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Vertex switching of a cycle in context of k-cordial labeling

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Abstract: Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the conditions $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(a) - e_f(b)| \leq 1$ for all $a, b \in A$, Where $v_f(a)$ and $e_f(a)$ denote the number of vertices with label a and the number of edges with label a respectively, when the edge $e = uv$ is labeled as $f(u) * f(v)$.In this paper we prove that the graph obtained by switching of a vertex in cycle is k-cordial for all odd k.

AMS Subject classification:05C78.
Keywords: Abelian Group; k-Cordial Labeling; Vertex Switching.

INTRODUCTION

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

The latest updates of various graph labeling techniques can be found in Gallian [1].

Definition 2. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where
$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

k-cordial labeling of a graph was introduced by M. Hovey [2]. He proved that All the connected graphs are 3-cordial, All the trees are $3, 4, 5$-cordial, Cycles are k-cordial for all odd k. Youssef [3] have derived the condition for a graph to admit k-cordial labeling. He proved that the *complete graph* K_n is 4-cordial $\iff n \leq 6$, the *complete bipartite graph* $K_{m,n}$ is 4-cordial $\iff m$ *or* $n \ncong 2(mod 4)$, the graph C_n^2 is 4-cordial $\iff n \ncong 2(mod 4)$. Kanani and Modha [4] proved that all the Fans f_n are k-cordial for all k. Kanani and Modha [5] proved that bistar graph $B_{m,n}$ is k-cordial for all k, restricted square graph $B^2_{n,n}$ of bistar is k-cordial for all odd k. They also prove that one point union of cycle C_3 with star graph $K_{1,n}$ and comb graph $P_n \bigodot K_1$ are k-cordial for all k.

Terms not defined here are used in the sense of Harary [6].

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Definition 3. A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all edges incident to v and adding edges joining v to every vertex not adjacent to v in G.

MAIN RESULTS

Theorem 4. Vertex switching of cycle C_n is 3-cordial.

Proof. Let $G = C_n$ and $v_1, v_2, v_3, \ldots, v_n$ be successive vertices of C_n. G_v denotes the vertex switching of G with respect to vertex v of G. Here we note that in each of the following cases the labeling pattern starts from the switching vertex which is denoted by v_1. It is noted that $|V(G)| = n$ and $|E(G)| = 2n - 5$.

The vertex labeling $f : V(G) \to Z_3$ is defined as follows.

Case 1. $n \equiv 0, 2(mod\ 3)$
$f(v_1) = 0;$
$f(v_i) = 2; \quad i \equiv 2(mod\ 3);$
$f(v_i) = 1; \quad i \equiv 0(mod\ 3);$
$f(v_i) = 0; \quad i \equiv 1(mod\ 3); \quad 2 \le i \le n.$

Case 2. $n \equiv 1(mod\ 3)$
$f(v_1) = 0;$
$f(v_i) = 2; \quad i \equiv 2(mod\ 3);$
$f(v_i) = 0; \quad i \equiv 0(mod\ 3);$
$f(v_i) = 1; \quad i \equiv 1(mod\ 3); \quad 2 \le i \le n.$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for 3-cordial labeling. Hence, vertex switching of cycle C_n is 3-cordial.

Illustration 2.2. The switching vertex of the cycle C_8 and its 3-cordial labeling is shown in *Figure 1*.

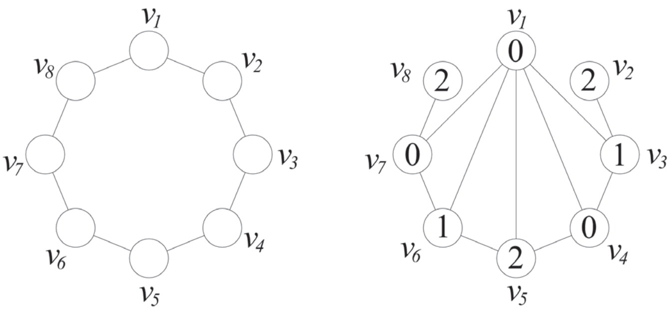

FIGURE 1. 3-Cordial labeling of the switching vertex of the cycle C_8.

Theorem 5. Vertex switching of cycle C_n is k-cordial for all odd k.

Proof. Let $G = C_n$ and $v_1, v_2, v_3, \ldots, v_n$ be successive vertices of C_n. G_v denotes the vertex switching of G with respect to vertex v of G. Here we note that in each of the following cases the labeling pattern starts from the switching vertex which is denoted by v_1. It is noted that $|V(G)| = n$ and $|E(G)| = 2n - 5$. Let $n = mk + j, 0 \le j \le k - 1$.

The vertex labeling $f : V(G) \to Z_k$ is defined as follows.

Case 1. $\frac{k+1}{2}$ is odd.

Subcase I. $m = 0$.

Subsubcase 1. $4 \le j \le \frac{k+3}{2}$.

$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = \frac{k+i}{2} - 1; \quad i$ is odd,
$= \frac{k-5}{4} + \frac{i}{2}; \quad i$ is even, $3 \le i \le j$.

Subsubcase 2. if $k > 5$ and $\frac{k+5}{2} \le j \le k - 1$.

$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = \frac{k+i}{2} - 1; \quad i$ is odd,
$= \frac{k-l}{4} + \frac{i}{2}; \quad i$ is even, $3 \le i \le j$, $\frac{k+l-4}{2} \le j \le \frac{k+l-2}{2}$.

where $l = 9, 13, ..., k$.

Subcase II. $m \ge 1$, m is odd.
Subsubcase 1. $j = 0$.
If $k = 5$,
$f(v_1) = 0$
$f(v_2) = 1$
$f(v_i) = p_i; \quad \frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i; \quad \frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \le i \le (m-1)k + 2$.
$f(v_i) = p_i; \quad \frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i; \quad \frac{i}{2} \equiv p_i(mod\ k)$, i is even, $(m-1)k+3 \le$

$i \leq mk$.
If $k > 5$,
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq (m-1)k + 2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} \equiv p_i(mod\ k)$, i is even, $(m-1)k+3 \leq i \leq mk - 1$.
$f(v_{mk}) = 1$
Subsubcase 2. $j = 1$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 1$.
Subsubcase 3. $j = 2$.
$f(v_1) = 0$
$f(v_i) = p_i$; $\frac{k+i-1}{2} \equiv p_i(mod\ k)$, i is even,
$= p_i$; $\frac{i-1}{2} \equiv p_i(mod\ k)$, i is odd, $2 \leq i \leq mk + 2$.
Subsubcase 4. $3 \leq j \leq \frac{k+3}{2}$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 2$.
$f(v_i) = \frac{k-5}{4} + p_i$; $\frac{k+i}{2} \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $mk+3 \leq i \leq mk + j$.
Subsubcase 5. If $k > 5$ and $\frac{k+5}{2} \leq j \leq k - 1$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 2$.
$f(v_i) = \frac{k-l}{4} + p_i$; $\frac{k+i}{2} \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $mk+3 \leq i \leq mk + j$.
where $l = 9, ..., k$, $\frac{k+l-4}{2} \leq j \leq \frac{k+l-2}{2}$.
Subcase III. $m \geq 1$, m is even.
Subsubcase 1. $j = 0$.
If $k = 5$,
$f(v_1) = 0$
$f(v_2) = 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq (m-1)k + 2$.
$f(v_i) = p_i$; $\frac{k+i}{2} \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even,
$(m-1)k + 3 \leq i \leq mk$.
If $k > 5$,
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq (m-1)k + 2$.
$f(v_i) = p_i$; $\frac{k+i}{2} \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even,
$(m-1)k + 3 \leq i \leq mk - 1$
$f(v_{mk}) = 1$
Subsubcase 2. $j = 1$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 1$.
Subsubcase 3. $j = 2$.
$f(v_1) = 0$
$f(v_i) = p_i$; $\frac{k+i-1}{2} \equiv p_i(mod\ k)$, i is even,
$= p_i$; $\frac{i-1}{2} \equiv p_i(mod\ k)$, i is odd, $2 \leq i \leq mk + 2$.
Subsubcase 4. $3 \leq j \leq \frac{k+3}{2}$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= \frac{k-5}{4} + p_i$; $\frac{i}{2} \equiv p_i(mod\ k)$, i is even, $mk + 3 \leq i \leq mk + j$.
Subsubcase 5. If $k > 5$ and $\frac{k+5}{2} \leq j \leq k - 1$.
$f(v_1) = 0$
$f(v_2) = k - 1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i(mod\ k)$, i is even, $3 \leq i \leq mk + 2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i(mod\ k)$, i is odd,
$= \frac{k-l}{4} + p_i$; $\frac{i}{2} \equiv p_i(mod\ k)$, i is even, $mk+3 \leq i \leq mk + j$.
where $l = 9, ..., k$, $\frac{k+l-4}{2} \leq j \leq \frac{k+l-2}{2}$.

Case 2. $\frac{k+1}{2}$ is even.
Subcase I: $m = 0$.
Subsubcase 1: $4 \leq j \leq \frac{k+5}{2}$.
$f(v_1) = 0$

$f(v_2) = k-1$
$f(v_i) = \frac{k+i}{2} - 1$; i is odd,
$= \frac{k-7}{4} + \frac{i}{2}$; i is even, $3 \leq i \leq j$.
Subsubcase 2. $\frac{k+7}{2} \leq j \leq k-1$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = \frac{k+i}{2} - 1$; i is odd,
$= \frac{k-l}{4} + \frac{i}{2}$; i is even, $3 \leq i \leq j$, $\frac{k+l-4}{2} \leq j \leq \frac{k+l-2}{2}$.
where $l = 11, 15, ..., k$.
Subcase II. $m \geq 1$, m is odd.
Subsubcase 1. $j = 0$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq (m-1)k+2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} \equiv p_i (mod\ k)$, i is even, $(m-1)k+3 \leq i \leq mk-1$.
$f(v_{mk}) = 1$
Subsubcase 2. $j = 1$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq mk+1$.
Subsubcase 3. $j = 2$.
$f(v_1) = 0$
$f(v_i) = p_i$; $\frac{k+i-1}{2} \equiv p_i (mod\ k)$, i is even,
$= p_i$; $\frac{i-1}{2} \equiv p_i (mod\ k)$, i is odd, $2 \leq i \leq mk+2$.
Subsubcase 4. $3 \leq j \leq \frac{k+5}{2}$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq mk+2$.
$f(v_i) = \frac{k-7}{4} + p_i$; $\frac{k+i}{2} \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $mk+3 \leq i \leq mk+j$.
Subsubcase 5. $\frac{k+7}{2} \leq j \leq k-1$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even,$3 \leq i \leq mk+2$.
$f(v_i) = \frac{k-l}{4} + p_i$; $\frac{k+i}{2} \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $mk+3 \leq i \leq mk+j$,
$\frac{k+l-4}{2} \leq j \leq \frac{k+l-2}{2}$, where $l = 11, 15, ..., k$.
Subcase III. $m \geq 1$, m is even.
Subsubcase 1. $j = 0$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq (m-1)k+2$.
$f(v_i) = p_i$; $\frac{k+i}{2} \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even,
$(m-1)k+3 \leq i \leq mk-1$.
$f(v_{mk}) = 1$
Subsubcase 2. $j = 1$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq mk+1$.
Subsubcase 3. $j = 2$.
$f(v_1) = 0$
$f(v_i) = p_i$; $\frac{k+i-1}{2} \equiv p_i (mod\ k)$, i is even,
$= p_i$; $\frac{i-1}{2} \equiv p_i (mod\ k)$, i is odd, $2 \leq i \leq mk+2$.
Subsubcase 4: $3 \leq j \leq \frac{k+5}{2}$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even, $3 \leq i \leq mk+2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= \frac{k-7}{4} + p_i$; $\frac{i}{2} \equiv p_i (mod\ k)$, i is even, $mk+3 \leq i \leq mk+j$.
Subsubcase 5. If $k > 7$ and $\frac{k+7}{2} \leq j \leq k-1$.
$f(v_1) = 0$
$f(v_2) = k-1$
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= p_i$; $\frac{i}{2} - 1 \equiv p_i (mod\ k)$, i is even,$3 \leq i \leq mk+2$.
$f(v_i) = p_i$; $\frac{k+i}{2} - 1 \equiv p_i (mod\ k)$, i is odd,
$= \frac{k-l}{4} + p_i$; $\frac{i}{2} \equiv p_i (mod\ k)$, i is even, $mk+3 \leq i \leq mk+j$,
$\frac{k+l-4}{2} \leq j \leq \frac{k+l-2}{2}$, where $l = 11, 15, ..., k$.

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence the switching vertex of the cycle graph G_v are k-cordial for all odd k.

Illustration 2.4.(a) The switching vertex of the cycle C_{13} and its 21-cordial labeling is shown in *Figure 2*.

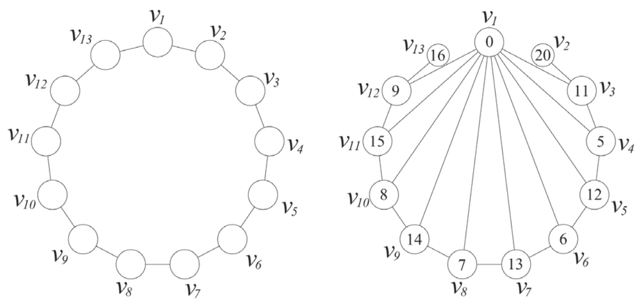

FIGURE 2. 21-Cordial labeling of the switching vertex of the cycle C_{13}.

Illustration 2.4.(b) The switching vertex of the cycle C_9 and its 7-cordial labeling is shown in *Figure 3*.

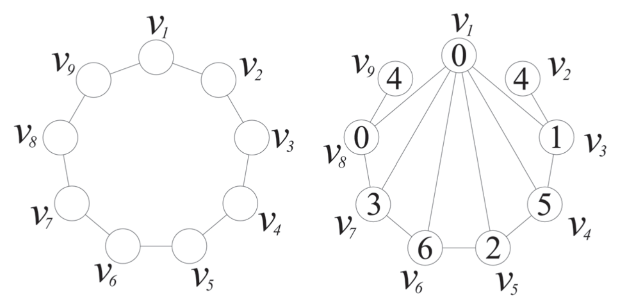

FIGURE 3. 7-Cordial labeling of the switching vertex of the cycle C_9.

Concluding Remarks

Here we have contributed general result for vertex switching of the cycle to the theory of k-cordial labeling. To derive similar results for other graph families is an open problem.

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