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X.

Hence (8) and (12) $\Rightarrow \tilde{M}[\delta]$ is an $(\in, \in \vee q)$-AIFSS of X. ■

Theorem 20. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ of BG-algebra X is an $(\in, \in \vee q)$-AIFSS of X and if $\sigma_{\tilde{M}[\delta]}(x) > 0.5, \tau_{\tilde{M}[\delta]}(x) < 0.5$ *for all* $x, y \in X$ *and* $\delta \in A$, then $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x) : x \in X \text{ and } \delta \in A\}$ is also an $(\in, \in \vee q)$-AIFSS of X.

Proof. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an $(\in, \in \vee q)$-AIFSS of X and $\sigma_{\tilde{M}[\delta]}(x) > 0.5$ and $\tau_{\tilde{M}[\delta]}(x) < 0.5$ for all $x, y \in X$. Let $x_t \in \sigma_{\tilde{M}[\delta]}, y_s \in \sigma_{\tilde{M}[\delta]}$. Then we have

$0.5 < \sigma_{\tilde{M}[\delta]}(x) \leq t \text{ and } 0.5 < \sigma_{\tilde{M}[\delta]}(y) \leq s$.

Therefore, $t \vee s > 0.5$. Also $\sigma_{\tilde{M}[\delta]}(x * y) > 0.5$. Thus, $\sigma_{\tilde{M}[\delta]}(x * y) + t \vee s > 0.5 + 0.5 = 1$.

Since $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q) - AIFSS$ of X, we have

either $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \text{ or } \sigma_{\tilde{M}[\delta]}(x * y)$

$+ t \vee s < 1$.

So we must have, $\sigma_{\tilde{M}[\delta]}(x*y) \leq t \vee s \Rightarrow (x*y)_{t \vee s} \in \sigma_{\tilde{M}[\delta]}$.

Therefore,

$$x_t \in \sigma_{\tilde{M}[\delta]}, y_s \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_{t \vee s} \in \sigma_{\tilde{M}[\delta]}. \quad (13)$$

Thus, $\sigma_{\tilde{M}[\delta]}$ is $(\in, \in)$-AIFSS of X.

Again, let $x_t \in \tau_{\tilde{M}[\delta]}, y_s \in \tau_{\tilde{M}[\delta]}$. Then we have

$t \leq \tau_{\tilde{M}[\delta]}(x) < 0.5 \text{ and } s \leq \tau_{\tilde{M}[\delta]}(y) < 0.5$.

Therefore, $t \wedge s < 0.5$ and also $\tau_{\tilde{M}[\delta]}(x * y) < 0.5$. Thus, $\tau_{\tilde{M}[\delta]}(x * y) + t \wedge s < 0.5 + 0.5 = 1$.

Since $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X, we have

either $\tau_{\tilde{M}[\delta]}(x*y) \geq t \wedge s \text{ or } \tau_{\tilde{M}[\delta]}(x*y) + t \wedge s > 1$.

So we must have, $\tau_{\tilde{M}[\delta]}(x*y) \geq t \wedge s \Rightarrow (x*y)_{t\wedge s} \in \tau_{\tilde{M}[\delta]}$. Therefore,

$$x_t \in \tau_{\tilde{M}[\delta]}, y_s \in \tau_{\tilde{M}[\delta]} \Rightarrow (x*y)_{t\wedge s} \in \tau_{\tilde{M}[\delta]}. \quad (14)$$

Thus, $\tau_{\tilde{M}[\delta]}$ is $(\in, \in)$-AIFSS of X.
Hence (13) and (14) $\Rightarrow \tilde{M}[\delta]$ is an $(\in, \in)$-AIFSS of X.

■

Theorem 21. An Anti intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ of BG-algebra X is an $(\in, \in \vee q)$-AIFSS of X if and only if the sets $(\sigma_{\tilde{M}[\delta]})_t = \{x : \sigma_{\tilde{M}[\delta]}(x) < t, \ where \ t \in (0.5, 1], \sigma_{\tilde{M}[\delta]}(0) < t\}$ and $(\tau_{\tilde{M}[\delta]})_s = \{x : \tau_{\tilde{M}[\delta]}(x) \geq s, \ where \ s \in (0, 0.5), \tau_{\tilde{M}[\delta]}(0) \geq s\}$ are subalgebra of X.

Proof. Assume $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X. Clearly,
$0 \in (\sigma_{\tilde{M}[\delta]})_t, 0 \in (\tau_{\tilde{M}[\delta]})_s$ $[since \ \sigma_{\tilde{M}[\delta]}(0) \leq t, \tau_{\tilde{M}[\delta]}(0) \geq s]$.
Let $x, y \in X \ and \ \delta \in A$ such that $x, y \in (\sigma_{\tilde{M}[\delta]})_t \ where \ t \in (0.5, 1]$.
Therefore $\sigma_{\tilde{M}[\delta]}(x) < t, \sigma_{\tilde{M}[\delta]}(y) < t$.
Now by theorem 3.8, we have
$\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 < t \vee s \vee 0.5 = t$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x*y) < t \Rightarrow x*y \in (\sigma_{\tilde{M}[\delta]})_t$.
Therefore, $x, y \in (\sigma_{\tilde{M}[\delta]})_t \Rightarrow x*y \in (\sigma_{\tilde{M}[\delta]})_t$.
Hence $(\sigma_{\tilde{M}[\delta]})_t$ is a subalgebra of X.

Again let $x, y \in X \ and \ \delta \in A$ such that $x, y \in (\tau_{\tilde{M}[\delta]})_s \ where \ s \in (0, 0.5)$.
Therefore $\tau_{\tilde{M}[\delta]}(x) \geq s, \tau_{\tilde{M}[\delta]}(y) \geq s$.
Now by theorem 3.8, we have
$\tau_{\tilde{M}[\delta]}(x*y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5 > s \wedge s \wedge 0.5 = s$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x*y) \geq s$
$\Rightarrow x*y \in (\tau_{\tilde{M}[\delta]})_s$.
Therefore, $x, y \in (\tau_{\tilde{M}[\delta]})_s \Rightarrow x*y \in (\tau_{\tilde{M}[\delta]})_s$.
Hence $(\tau_{\tilde{M}[\delta]})_s$ is a subalgebra of X.

Conversely, $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ be an intuitionistic fuzzy soft subset of X and the sets $(\sigma_{\tilde{M}[\delta]})_t = \{x : \sigma_{\tilde{M}[\delta]}(x) < t\}$, where $t \in (0.5, 1]\}$ and $(\tau_{\tilde{M}[\delta]})_s = \{x : \tau_{\tilde{M}[\delta]}(x) \geq s, \ where \ s \in (0, 0.5)\}$ are subalgebra of X, to prove $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is an $(\in, \in \vee q) - AIFSS$ of X. Suppose $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is not an $(\in, \in \vee q) - AIFSS$ of X, then there exists $a, b \in X$ such that at least one of $\sigma_{\tilde{M}[\delta]}(a*b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5$ and $\tau_{\tilde{M}[\delta]}(a*b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5$ hold. Suppose $\sigma_{\tilde{M}[\delta]}(a*b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5$ holds.
Let $t = \{\sigma_{\tilde{M}[\delta]}(a*b) + (\sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5)\}/2$. Then $t \in (0.5, 1]$ and

$$\sigma_{\tilde{M}[\delta]}(a*b) > t > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5 \quad (15)$$

$\Rightarrow \sigma_{\tilde{M}[\delta]}(a) < t, \sigma_{\tilde{M}[\delta]}(b) < t$
$\Rightarrow a \in (\sigma_{\tilde{M}[\delta]})_t, b \in (\sigma_{\tilde{M}[\delta]})_t$
$\Rightarrow a*b \in (\sigma_{\tilde{M}[\delta]})_t$ $[since \ (\sigma_{\tilde{M}[\delta]})_t \ is \ a \ subalgebra \ of \ X]$.
Therefore, $\sigma_{\tilde{M}[\delta]}(a*b) < t$, which contradicts (15). Hence we must have

$$\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 \quad (16)$$

Next let $\tau_{\tilde{M}[\delta]}(a*b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5$ holds.
Let $s = \{\tau_{\tilde{M}[\delta]}(a*b) + (\tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5)\}/2$. Then $s \in (0, 0.5)$ and

$$\tau_{\tilde{M}[\delta]}(a*b) < s < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5 \quad (17)$$

$\Rightarrow \tau_{\tilde{M}[\delta]}(a) > s, \tau_{\tilde{M}[\delta]}(b) > s$
$\Rightarrow a \in (\tau_{\tilde{M}[\delta]})_s, b \in (\tau_{\tilde{M}[\delta]})_s$
$\Rightarrow a*b \in (\tau_{\tilde{M}[\delta]})_s$ $[since \ (\tau_{\tilde{M}[\delta]})_s \ is \ a \ subalgebra \ of \ X]$.
Therefore, $\tau_{\tilde{M}[\delta]}(a*b) > s$, which contradicts (17). Hence we must have

$$\tau_{\tilde{M}[\delta]}(x*y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5. \quad (18)$$

Hence (16) and (18) $\Rightarrow \tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X. ■

Definition 22. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ be an intuitionistic fuzzy soft subset of BG-algebra X and $t \in (0, 1]$. Then let
$(\sigma_{\tilde{M}[\delta]})_t = \{x : x_t \in \sigma_{\tilde{M}[\delta]}\} = \{x : \sigma_{\tilde{M}[\delta]}(x) \geq t\}$,
$\langle\sigma_{\tilde{M}[\delta]}\rangle_t = \{x : x_t q \sigma_{\tilde{M}[\delta]}\} = \{x : \sigma_{\tilde{M}[\delta]}(x) + t > 1\}$,
$[\sigma_{\tilde{M}[\delta]}]_t = \{x : x_t \in \vee q \sigma_{\tilde{M}[\delta]}\} = \{x : \sigma_{\tilde{M}[\delta]} \geq t \ or \ \sigma_{\tilde{M}[\delta]}(x) + t > 1\}$,
where $(\sigma_{\tilde{M}[\delta]})_t$ is called t level set of $\sigma_{\tilde{M}[\delta]}$, $\langle\sigma_{\tilde{M}[\delta]}\rangle_t$ is called q level set of $\sigma_{\tilde{M}[\delta]}$ and $[\sigma_{\tilde{M}[\delta]}]_t$ is called $\in \vee q$ level set of $\sigma_{\tilde{M}[\delta]}$,
clearly,
$[\sigma_{\tilde{M}[\delta]}]_t = \langle\sigma_{\tilde{M}[\delta]}\rangle_t \cup (\sigma_{\tilde{M}[\delta]})_t$ and
$(\tau_{\tilde{M}[\delta]})_t = \{x : x_t \in \tau_{\tilde{M}[\delta]}\} = \{x : \tau_{\tilde{M}[\delta]}(x) \leq t\}$,
$\langle\tau_{\tilde{M}[\delta]}\rangle_t = \{x : x_t q \tau_{\tilde{M}[\delta]}\} = \{x : \tau_{\tilde{M}[\delta]} + t < 1\}$,
$[\tau_{\tilde{M}[\delta]}]_t = \{x : x_t \in \vee q \tau_{\tilde{M}[\delta]}\} = \{x : \tau_{\tilde{M}[\delta]} \leq t \ or \ \tau_{\tilde{M}[\delta]}(x) + t < 1\}$,
where $(\tau_{\tilde{M}[\delta]})_t$ is called t level set of $\tau_{\tilde{M}[\delta]}$, $\langle\tau_{\tilde{M}[\delta]}\rangle_t$ is called q level set of $\tau_{\tilde{M}[\delta]}$ and $[\tau_{\tilde{M}[\delta]}]_t$ is called $\in \vee q$ level set of $\tau_{\tilde{M}[\delta]}$,
clearly, $[\tau_{\tilde{M}[\delta]}]_t = \langle\tau_{\tilde{M}[\delta]}\rangle_t \cup (\tau_{\tilde{M}[\delta]})_t$.

Theorem 23. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ be an intuitionistic fuzzy soft subset of BG-algebra X. Then $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is an $(\in, \in \vee q)$- AIFSS of X if and only if $[\sigma_{\tilde{M}[\delta]}]_t \; and \; [\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X for all $t \in (0, 1]$. We call $[\sigma_{\tilde{M}[\delta]}]_t \; and \; [\tau_{\tilde{M}[\delta]}]_t \; as \; \in \vee q$ level subalgebra of X.

Proof. Assume that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X, to prove $[\sigma_{\tilde{M}[\delta]}]_t \; and \; [\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X. Let $x, y \in [\sigma_{\tilde{M}[\delta]}]_t$ for $t \in (0, 1]$. Then $x_t, y_t \in \vee q \sigma_{\tilde{M}[\delta]}$,
i.e., $\sigma_{\tilde{M}[\delta]}(x) \leq t \; or \; \sigma_{\tilde{M}[\delta]}(x) + t < 1 \; and \; \sigma_{\tilde{M}[\delta]}(y) \leq t \; or \; \sigma_{\tilde{M}[\delta]}(y) + t < 1$.

Since $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X, we have

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$ for any $x, y \in X \; and \; \delta \in A$.

Now we have the following cases.

Case 1. $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq t$, let $t < 0.5$. Then

$\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 = t \vee t \vee 0.5 = 0.5$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq 0.5 \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 0.5 + 0.5 = 1 \Rightarrow (x * y)_t q \sigma_{\tilde{M}[\delta]}$.

Again if $t \geq 0.5$, then

$\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 \leq t \vee t \vee 0.5 = t$,

$\sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y)_t \in \sigma_{\tilde{M}[\delta]}$.

Hence $(x * y)_t \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_t \in [\sigma_{\tilde{M}[\delta]}]_t$.

Case 2. $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) + t < 1$, let $t < 0.5$. Then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 < t \vee (1 - t) \vee 0.5 = 1 - t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 \Rightarrow (x * y)_t q \sigma_{\tilde{M}[\delta]}$.

Again if $t \geq 0.5$, then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 \leq t \vee (1 - t) \vee 0.5 = t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y)_t \in \sigma_{\tilde{M}[\delta]}$.

Hence $(x * y)_t \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_t \in [\sigma_{\tilde{M}[\delta]}]_t$.

Case 3. $\sigma_{\tilde{M}[\delta]}(x) + t < 1, \sigma_{\tilde{M}[\delta]}(y) \leq t$, let $t < 0.5$. Then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 < (1 - t) \vee t \vee 0.5 = 1 - t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 \Rightarrow (x * y)_t q \sigma_{\tilde{M}[\delta]}$.

Again if $t \geq 0.5$, then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 = (1 - t) \vee t \vee 0.5 = t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y)_t \in \sigma_{\tilde{M}[\delta]}$.

Hence $(x * y)_t \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_t \in [\sigma_{\tilde{M}[\delta]}]_t$.

Case 4. $\sigma_{\tilde{M}[\delta]}(x) + t < 1, \sigma_{\tilde{M}[\delta]}(y) + t < 1$, let $t < 0.5$. Then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 < (1 - t) \vee (1 - t) \vee 0.5 = 1 - t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 \Rightarrow (x * y)_t q \sigma_{\tilde{M}[\delta]}$.

Again if $t \geq 0.5$, then

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5 = (1 - t) \vee (1 - t) \vee 0.5 = 0.5 \leq t$,

$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y)_t \in \sigma_{\tilde{M}[\delta]}$.

Hence $(x * y)_t \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y)_t \in [\sigma_{\tilde{M}[\delta]}]_t$.
Hence from above four cases $x, y \in [\sigma_{\tilde{M}[\delta]}]_t \Rightarrow (x * y)_t \in [\sigma_{\tilde{M}[\delta]}]_t$.
Hence $[\sigma_{\tilde{M}[\delta]}]_t$ is a subalgebra of X. Similarly, we can prove $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X. Conversely, let $\tilde{M}[\delta] = \{(X, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ be an AIFSS in X such that $[\sigma_{\tilde{M}[\delta]}]_t$ and $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X for all $t \in (0, 1]$, to prove $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X. Suppose $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is not an $(\in, \in \vee q)$-AIFSS of X, then there exist $a, b \in X$ such that at least one of $\sigma_{\tilde{M}[\delta]}(a * b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$ and $\tau_{\tilde{M}[\delta]}(a * b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5$ hold. Suppose $\sigma_{\tilde{M}[\delta]}(a*b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5$ is true, then choose $t \in (0, 1]$ such that

$$\sigma_{\tilde{M}[\delta]}(a * b) > t > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee 0.5 \tag{19}$$

Then $\sigma_{\tilde{M}[\delta]}(a) < t, \sigma_{\tilde{M}[\delta]}(b) < t \Rightarrow x, y \in (\sigma_{\tilde{M}[\delta]})_t \subset [\sigma_{\tilde{M}[\delta]}]_t$ which is a subalgebra.
Therefore, $(a*b) \in [\sigma_{\tilde{M}[\delta]}]_t \Rightarrow \sigma_{\tilde{M}[\delta]}(a*b) \leq t \; or \; \sigma_{\tilde{M}[\delta]}(a*b) + t < 1$ which contradict (19).

Again, if $\tau_{\tilde{M}[\delta]}(a * b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5$ is true, then choose $t \in (0, 1]$, such that

$$\tau_{\tilde{M}[\delta]}(a * b) < t < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge 0.5 \tag{20}$$

Then $\tau_{\tilde{M}[\delta]}(a) > t, \tau_{\tilde{M}[\delta]}(b) > t \Rightarrow a, b \in (\tau_{\tilde{M}[\delta]})_t \subset [\tau_{\tilde{M}[\delta]}]_t$ which is a subalgebra.
Therefore, $(a*b) \in [\tau_{\tilde{M}[\delta]}]_t \Rightarrow \tau_{\tilde{M}[\delta]}(a*b) \geq t \; or \; \tau_{\tilde{M}[\delta]}(a*b) + t > 1$ which contradict (20).
Hence we must have

$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5$

$\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5$ for all $x, y \in X \; and \; \delta \in A$.

Hence $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of X. ■

Definition 24. If $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ and $\tilde{N}[\eta] = \{(X, \sigma_{\tilde{N}[\eta]}(x), \tau_{\tilde{N}[\eta]}(x)) : x \in X \; and \; \eta \in B\}$ are any two $(\in, \in \vee q)$-IFSSs of a set X,then

$$\tilde{M}[\delta] \cap \tilde{N}[\eta] = \{(x, \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x), \tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x)) : x \in X \; and \; (\delta, \eta) \in A \cap B\},$$

where $\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x) = \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{N}[\eta]}(x)$ and $\tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x) = \tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{N}[\eta]}(x)$.

Theorem 25. Let $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q) - AIFSSs$ of X. Then $\tilde{M}[\delta] \cap \tilde{N}[\eta](x) = \{(x, \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x), \tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x)) : x \in X \; and \; (\delta, \eta) \in A \cap B\}$ is also an $(\in, \in \vee q) - AIFSS$ of X.

Proof. Let $x, y \in X \; and \; (\delta, \eta) \in A \cap B$. Since $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q) - AIFSS$ of X, we have $\tilde{M}[\delta] \cap \tilde{N}[\eta](x) = \{\langle x, \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x), \tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x)\rangle : x \in X \; and \; \alpha \in A \cap B\}$,

$$\begin{aligned}
\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x * y) &= \sigma_{\tilde{M}[\delta]}(x * y) \vee \sigma_{\tilde{N}[\eta]}(x * y) \\
&\leq (\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5) \vee (\sigma_{\tilde{N}[\eta]}(x) \vee \sigma_{\tilde{N}[\eta]}(y) \vee 0.5) \\
&= (\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{N}[\eta]}(x)) \vee (\sigma_{\tilde{M}[\delta]}(y) \vee \sigma_{\tilde{N}[\eta]}(y)) \vee 0.5 \\
\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x * y) &\leq \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x) \vee \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(y) \vee 0.5. \qquad (21)
\end{aligned}$$

Again,

$$\begin{aligned}
\tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x * y) &= \tau_{\tilde{M}[\delta]}(x * y) \wedge \tau_{\tilde{N}[\eta]}(x * y) \\
&\geq (\tau_{\tilde{M}[\alpha]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5) \wedge (\tau_{\tilde{N}[\eta]}(x) \wedge \tau_{\tilde{N}[\eta]}(y) \wedge 0.5) \\
&= (\tau_{\tilde{M}[\alpha]}(x) \wedge \tau_{\tilde{M}[\delta]}(x)) \wedge (\tau_{\tilde{N}[\eta]}(y) \wedge \tau_{\tilde{N}[\eta]}(y)) \wedge 0.5 \\
\tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x * y) &\geq \tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(x) \wedge \tau_{\tilde{M}[\delta]\cap\tilde{N}[\eta]}(y) \wedge 0.5. \qquad (22)
\end{aligned}$$

Hence (21) and (22) $\Rightarrow \tilde{M}[\delta] \cap \tilde{N}[\eta]$ is also an $(\in, \in \vee q) - AIFSS$ of X. ■

Theorem 26. Let $\tilde{M}[\delta_i] = \{(\sigma_{\tilde{M}[\delta_i]}, \tau_{\tilde{M}[\delta_i]}) : i = 1, 2, ...\}$ be a family of $(\in, \in \vee q)$-AIFSSs of a BG-algebra X. Then $\cap_i^n \tilde{M}[\delta_i]$ is also an $(\in, \in \vee q)$-AIFSS of X, where $\cap_i^n \tilde{M}[\delta_i](x) = \{(x, (\vee \sigma_{\tilde{M}[\delta_i]}(x) : i = 1, 2, ...), (\wedge \tau_{\tilde{M}[\delta_i]}(x) : i = 1, 2, ...)) : x \in X \; and \; \delta_i \in \cap_i^n A_i\}$.

We define two operators $\oplus\tilde{M}[\delta]$ and $\otimes\tilde{M}[\delta]$ on intuitionistic fuzzy soft sets as follows.

Definition 27. Let $\tilde{M}[\delta] = \{(X, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ be an intuitionistic fuzzy soft set defined on X. The operators $\oplus\tilde{M}[\delta]$ and $\otimes\tilde{M}[\delta]$ are defined as follows:
$\oplus\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \bar{\sigma}_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ and
$\otimes\tilde{M}[\delta] = \{(x, \bar{\tau}_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ respectively.

Theorem 28. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ is an $(\in, \in \vee q)$-AIFSS of a BG-algebra X. Then (i) $\oplus\tilde{M}[\delta]$, (ii) $\otimes\tilde{M}[\delta]$ both are $(\in, \in \vee q)$-AIFSS of X.

Proof. For (i), it is sufficient to show that $\bar{\sigma}_{\tilde{M}[\delta]}$ satisfies
$\bar{\sigma}_{\tilde{M}[\delta]}(0) \geq \bar{\sigma}_{\tilde{M}[\delta]}(x) \; and \; \bar{\sigma}_{\tilde{M}[\delta]}(x * y) \geq \bar{\sigma}_{\tilde{M}[\delta]}(x) \vee \bar{\sigma}_{\tilde{M}[\delta]}(y) \vee 0.5$. We have
$\bar{\sigma}_{\tilde{M}[\delta]}(0) = 1 - \sigma_{\tilde{M}[\delta]}(0) \leq 1 - \sigma_{\tilde{M}[\delta]}(x) \geq \bar{\sigma}_{\tilde{M}[\delta]}(x)$.
Let $x, y \in X \; and \; \delta \in A$. Then

$$\begin{aligned}
\bar{\sigma}_{\tilde{M}[\delta]}(x * y) &= 1 - \sigma_{\tilde{M}[\delta]}(x * y) \\
&\leq 1 - (\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee 0.5) \\
&= (1 - \sigma_{\tilde{M}[\delta]}(x)) \wedge (1 - \sigma_{\tilde{M}[\delta]}(y)) \wedge (1 - 0.5) \\
\bar{\sigma}_{\tilde{M}[\delta]}(x * y) &\geq \bar{\sigma}_{\tilde{M}[\delta]}(x) \wedge \bar{\sigma}_{\tilde{M}[\delta]}(y) \wedge 0.5.
\end{aligned}$$

Hence, $\oplus\tilde{M}[\delta]$ is an $(\in, \in \vee q)$-AIFSS of X.
For (ii), it is sufficient to show that $\bar{\tau}_{\tilde{M}[\delta]}$ satisfies
$\bar{\tau}_{\tilde{M}[\delta]}(0) \leq \bar{\tau}_{\tilde{M}[\delta]}(x)$ and $\bar{\tau}_{\tilde{M}[\delta]}(x * y) \leq \bar{\tau}_{\tilde{M}[\delta]}(x) \vee \bar{\tau}_{\tilde{M}[\delta]}(y) \vee 0.5$. We have
$\bar{\tau}_{\tilde{M}[\delta]}(0) = 1 - \tau_{\tilde{M}[\delta]}(0) \geq 1 - \tau_{\tilde{M}[\delta]}(x) \leq \bar{\tau}_{\tilde{M}[\delta]}(x)$.
Let $x, y \in X$ *and* $\delta \in A$. Then

$$\begin{aligned}\bar{\tau}_{\tilde{M}[\delta]}(x * y) &= 1 - \tau_{\tilde{M}[\delta]}(x * y) \\ &\geq 1 - (\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge 0.5) \\ &= (1 - \tau_{\tilde{M}[\delta]}(x)) \vee (1 - \tau_{\tilde{M}[\delta]}(y)) \vee (1 - 0.5) \\ \bar{\tau}_{\tilde{M}[\delta]}(x * y) &\leq \bar{\tau}_{\tilde{M}[\delta]}(x) \vee \bar{\tau}_{\tilde{M}[\delta]}(y) \vee 0.5.\end{aligned}$$

Hence, $\otimes\tilde{M}[\delta]$ is an $(\in, \in \vee q)$-AIFSS of X. ■

CARTESIAN PRODUCT OF $(\in, \in \vee q)$-ANTI INTUITIONISTIC FUZZY SOFT SUBALGEBRAS OF BG-ALGEBRAS

In this section, we consider cartesian product of $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebras of BG-algebras.

Definition 29. Let $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q)$-AIFSSs of a BG-algebra X. Then their cartesian product $\tilde{M}[\delta] \times \tilde{N}[\eta] : X \times X \to [0, 1]$ is defined by $\tilde{M}[\delta] \times \tilde{N}[\eta](x, y) = \{((x, y), \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, y), \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, y)) : (x, y) \in X \times X \text{ and } (\delta, \eta) \in A \times B\}$, where $\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, y) = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{N}[\eta]}(y)$ and $\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, y) = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{N}[\eta]}(y)$.

Theorem 30. Let $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q)$-AIFSSs of a BG-algebra X. Then $(\tilde{M}, A) \times (\tilde{N}, B)$ is also an $(\in, \in \vee q)$-AIFSS of $X \times X$.

Proof. For any $(x, y) \in X \times X$ *and* $(\delta, \eta) \in A \times B$, we have
$\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(0, 0) = \sigma_{\tilde{M}[\delta]}(0) \vee \sigma_{\tilde{N}[\eta]}(0) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{N}[\eta]}(x)$
$\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(0, 0) \leq \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, x)$ and
$\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(0, 0) = \tau_{\tilde{M}[\delta]}(0) \wedge \tau_{\tilde{N}[\eta]}(0) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{N}[\eta]}(x)$
$\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(0, 0) \geq \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x, x)$.
Let (x_1, y_1) and $(x_2, y_2) \in X \times X$ *and* $(\delta, \eta) \in A \times B$. Then

$$\begin{aligned}\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1, y_1) * (x_2, y_2)) &= \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1 * x_2, y_1 * y_2) \\ &= \sigma_{\tilde{M}[\delta]}(x_1 * x_2) \vee \sigma_{\tilde{M}[\eta]}(y_1 * y_2) \\ &\leq (\sigma_{\tilde{M}[\delta]}(x_1) \vee \sigma_{\tilde{M}[\delta]}(x_2) \vee 0.5) \vee (\sigma_{\tilde{N}[\eta]}(y_1) \vee \sigma_{\tilde{N}[\eta]}(y_2) \vee 0.5) \\ &= (\sigma_{\tilde{M}[\delta]}(x_1) \vee \sigma_{\tilde{N}[\eta]}(y_1)) \vee (\sigma_{\tilde{M}[\delta]}(x_2) \vee \sigma_{\tilde{N}[\eta]}(y_2)) \vee 0.5 \\ \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1, y_1) * (x_2, y_2)) &\leq \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1, y_1) \vee \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2, y_2) \vee 0.5. \qquad (23)\end{aligned}$$

Again,

$$\begin{aligned}\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1, y_1) * (x_2, y_2)) &= \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1 * x_2, y_1 * y_2) \\ &= \tau_{\tilde{M}[\delta]}(x_1 * x_2) \wedge \tau_{\tilde{M}[\eta]}(y_1 * y_2) \\ &\geq (\tau_{\tilde{M}[\delta]}(x_1) \wedge \tau_{\tilde{M}[\delta]}(x_2) \wedge 0.5) \wedge (\tau_{\tilde{N}[\eta]}(y_1) \wedge \tau_{\tilde{N}[\eta]}(y_2) \wedge 0.5) \\ &= (\tau_{\tilde{M}[\delta]}(x_1) \wedge \tau_{\tilde{N}[\eta]}(y_1)) \wedge (\tau_{\tilde{M}[\delta]}(x_2) \wedge \tau_{\tilde{N}[\eta]}(y_2)) \wedge 0.5 \\ \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1, y_1) * (x_2, y_2)) &\geq \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1, y_1) \wedge \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2, y_2) \wedge 0.5. \qquad (24)\end{aligned}$$

Hence (23) and (24) $\Rightarrow \tilde{M}[\delta] \times \tilde{N}[\eta]$ is also an $(\in, \in \vee q)$-AIFSS of $X \times X$. ■

Example 31. Let $X = \{0, a, b\}$ be a set with the following Cayley table:

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*	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

Then X is a BG-algebra. We consider two $(\in, \in \vee q)$-intuitionistic fuzzy soft sets $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \textit{ and } \delta \in A\}$ as

$\tilde{M}$	0	a	b
δ	[0.2,0.7]	[0.2,0.7]	[0.5,0.5]

and $\tilde{N}[\eta] = \{(x, \sigma_{\tilde{N}[\eta]}(x), \tau_{\tilde{N}[\eta]}(x)) : x \in X \textit{ and } \eta \in B\}$ as

$\tilde{N}$	0	a	b
η	[0.3,0.6]	[0.4,0.5]	[0.3,0.6]

Therefore $\tilde{M}[\delta]$ and $\tilde{N}[\eta]$ are an $(\in, \in \vee q)$-AIFSSs of X.

Now, $\sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,0) = 0.3, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,a) = 0.4, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,b) = 0.3, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(a,0) = 0.3, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(a,a) = 0.4, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(a,b) = 0.3, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(b,0) = 0.5, \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(b,a) = 0.5$ and $\sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(b,b) = 0.5$.

$\tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(0,0) = 0.6, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(0,a) = 0.5, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(0,b) = 0.6, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(a,0) = 0.6, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(a,a) = 0.5, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(a,b) = 0.6, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(b,0) = 0.5, \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(b,a) = 0.5$ and $\tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(b,b) = 0.5$.
Then $\tilde{M}[\delta] \times \tilde{N}[\eta]$ is an $(\in, \in \vee q)$-AIFSS of $X \times X$. Some results are calculated below:

$$\begin{aligned}
\sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}((0,a) * (0,b)) &\leq \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,a) \vee \sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,b) \vee 0.5 \\
\sigma_{\tilde{M}[\delta]} \times \sigma_{\tilde{N}[\eta]}(0,b) &\leq 0.4 \vee 0.3 \vee 0.5 \\
0.3 &\leq 0.5 \textit{ and} \\
\tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}((a,0) * (a,b)) &\geq \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(a,0) \wedge \tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(a,b) \wedge 0.5 \\
\tau_{\tilde{M}[\delta]} \times \tau_{\tilde{N}[\eta]}(0,a) &\geq 0.6 \wedge 0.6 \wedge 0.5 \\
0.5 &\geq 0.5.
\end{aligned}$$

■

Theorem 32. Let $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q)$-AIFSSs of a BG-algebra X. Then $\oplus(\tilde{M}[\delta] \times \tilde{N}[\eta]) = \{((x,y), \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x,y), \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x,y)) : (x,y) \in X \times X \textit{ and } (\delta, \eta) \in A \times B\}$ is also an $(\in, \in \vee q)$-AIFSS of $X \times X$.

Proof. Now, $\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1) * (x_2,y_2)) \leq \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1) \vee \sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2) \vee 0.5$
$\Rightarrow 1 - \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1) * (x_2,y_2)) = (1 - \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1)) \vee (1 - \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2)) \vee (1 - 0.5)$
$\Rightarrow \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1) * (x_2,y_2)) \geq 1 - [(1 - \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1) \wedge 1 - \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2) \wedge (1 - 0.5)]$
$\Rightarrow \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1) * (x_2,y_2)) \geq \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1) \wedge \overline{\sigma}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2) \wedge 0.5.$
Hence, $\oplus(\tilde{M}[\delta] \times \tilde{N}[\eta])$ is also an $(\in, \in \vee q)$-AIFSS of $X \times X$. ■

Theorem 33. Let $(\tilde{M}, A)$ and $(\tilde{N}, B)$ be two $(\in, \in \vee q)$-AIFSSs of a BG-algebra X. Then $\otimes(\tilde{M}[\delta] \times \tilde{N}[\eta]) = \{((x,y), \overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x,y), \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x,y)) : (x,y) \in X \times X \textit{ and } (\delta, \eta) \in A \times B\}$ is also an $(\in, \in \vee q)$-AIFSSs of $X \times X$.

Proof. Now, $\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1)*(x_2,y_2)) \geq \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1) \wedge \tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2) \wedge 0.5$
$\Rightarrow 1-\overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1)*(x_2,y_2)) = (1-\overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1)) \wedge (1-\overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2)) \wedge (1-0.5)$
$\Rightarrow \overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1)*(x_2,y_2)) \geq 1-[(1-\overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1)) \vee (1-\overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2)) \vee (1-0.5)]$
$\Rightarrow \overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}((x_1,y_1)*(x_2,y_2)) \leq \overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_1,y_1) \vee \overline{\tau}_{\tilde{M}[\delta]\times\tilde{N}[\eta]}(x_2,y_2) \vee 1-0.5.$
Hence, $\otimes(\tilde{M}[\delta]\times\tilde{N}[\eta])$ is also an $(\in,\in\vee q)$-AIFSSs of $X\times X$. ■

HOMOMORPHISM AND $(\in,\in\vee q)$-ANTI INTUITIONISTIC FUZZY SOFT IDEALS OF BG-ALGEBRAS

In this section, homomorphism and $(\in,\in\vee q)$-anti intuitionistic fuzzy soft ideals of BG-algebras are defined and some results are studied.

Definition 34. Let X and X' be two BG-algebras. Then a mapping $f: X\rightarrow X'$ is said to be homomorphism if $f(x*y) = f(x)*f(y)$ *for all* $x,y\in X$.

Definition 35. A fuzzy soft subset $\sigma_{\tilde{M}[\delta]}$ of a BG-algebra X is said to be an $(\in,\in\vee q)$-AFSI of X if $(x*y)_t, y_s\in\sigma_{\tilde{M}[\delta]} \Rightarrow x_{t\vee s}\in\sigma_{\tilde{M}[\delta]}$ for all $x,y\in X$ *and* $\delta\in A$.

Definition 36. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x,\sigma_{\tilde{M}[\delta]}(x),\tau_{\tilde{M}[\delta]}(x)) : x\in X\}$ in a BG-algebra X is said to be an $(\in,\in\vee q)$-AIFSI of X if it satisfies the following conditions:

(i) $(x*y)_t, y_s\in\sigma_{\tilde{M}[\delta]}(x) \Rightarrow x_{t\vee s}\in\vee q\sigma_{\tilde{M}[\delta]}(x)$,
i.e., $\sigma_{\tilde{M}[\delta]}(x*y)\leq t, \sigma_{\tilde{M}[\delta]}(y)\leq s \Rightarrow \sigma_{\tilde{M}[\delta]}(x)\leq t\vee s$ *or* $\sigma_{\tilde{M}[\delta]}(x)+t\vee s<1$, for all $x,y\in X$ and $\delta\in A$, where $t\vee s = max\{t,s\}$.

(ii) $(x*y)_t, y_s\in\tau_{\tilde{M}[\delta]}(x) \Rightarrow x_{t\wedge s}\in\vee q\tau_{\tilde{M}[\delta]}(x)$,
i.e., $\tau_{\tilde{M}[\delta]}(x*y)\geq t, \tau_{\tilde{M}[\delta]}(y)\geq s \Rightarrow \tau_{\tilde{M}[\delta]}(x)\geq t\wedge s$ *or* $\tau_{\tilde{M}[\delta]}(x)+t\wedge s>1$, for all $x,y\in X$ and $\delta\in A$, where $t\wedge s = min\{t,s\}$.

Theorem 37. Let X and X' be two BG-algebras and $f: X\rightarrow X'$ be an homomorphism. If $\tilde{M}[\delta] = \{(x,\sigma_{\tilde{M}[\delta]}(x),\tau_{\tilde{M}[\delta]}(x)) : x\in X \text{ and } \delta\in A\}$ is an $(\in,\in\vee q)$-AIFSI of X', then $f^{-1}(\tilde{M}[\delta])$ is also an $(\in,\in\vee q)$-AIFSI of X.

Proof. $f^{-1}(\tilde{M}[\delta])(x) = f^{-1}(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]})(x)$ is defined as
$f^{-1}(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]})(x) = (\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]})(f^{-1}(x))$ for every $x\in X$ *and* $\delta\in A$.
Let $\tilde{M}[\delta] = \{(x,\sigma_{\tilde{M}[\delta]}(x),\tau_{\tilde{M}[\delta]}(x)) : x\in X \text{ and } \delta\in A\}$ is an $(\in,\in\vee q)$-AIFSI of X', let $x,y\in X$ such that $(x*y)_t, y_s\in f^{-1}(\tilde{M}[\delta]) = f^{-1}(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}) = (f^{-1}\sigma_{\tilde{M}[\delta]}, f^{-1}\tau_{\tilde{M}[\delta]})$. Then $(x*y)_t, y_s\in f^{-1}(\sigma_{\tilde{M}[\delta]})$ and $(x*y)_t, y_s\in f^{-1}(\tau_{\tilde{M}[\delta]})$.

Case 1. Let $(x*y)_t, y_s\in f^{-1}(\sigma_{\tilde{M}[\delta]})$
$\Rightarrow f^{-1}(\sigma_{\tilde{M}[\delta]})(x*y)\leq t$ *and* $f^{-1}(\sigma_{\tilde{M}[\delta]})(y)\leq s$
$\Rightarrow \sigma_{\tilde{M}[\delta]}f(x*y)\leq t$ *and* $\sigma_{\tilde{M}[\delta]}f(y)\leq s$
$\Rightarrow (f(x*y))_t\in\sigma_{\tilde{M}[\delta]}$ *and* $(f(y))_s\in\sigma_{\tilde{M}[\delta]}$
$\Rightarrow (f(x)*f(y))_t\in\sigma_{\tilde{M}[\delta]}$ *and* $(f(y))_s\in\sigma_{\tilde{M}[\delta]}$ [Since f is a homomorphism]
$\Rightarrow (f(x))_{t\vee s}\in\sigma_{\tilde{M}[\delta]}$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(f(x))\leq t\vee s$ *or* $\sigma_{\tilde{M}[\delta]}(f(x))+t\vee s<1$

$\Rightarrow f^{-1}(\sigma_{\tilde{M}[\delta]}(x)) \leq t \vee s \; or \; f^{-1}(\sigma_{\tilde{M}[\delta]}(x)) + t \vee s < 1$
$\Rightarrow x_{t\vee s} \in f^{-1}(\sigma_{\tilde{M}[\delta]}) \; or \; x_{t\vee s} \in qf^{-1}(\sigma_{\tilde{M}[\delta]})$
$\Rightarrow x_{t\vee s} \in \vee q f^{-1}(\sigma_{\tilde{M}[\delta]})$.
Therefore,

$$(x * y)_t, y_s \in f^{-1}(\sigma_{\tilde{M}[\delta]}) \Rightarrow x_{t\vee s} \in \vee q f^{-1}(\sigma_{\tilde{M}[\delta]}) \tag{25}$$

Case 2. Let $(x * y)_t, y_s \in f^{-1}(\tau_{\tilde{M}[\delta]})$
$\Rightarrow f^{-1}(\tau_{\tilde{M}[\delta]})(x * y) \geq t \; and \; f^{-1}(\tau_{\tilde{M}[\delta]})(y) \geq s$
$\Rightarrow \tau_{\tilde{M}[\delta]} f(x * y) \geq t \; and \; \tau_{\tilde{M}[\delta]} f(y) \geq s$
$\Rightarrow (f(x * y))_t \in \tau_{\tilde{M}[\delta]} \; and \; (f(y))_s \in \tau_{\tilde{M}[\delta]}$
$\Rightarrow (f(x) * f(y))_t \in \tau_{\tilde{M}[\delta]} \; and \; (f(y))_s \in \tau_{\tilde{M}[\delta]}$ [Since f is a homomorphism]
$\Rightarrow (f(x))_{t\wedge s} \in \tau_{\tilde{M}[\delta]}$
$\Rightarrow \tau_{\tilde{M}[\delta]}(f(x)) \geq t \wedge s \; or \; \tau_{\tilde{M}[\delta]}(f(x)) + t \wedge s > 1$
$\Rightarrow f^{-1}(\tau_{\tilde{M}[\delta]}(x)) \geq t \wedge s \; or \; f^{-1}(\tau_{\tilde{M}[\delta]}(x)) + t \wedge s > 1$
$\Rightarrow x_{t\wedge s} \in f^{-1}(\tau_{\tilde{M}[\delta]}) \; or \; x_{t\wedge s} \in qf^{-1}(\tau_{\tilde{M}[\delta]})$
$\Rightarrow x_{t\wedge s} \in \vee q f^{-1}(\tau_{\tilde{M}[\delta]})$.
Therefore,

$$(x * y)_t, y_s \in f^{-1}(\tau_{\tilde{M}[\delta]}) \Rightarrow x_{t\wedge s} \in \vee q f^{-1}(\sigma_{\tilde{M}[\delta]}) \tag{26}$$

Hence (25) and (26) $\Rightarrow f^{-1}(\tilde{M}[\delta]) = f^{-1}(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}) = (f^{-1}\sigma_{\tilde{M}[\delta]}, f^{-1}\tau_{\tilde{M}[\delta]})$ is an $(\in, \in \vee q)$-AIFSI of X. ■

Theorem 38. Let X and X' be two BG-algebras and $f : X \to X'$ be an onto homomorphism. If $\tilde{M}[\alpha] = \{(x, \sigma_{\tilde{M}[\alpha]}(x), \tau_{\tilde{M}[\alpha]}(x)) : x \in X \; and \; \alpha \in A\}$ is an intuitionistic fuzzy soft subset of X' such that $f^{-1}(\tilde{M}[\alpha])$ is an $(\in, \in \vee q)$-AIFSI of X, then $\tilde{M}[\alpha]$ is also an $(\in, \in \vee q)$-AIFSI of X.

Proof. Let $x', y' \in X' \; and \; \alpha \in A$ such that $(x' * y')_t, y'_s \in \tilde{M}[\alpha] = (\sigma_{\tilde{M}[\alpha]}(x), \tau_{\tilde{M}[\alpha]}(x))$, where $t, s \in [0, 1]$.
$\Rightarrow (x' * y')_t, y'_s \in \sigma_{\tilde{M}[\alpha]} \; and \; (x' * y')_t, y'_s \in \tau_{\tilde{M}[\alpha]}$. Then $\sigma_{\tilde{M}[\alpha]}(x' * y') \leq t, \sigma_{\tilde{M}[\alpha]}(y') \leq s \; and \; \tau_{\tilde{M}[\alpha]}(x' * y') \geq t, \tau_{\tilde{M}[\alpha]}(y') \geq s$.
Since f is onto, so there exists $x, y \in X$ such that $f(x) = x', f(y) = y'$, also f is homomorphism so $f(x * y) = f(x) * f(y) = x' * y'$.
Now $(x' * y')_t, y'_s \in \sigma_{\tilde{M}[\alpha]}$
$\Rightarrow \sigma_{\tilde{M}[\alpha]}(f(x * y)) \leq t \; and \; \sigma_{\tilde{M}[\alpha]}(f(y)) \leq s$
$\Rightarrow f^{-1}(\sigma_{\tilde{M}[\alpha]})(x * y) \leq t \; and \; f^{-1}(\sigma_{\tilde{M}[\alpha]})(y) \leq s$
$\Rightarrow (x * y)_t \in f^{-1}(\sigma_{\tilde{M}[\alpha]}) \; and \; y_s \in f^{-1}(\sigma_{\tilde{M}[\alpha]})$
$\Rightarrow x_{t\vee s} \in \vee q f^{-1}(\sigma_{\tilde{M}[\alpha]})$
$\Rightarrow f^{-1}(\sigma_{\tilde{M}[\alpha]})(x) \leq t \vee s \; or \; f^{-1}(\sigma_{\tilde{M}[\alpha]})(x) + t \vee s < 1$
$\Rightarrow \sigma_{\tilde{M}[\alpha]}(f(x)) \leq t \vee s \; or \; \sigma_{\tilde{M}[\alpha]}(f(x)) + t \vee s < 1$
$\Rightarrow \sigma_{\tilde{M}[\alpha]}(x') \leq t \vee s \; or \; \sigma_{\tilde{M}[\alpha]}(x') + t \vee s < 1$
$\Rightarrow x'_{t\vee s} \in \vee q \sigma_{\tilde{M}[\alpha]}$.
Therefore,

$$(x' * y')_t, y'_s \in \sigma_{\tilde{M}[\alpha]} \Rightarrow x'_{t\vee s} \in \vee q \sigma_{\tilde{M}[\alpha]} \tag{27}$$

Again $(x' * y')_t, y'_s \in \tau_{\tilde{M}[\alpha]}$
$\Rightarrow \tau_{\tilde{M}[\alpha]}(f(x * y)) \geq t \; and \; \tau_{\tilde{M}[\alpha]}(f(y)) \geq s$
$\Rightarrow f^{-1}(\tau_{\tilde{M}[\alpha]})(x * y) \geq t \; and \; f^{-1}(\tau_{\tilde{M}[\alpha]})(y) \geq s$
$\Rightarrow (x * y)_t \in f^{-1}(\tau_{\tilde{M}[\alpha]}) \; and \; y_t \in f^{-1}(\tau_{\tilde{M}[\alpha]})$
$\Rightarrow x_{t\wedge s} \in \vee q f^{-1}(\tau_{\tilde{M}[\alpha]})$
$\Rightarrow f^{-1}(\tau_{\tilde{M}[\alpha]})(x) \geq t \wedge s \; or \; f^{-1}(\tau_{\tilde{M}[\alpha]})(x) + t \wedge s > 1$

$\Rightarrow \tau_{\tilde{M}[\alpha]}(f(x)) \geq t \vee s$ or $\tau_{\tilde{M}[\alpha]}(f(x)) + t \wedge s > 1$
$\Rightarrow \tau_{\tilde{M}[\alpha]}(x') \geq t \wedge s$ or $\tau_{\tilde{M}[\alpha]}(x') + t \wedge s > 1$
$\Rightarrow x'_{t \wedge s} \in \vee q \tau_{\tilde{M}[\alpha]}$.
Therefore,

$$(x' * y')_t, y'_s \in \tau_{\tilde{M}[\alpha]} \Rightarrow x'_{t \wedge s} \in \vee q \tau_{\tilde{M}[\alpha]} \tag{28}$$

Hence (27) and (28) $\Rightarrow \tilde{M}[\alpha]$ is also an $(\in, \in \vee q)$-AIFSI of X. ■

GENERALIZATIONS OF $(\in, \in \vee q)$-ANTI INTUITIONISTIC FUZZY SOFT SUBALGEBRAS OF BG-ALGEBRAS

Generalizations of $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebras of BG-algebras are defined and some important properties are presented in this section.

Definition 39. A fuzzy soft point (x, t) is said to belong to a fuzzy soft set $\sigma_{\tilde{M}[\delta]}$ written as $(x, t) \in \vee q \sigma_{\tilde{M}[\delta]}$ or $(x, t) q \sigma_{\tilde{M}[\delta]}$.

Let k denote an arbitrary element of [0, 1) unless otherwise specified. To say that $(x, t) q_k \sigma_{\tilde{M}[\delta]}$, we mean $\sigma_{\tilde{M}[\delta]}(x) + t + k > 1$. To say that $(x, t) \in \vee q_k \sigma_{\tilde{M}[\delta]}$, we mean $(x, t) \in \sigma_{\tilde{M}[\delta]}$ or $\sigma_{\tilde{M}[\delta]} q_k \sigma_{\tilde{M}[\delta]}$.

Definition 40. A fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]})(x) : x \in X \; and \; \delta \in A\}$ in a BG-algebra X is said to be an $(\in, \in \vee q_k)$-AIFSS of X if for every $x, y \in X, \delta \in A$ and $t, s \in (0, 1]$ satisfies the following conditions:
$(x, t), (y, s) \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t \vee s) \in q_k \sigma_{\tilde{M}[\delta]}$, where $t \vee s = max\{t, s\}$.

Definition 41. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ in a BG-algebra X is said to be an $(\in, \in \vee q_k)$-AIFSS of X if for every $x, y \in X, \delta \in A$ and $t, s \in (0, 1]$ satisfies the following conditions:

(i) $(x, t), (y, s) \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t \vee s) \in \vee q_k \sigma_{\tilde{M}[\delta]}$, where $t \vee s = max\{t, s\}$,
(ii) $(x, t), (y, s) \in \tau_{\tilde{M}[\delta]} \Rightarrow (x * y, t \wedge s) \in \vee q_k \tau_{\tilde{M}[\delta]}$, where $t \wedge s = min\{t, s\}$.

We give characterizations of an $(\in, \in \vee q_k)$-anti intuitionistic fuzzy soft subalgebra.

Theorem 42. An intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ of a BG-algebra X is an $(\in, \in \vee q_k)$-AIFSS of X if and only if for any $x, y \in X, \; and \; \delta \in A$ satisfies the following conditions:

(i) $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$,
(ii) $\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}$.

Proof. (i) First, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \; and \; \delta \in A\}$ be an $(\in, \in \vee q_k)$-AIFSS of X.

Case 1. Let $\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) > \frac{1-k}{2}$ for all $x, y \in X \; and \; \delta \in A$. Then
$\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$.
If possible, let $\sigma_{\tilde{M}[\delta]}(x * y) > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$. Choose a real number t such that $\sigma_{\tilde{M}[\delta]}(x * y) > t > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x) < t, \sigma_{\tilde{M}[\delta]}(y) < t$ for some $t \in (0, 1)$.
It follows that $(x, t) \in \sigma_{\tilde{M}[\delta]}, (y, t) \in \sigma_{\tilde{M}[\delta]}$.
But $\sigma_{\tilde{M}[\delta]}(x * y) > t$
$\Rightarrow (x * y, t) \overline{\in} \sigma_{\tilde{M}[\delta]}$ and $\sigma_{\tilde{M}[\delta]}(x * y) + t > 2t$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t > 2(\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y)) > 2 \times \frac{1-k}{2} = 1 - k$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 - k \Rightarrow (x * y, t) \overline{q_k} \sigma_{\tilde{M}[\delta]}$. Hence $(x * y, t) \overline{\in \vee q_k} \sigma_{\tilde{M}[\delta]}$, which contradicts the fact that $\tilde{M}[\delta]$ is

an $(\in, \in \vee q_k)$-AIFSS of X.
Therefore, $\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$.

Case 2. Let $\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \leq \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$. Then
$\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) = \frac{1-k}{2}$.
If possible, let $\sigma_{\tilde{M}[\delta]}(x * y) > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} = \frac{1-k}{2}$. Then
$\sigma_{\tilde{M}[\delta]}(x) \leq \frac{1-k}{2}$ and $\sigma_{\tilde{M}[\delta]}(y) \leq \frac{1-k}{2}$. Thus $(x, \frac{1-k}{2}) \in \sigma_{\tilde{M}[\delta]}, (y, \frac{1-k}{2}) \in \sigma_{\tilde{M}[\delta]}$. But $(x * y, \frac{1-k}{2}) \overline{\in} \sigma_{\tilde{M}[\delta]}$. Also $\sigma_{\tilde{M}[\delta]}(x * y) + \frac{1-k}{2} > \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$,
i.e., $(x * y, \frac{1-k}{2}) \overline{q_k} \sigma_{\tilde{M}[\delta]}$. Hence $(x * y, \frac{1-k}{2}) \overline{\in \vee q_k} \sigma_{\tilde{M}[\delta]}$, which again a contradiction that $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X.
Therefore, $\sigma_{\tilde{M}[\delta]}(x * y) \leq \frac{1-k}{2} = \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$.

Converse part:
Let

$$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}. \tag{29}$$

Let $x, y \in X,\ \delta \in A$ *and* $t, s \in (0, 1]$ such that $(x, t), (y, t) \in \sigma_{\tilde{M}[\delta]}$. Then $\sigma_{\tilde{M}[\delta]}(x) \leq t$ *and* $\sigma_{\tilde{M}[\delta]}(y) \leq s$.
Therefore, $\sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \leq t \vee s$. By Eq. (29), we have $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \vee \frac{1-k}{2}$.
Now, if $t \vee s \geq \frac{1-k}{2}$, then $t \vee s \vee \frac{1-k}{2} = t \vee s$.
Therefore, $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s$

$$\Rightarrow (x * y, t \vee s) \in \sigma_{\tilde{M}[\delta]}. \tag{30}$$

Again, if $t \vee s > \frac{1-k}{2}$, then $t \vee s \vee \frac{1-k}{2} = \frac{1-k}{2}$.
Therefore, $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \vee \frac{1-k}{2} = \frac{1-k}{2}$
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t \vee s < \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$

$$\Rightarrow (x * y, t \vee s) q_k \sigma_{\tilde{M}[\delta]}. \tag{31}$$

From Eqs. (30) and (31), we have

$$(x, t), (y, t) \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t \vee s) \in \vee q_k \sigma_{\tilde{M}[\delta]}. \tag{32}$$

Therefore, $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X.

(ii) First, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an $(\in, \in \vee q_k)$-AIFSS of X.

Case 1. Let $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) < \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$. Then
$\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2} = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$.
If possible, let $\tau_{\tilde{M}[\delta]}(x * y) < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$. Choose a real number t such that
$\tau_{\tilde{M}[\delta]}(x * y) < t < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x) > t, \tau_{\tilde{M}[\delta]}(y) > t$ for some $t \in (0, 1)$.
It follows that $(x, t) \in \tau_{\tilde{M}[\delta]}, (y, t) \in \tau_{\tilde{M}[\delta]}$.
But $\tau_{\tilde{M}[\delta]}(x * y) < t$
$\Rightarrow (x * y, t) \overline{\in} \tau_{\tilde{M}[\delta]}$ *and* $\tau_{\tilde{M}[\delta]}(x * y) + t < 2t$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + t < 2(\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)) < 2 \times \frac{1-k}{2} = 1 - k$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + t < 1 - k \Rightarrow (x * y, t) \overline{q_k} \tau_{\tilde{M}[\delta]}$. Hence $(x * y, t) \overline{\in \vee q_k} \tau_{\tilde{M}[\delta]}$, which contradicts the fact that $\tilde{M}[\delta]$ is

an $(\in, \in \vee q_k)$-AIFSS of X.
Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}$.

Case 2. Let $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \geq \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$. Then $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) = \frac{1-k}{2}$.
If possible, let $\tau_{\tilde{M}[\delta]}(x * y) < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2} = \frac{1-k}{2}$. Then
$\tau_{\tilde{M}[\delta]}(x) \geq \frac{1-k}{2}$ and $\tau_{\tilde{M}[\delta]}(y) \geq \frac{1-k}{2}$.
Thus $(x, \frac{1-k}{2}) \in \tau_{\tilde{M}[\delta]}, (y, \frac{1-k}{2}) \in \tau_{\tilde{M}[\delta]}$.
But $(x * y, \frac{1-k}{2})\overline{\in}\tau_{\tilde{M}[\delta]}$. Also $\tau_{\tilde{M}[\delta]}(x * y) + \frac{1-k}{2} < \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$,
i.e., $(x * y, \frac{1-k}{2})\overline{q_k}\tau_{\tilde{M}[\delta]}$. Hence $(x * y, \frac{1-k}{2})\overline{\in \vee q_k}\tau_{\tilde{M}[\delta]}$, which again a contradiction that $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X.
Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq \frac{1-k}{2} = \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}$.

Converse part:
Let

$$\tau_{\tilde{M}[\delta]}(x * y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}. \tag{33}$$

Let $x, y \in X, \delta \in A$ *and* $t, s \in (0, 1]$ such that $(x, t), (y, t) \in \tau_{\tilde{M}[\delta]}$. Then $\tau_{\tilde{M}[\delta]}(x) \geq t$ and $\tau_{\tilde{M}[\delta]}(y) \geq s$.
Therefore, $\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \geq t \wedge s$. By Eq. (33), we have $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \wedge \frac{1-k}{2}$.
Now, if $t \wedge s \leq \frac{1-k}{2}$, then $t \wedge s \wedge \frac{1-k}{2} = t \wedge s$.
Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s$

$$\Rightarrow (x * y, t \wedge s) \in \tau_{\tilde{M}[\delta]}. \tag{34}$$

Again, if $t \wedge s > \frac{1-k}{2}$, then $t \wedge s \wedge \frac{1-k}{2} = \frac{1-k}{2}$.
Therefore, $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \wedge \frac{1-k}{2} = \frac{1-k}{2}$
$\Rightarrow \tau_{\tilde{M}[\delta]}(x * y) + t \wedge s > \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$

$$\Rightarrow (x * y, t \wedge s)q_k\tau_{\tilde{M}[\delta]}. \tag{35}$$

From Eqs. (34) and (35), we have

$$(x, t), (y, t) \in \tau_{\tilde{M}[\delta]} \Rightarrow (x * y, t \wedge s) \in \vee q_k\tau_{\tilde{M}[\delta]}. \tag{36}$$

Therefore, $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X.
Hence (32) and (36) $\Rightarrow \tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ *and* $\delta \in A\}$ is an $(\in, \in \vee q_k)$-AIFSS of X. ■

Theorem 43. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ *and* $\delta \in A\}$ be an intuitionistic fuzzy set in X if and only if the level subset $U(\tilde{M}[\delta]; t) := \{x \in X : \sigma_{\tilde{M}[\delta]} \leq t$ *and* $\tau_{\tilde{M}[\delta]} \geq t\}$ is a subalgebra of X for all $t \in (0, \frac{1-k}{2})$.

Proof. Assume that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ *and* $\delta \in A\}$ is an $(\in, \in \vee q_k)$-AIFSS of X.
(i) Let $t \in (0, \frac{1-k}{2})$ and $x, y \in U(\tilde{M}[\delta]; t)$. Then $\sigma_{\tilde{M}[\delta]}(x) \leq t$ *and* $\tau_{\tilde{M}[\delta]}(x) \leq t\}$. It follows from (29) that
$\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee t \vee \frac{1-k}{2} = t$
so that $x * y \in U(\tilde{M}[\delta]; t)$. Hence $U(\tilde{M}[\delta]; t)$ is a subalgebra of X.

Conversely, suppose that $U(\tilde{M}[\delta]; t)$ is a subalgebra of X for all $t \in (0, \frac{1-k}{2})$. If (29) is not valid, then there exist $x, y \in X$ *and* $\delta \in A$ such that
$\sigma_{\tilde{M}[\delta]}(x * y) > \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$.

Hence we can that $t \in (0,1)$ such that
$\sigma_{\tilde{M}[\delta]}(x * y) > t \geq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$.
Then $t \in (0, \frac{1-k}{2})$ and $x, y \in U(\tilde{M}[\delta]; t)$. Since $U(\tilde{M}[\delta]; t)$ is a subalgebra of X, it follows that $x * y \in U(\tilde{M}[\delta]; t)$ so that $\sigma_{\tilde{M}[\delta]}(x * y) \geq t$. This is contradiction. Therefore (29) is valid, and $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X.

(ii) Let $t \in (0, \frac{1-k}{2})$ and $x, y \in U(\tilde{M}[\delta]; t)$. Then $\tau_{\tilde{M}[\delta]}(x) \geq t$ *and* $\tau_{\tilde{M}[\delta]}(x) \geq t\}$. It follows from (33) that
$\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge t \wedge \frac{1-k}{2} = t$
so that $x * y \in U(\tilde{M}[\delta]; t)$. Hence $U(\tilde{M}[\delta]; t)$ is a subalgebra of X.

Conversely, suppose that $U(\tilde{M}[\delta]; t)$ is a subalgebra of X for all $t \in (0, \frac{1-k}{2})$. If (33) is not valid, then there exist $x, y \in X$ *and* $\delta \in A$ such that
$\tau_{\tilde{M}[\delta]}(x * y) < \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}$.
Hence we can that $t \in (0,1)$ such that
$\tau_{\tilde{M}[\delta]}(x * y) < t \leq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y) \wedge \frac{1-k}{2}$.
Then $t \in (0, \frac{1-k}{2})$ and $x, y \in U(\tilde{M}[\delta]; t)$. Since $U(\tilde{M}[\delta]; t)$ is a subalgebra of X, it follows that $x * y \in U(\tilde{M}[\delta]; t)$ so that $\tau_{\tilde{M}[\delta]}(x * y) \leq t$. This is contradiction. Therefore (33) is valid, and $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q_k)$-AIFSS of X. ■

Theorem 44. An Anti-intuitionistic fuzzy soft subset $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ of a BG-algebra X is an $(\in, \in \vee q_k)$-AIFSS of X and if $\sigma_{\tilde{M}[\delta]}(x) > \frac{1-k}{2}, \tau_{\tilde{M}[\delta]}(x) < \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$, then $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is also an $(\in, \in)$-AIFSS of X.

Proof. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an $(\in, \in \vee q_k)$-AIFSS of X and $\sigma_{\tilde{M}[\delta]}(x) > \frac{1-k}{2}$ and $\tau_{\tilde{M}[\delta]}(x) < \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$. Let $(x,t) \in \sigma_{\tilde{M}[\delta]}, (y,s) \in \sigma_{\tilde{M}[\delta]}$. Then we have
$\frac{1-k}{2} < \sigma_{\tilde{M}[\delta]}(x) \leq t \text{ and } \frac{1-k}{2} < \sigma_{\tilde{M}[\delta]}(y) \leq s$.
Therefore, $t \vee s > \frac{1-k}{2}$. Also $\sigma_{\tilde{M}[\delta]}(x * y) > \frac{1-k}{2}$. Thus, $\sigma_{\tilde{M}[\delta]}(x * y) + t \vee s > \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$.

Since $\sigma_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X, we have
either $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \text{ or } \sigma_{\tilde{M}[\delta]}(x * y) + t \vee s < 1 - k$.
So we must have, $\sigma_{\tilde{M}[\delta]}(x * y) \leq t \vee s \Rightarrow (x * y, t \vee s) \in \sigma_{\tilde{M}[\delta]}$. Therefore,

$$(x,t) \in \sigma_{\tilde{M}[\delta]}, (y,s) \in \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t \vee s) \in \sigma_{\tilde{M}[\delta]}. \tag{37}$$

Thus, $\sigma_{\tilde{M}[\delta]}$ is $(\in, \in)$-AIFSS of X.

Again, let $(x,t) \in \tau_{\tilde{M}[\delta]}, (y,s) \in \tau_{\tilde{M}[\delta]}$. Then we have
$t \leq \tau_{\tilde{M}[\delta]}(x) < \frac{1-k}{2} \text{ and } s \leq \tau_{\tilde{M}[\delta]}(y) < \frac{1-k}{2}$.
Therefore, $t \wedge s < \frac{1-k}{2}$ and also $\tau_{\tilde{M}[\delta]}(x * y) < \frac{1-k}{2}$. Thus, $\tau_{\tilde{M}[\delta]}(x * y) + t \wedge s < \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$.

Since $\tau_{\tilde{M}[\delta]}$ is an $(\in, \in \vee q)$-AIFSS of X, we have
either $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \text{ or } \tau_{\tilde{M}[\delta]}(x * y) + t \wedge s > 1 - k$.
So we must have, $\tau_{\tilde{M}[\delta]}(x * y) \geq t \wedge s \Rightarrow (x * y, t \wedge s) \in \tau_{\tilde{M}[\delta]}$. Therefore,

$$(x,t) \in \tau_{\tilde{M}[\delta]}, (y,s) \in \tau_{\tilde{M}[\delta]} \Rightarrow (x * y, t \wedge s) \in \tau_{\tilde{M}[\delta]}. \tag{38}$$

Thus, $\tau_{\tilde{M}[\delta]}$ is $(\in, \in)$-AIFSS of X.
Hence (37) and (38) $\Rightarrow \tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is also an $(\in, \in)$-AIFSS of X. ■

Theorem 45. Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ be an intuitionistic fuzzy soft subset of BG-algebra X. Then $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an $(\in, \in \vee q_k)$-AIFSS of X if and only if $[\sigma_{\tilde{M}[\delta]}]_t$ and $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X for all $t \in (0,1]$. We call $[\sigma_{\tilde{M}[\delta]}]_t$ and $[\tau_{\tilde{M}[\delta]}]_t$ as an $(\in \vee q_k)$-level subalgebras of X.

Proof. Assume that $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an $(\in, \in \vee q_k)$ -AIFSS of X, to prove $[\sigma_{\tilde{M}[\delta]}]_t$ and $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X. Let $x, y \in [\sigma_{\tilde{M}[\delta]}]_t$ for $t \in (0,1]$. Then $(x,t), (y,t) \in q_k \sigma_{\tilde{M}[\delta]}$,

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i.e.,$\sigma_{\tilde{M}[\delta]}(x) \leq t \ or \ \sigma_{\tilde{M}[\delta]}(x) + t < 1$ and $\sigma_{\tilde{M}[\delta]}(y) \leq t \ or \ \sigma_{\tilde{M}[\delta]}(y) + t < 1$.
Since $\tilde{M}[\delta]$ is an $(\in, \in \vee q_k)$-AIFSS of X, we have
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2}$ for any $x, y \in X$ and $\delta \in A$.
Now we have the following cases.

Case 1. $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) \leq t, \ let \ t < \frac{1-k}{2}$. Then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} = t \vee t \vee \frac{1-k}{2} = \frac{1-k}{2}$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq \frac{1-k}{2} \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < \frac{1-k}{2} + \frac{1-k}{2} = 1 - k \Rightarrow (x * y, t) q_k \sigma_{\tilde{M}[\delta]}$.
Again if $t \geq \frac{1-k}{2}$, then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} \leq t \vee t \vee \frac{1-k}{2} = t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y, t) \in \sigma_{\tilde{M}[\delta]}$.
Hence $(x * y, t) \in \vee q_k \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t) \in [\sigma_{\tilde{M}[\delta]}]_t$.

Case 2. $\sigma_{\tilde{M}[\delta]}(x) \leq t, \sigma_{\tilde{M}[\delta]}(y) + t < 1 - k, \ let \ t < \frac{1-k}{2}$. Then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} < t \vee (1 - k - t) \vee \frac{1-k}{2} = 1 - k - t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - k - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 - k \Rightarrow (x * y, t) q_k \sigma_{\tilde{M}[\delta]}$.
Again if $t \geq \frac{1-k}{2}$, then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} \leq t \vee (1 - k - t) \vee \frac{1-k}{2} = t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y, t) \in \sigma_{\tilde{M}[\delta]}$.
Hence $(x * y, t) \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t) \in [\sigma_{\tilde{M}[\delta]}]_t$.

Case 3. $\sigma_{\tilde{M}[\delta]}(x) + t < 1 - k, \sigma_{\tilde{M}[\delta]}(y) \leq t, \ let \ t < \frac{1-k}{2}$. Then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} < (1 - k - t) \vee t \vee \frac{1-k}{2} = 1 - k - t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - k - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 - k \Rightarrow (x * y, t) q_k \sigma_{\tilde{M}[\delta]}$.
Again if $t \geq \frac{1-k}{2}$, then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} = (1 - k - t) \vee t \vee \frac{1-k}{2} = t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y, t) \in \sigma_{\tilde{M}[\delta]}$.
Hence $(x * y, t) \in \vee q \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t) \in [\sigma_{\tilde{M}[\delta]}]_t$,

Case 4. $\sigma_{\tilde{M}[\delta]}(x) + t < 1 - k, \sigma_{\tilde{M}[\delta]}(y) + t < 1 - k, \ let \ t < \frac{1-k}{2}$. Then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} < (1 - k - t) \vee (1 - k - t) \vee \frac{1-k}{2} = 1 - k - t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) < 1 - k - t \Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) + t < 1 - k \Rightarrow (x * y, t) q_k \sigma_{\tilde{M}[\delta]}$.
Again if $t \geq \frac{1-k}{2}$, then
$\sigma_{\tilde{M}[\delta]}(x * y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(y) \vee \frac{1-k}{2} = (1 - k - t) \vee (1 - k - t) \vee \frac{1-k}{2} = \frac{1-k}{2} \leq t$,
$\Rightarrow \sigma_{\tilde{M}[\delta]}(x * y) \leq t \Rightarrow (x * y, t) \in \sigma_{\tilde{M}[\delta]}$.
Hence $(x * y, t) \in \vee q_k \sigma_{\tilde{M}[\delta]} \Rightarrow (x * y, t) \in [\sigma_{\tilde{M}[\delta]}]_t$.
Hence from above four cases $x, y \in [\sigma_{\tilde{M}[\delta]}]_t \Rightarrow (x * y, t) \in [\sigma_{\tilde{M}[\delta]}]_t$.
Hence $[\sigma_{\tilde{M}[\delta]}]_t$ is a subalgebra of X. Similarly, we can prove $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X.

Conversely, let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ be an AIFSS in X such that $[\sigma_{\tilde{M}[\delta]}]_t$ and $[\tau_{\tilde{M}[\delta]}]_t$ is a subalgebra of X for all $t \in (0, 1]$, to prove $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is an $(\in, \in \vee q_k)$-AIFSS of X. Suppose $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \ and \ \delta \in A\}$ is not an $(\in, \in \vee q_k)$-AIFSS of X, then there exists $a, b \in X$ such that at least one of $\sigma_{\tilde{M}[\delta]}(a * b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee \frac{1-k}{2}$ and $\tau_{\tilde{M}[\delta]}(a * b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge \frac{1-k}{2}$ hold. Suppose $\sigma_{\tilde{M}[\delta]}(a * b) > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee \frac{1-k}{2}$ is true, then choose $t \in (0, 1]$ such that

$$\sigma_{\tilde{M}[\delta]}(a * b) > t > \sigma_{\tilde{M}[\delta]}(a) \vee \sigma_{\tilde{M}[\delta]}(b) \vee \frac{1-k}{2} \tag{39}$$

Then $\sigma_{\tilde{M}[\delta]}(a) < t, \sigma_{\tilde{M}[\delta]}(b) < t \Rightarrow a, b \in (\sigma_{\tilde{M}[\delta]})_t \subset [\sigma_{\tilde{M}[\delta]}]_t$ which is a subalgebra.
Therefore, $(a * b) \in [\sigma_{\tilde{M}[\delta]}]_t \Rightarrow \sigma_{\tilde{M}[\delta]}(a * b) \leq t \ or \ \sigma_{\tilde{M}[\delta]}(a * b) + t < 1 - k$ which contradict (39).

Again, if $\tau_{\tilde{M}[\delta]}(a*b) < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge \frac{1-k}{2}$ is true, then choose $t \in (0, 1]$ such that

$$\tau_{\tilde{M}[\delta]}(a*b) < t < \tau_{\tilde{M}[\delta]}(a) \wedge \tau_{\tilde{M}[\delta]}(b) \wedge \frac{1-k}{2} \qquad (40)$$

Then $\tau_{\tilde{M}[\delta]}(a) > t, \tau_{\tilde{M}[\delta]}(b) > t \Rightarrow x, y \in (\tau_{\tilde{M}[\delta]})_t \subset [\tau_{\tilde{M}[\delta]}]_t$ which is a subalgebra.
Therefore, $(a*b) \in [\tau_{\tilde{M}[\delta]}]_t \Rightarrow \tau_{\tilde{M}[\delta]}(a*b) \geq t$ *or* $\tau_{\tilde{M}[\delta]}(a*b) + t > 1 - k$ which contradict (40).
Hence we must have

$\sigma_{\tilde{M}[\delta]}(x*y) \leq \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{M}[\delta]}(x) \vee \frac{1-k}{2}$

$\tau_{\tilde{M}[\delta]}(x*y) \geq \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(x) \wedge \frac{1-k}{2}$ for all $x, y \in X$ *and* $\delta \in A$.
Hence $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}$ is an $(\in, \in \vee q_k)$-AIFSS of X. ■

CONCLUSION

In this paper, we introduce the notions of $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebras of BG-algebra and investigate some of their usual properties. We can introduce $(\in, \in \vee q)$-anti intuitionistic fuzzy soft ideals. We also characterize the $(\in, \in \vee q_k)$-anti intuitionistic fuzzy soft subalgebra of BG-algebra which is a generalization of $(\in, \in \vee q)$-anti intuitionistic fuzzy soft subalgebra. In the future, we may carry out translations of intuitionistic fuzzy soft subalgebras of B-algebras.

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