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H_2-H_∞ Robust Static Output feedback control for Vehicle Chassis Stability with input saturation

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Abstract

This paper describes a robust output feedback control with input constraints for a lateral and roll vehicle dynamics model under critical motions. The variation of the road adhesion coefficient has been considered. To this, a T-S disturbed model has been studied. The lateral velocity and the roll angle are not measured. In a second study, the uncertainties of stiffness coefficients have been taken into account, to which, a robust $H2 - H\infty$ output feedback controller has been designed. Based on Lyapunov theory, the controller gains are obtained by solving a set of Linear Matrix Inequalities (LMIs) constraints. The proposed techniques have been evaluated using a bicycle model with four degrees of freedom on Matlab simulator.

Keywords: Vehicle Dynamics, T-S model, output feedback control, $H2 - H\infty$, input saturation, Lyapunov function, LMIs

INTRODUCTION

Recently, several researches have been made for automotive vehicles in order to improve their response under critical situations, to ensure vehicle stability and reduce the drivers workload. There are many systems, which are already available in vehicles production, like ABS, DYC, TCS, etc. In this work, we will design a robust output feedback controller for the vehicle system modelized under a Takagi-Sugeno (T-S) model. It is based on a $H2 - H\infty$ approach and a T-S representation of the lateral and roll vehicle dynamics. This modelling is largely used in [1][2].

For technological or security reasons, the actuator saturation is one of the most nonlinear phenomena in real application. The control input saturation may notably weaken closed-loop fulfillment and harm the stability of the system. Hence, looking to de- sign stabilizing controllers via input saturation is very interesting like the work of [3] and [4]. In work of [5], the authors have considered the T-S approach to design an observer-based controller that improves the stability and the behaviour of the vehicle in critical situations without considering saturation constraints. For this, it is necessary to add constraints in order to avoid this problem solved in terms of LMIs. There have been several studies concerning the stability and the stabilization using observers and stale-feedback controllers for T-S models such as the work in [6] and [7]. All studies focus on state-feedback control in case of measurable states and observer based control in case of non measurable states (either some or all of them).

Moreover, the stability and stabilization of vehicles have been frequently and recently guaranteed through using observer-based controllers or state-feedback controllers like [8], but looking to stabilize this type of systems by a robust output feedback controller has a little bit treated in literature. Robust output feedback controllers have recently been studied and designed using Lyapunov function to solve problems of non-measurable states as the works of [9] and [10], but it is not yet feasible for real systems especially for vehicles.

However, H_2 -H_∞ approach is less used and it is used to minimize the effect of the disturbances, which has been studied in [11]. But, in his work, the author has chosen a bicycle as a vehicle model with two degrees of freedom.

The main contribution of this work is to limit the system inputs by imposing constraints and to use H_2 -H_∞ instead of H_∞ for lateral and roll dynamics of a bicycle model with four degrees of freedom.

In fact, when the infinity norm of the controlled output is required to be constrained under a certain level, the robust energy-to-peak strategy or $H_2 - H_\infty$ approach is an excellent choice in robust control. A recent study on energy-to-peak performance conditions for both continuous-time and discrete-time systems can be seen in [12]. From control appearance, the transient response is an important indication. Despite the fact that robust $H_2 - H_\infty$ control has many applications, there are few results considering the transient response in the existing literature [13]-[14].

This paper is organized as follows: After the introduction, section 2 describes the used model and the design of the output feedback controller with saturation. Then, section 3 shows the founded results by the output feedback control for the vehicle dynamics in nominal and uncertain cases. Next, some simulation results are given in Section 4. Finally, some conclusions are given in section 5.

PROBLEM STATEMENTS

Model's presentation

The model used in this work considers the lateral and roll dynamics, which are obtained by considering the well known bicycle model with a roll degree of freedom. The yaw, roll acceleration and the sideslip speed are given by the following expressions which are derived from the vehicle dynamic model as it is shown in Figure 1 according to the work given in [5].

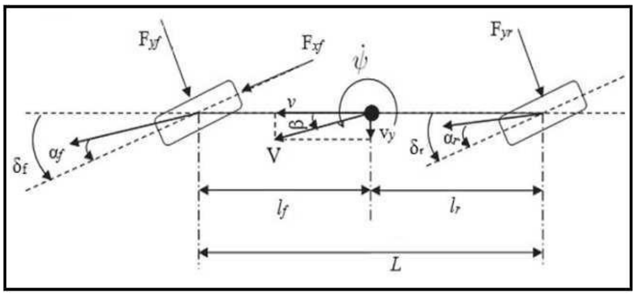

Figure 1: Lateral and roll dynamics bicycle model

$$\begin{cases} m_s a_{ys} + m_u a_{yu} = 2F_{yf} + 2F_{yr} \\ I_z \ddot{\psi} = 2l_f F_{yf} - 2l_r F_{yr} \\ I_x \ddot{\varphi}_v = m_s g h \varphi_v + m_s h a_{ys} - k_\varphi \varphi_v - C_\varphi \dot{\varphi}_v + M_x \end{cases}$$

where all paraeters are given in Table 1.

Table 1: Vehicle parameters

Parameter	Definition	Value	Unit
m_x	Sprung vehicle mass	1592	$[Kg]$
m_u	Unsprung vehicle mass	240	$[Kg]$
M	total vehicle mass	1832	$[Kg]$
V	vehicle speed	50	$[m.s^{-1}]$
I_x	roll moment of inertia at center of gravity G	614	$[Kgm^2]$
I_z	yaw moment of inertia at CG	2988	$[Kgm^2]$
l_r	Distance from CG to rear axles	1.77	$[m]$
l_f	Distance from CG to rear axles	1.18	$[m]$
h	CG height from roll axis	0.9	$[m]$
C_φ	combined roll damping coefficient	6000	[N ms.rad^{-1}]
k_φ	combined roll stiffness coefficient	14000	[N ms.rad^{-1}]

T-S representation of the vehicle model

Let us consider fuzzy sets M_{fi} and M_{ri}, i = 1, 2, defined for the two tire slip angle regions, i.e., M_{f1} and M_{r1} are the fuzzy symbols for the low front and rear tire slip angle regions, respectively, M_{f2} and M_{r2} for the high front and rear tire slip angle regions, respectively. The front and rear cornering forces may be described by the following T-S fuzzy rules:

$$\begin{aligned} &If\ |\alpha_f|\ is\ M_{f1}\ then\ F_{yf} = C_{f1}\alpha_f \\ &If\ |\alpha_f|\ is\ M_{f2}\ then\ F_{yf} = C_{f2}\alpha_f \\ &If\ |\alpha_r|\ is\ M_{r1}\ then\ F_{yr} = C_{r1}\alpha_r \\ &If\ |\alpha_r|\ is\ M_{r2}\ then\ F_{yr} = C_{r2}\alpha_r \end{aligned} \quad (1)$$

where C_{fi} and C_{ri} are the front and rear tire cornering stiffness, respectively, which depend on the road friction coefficient and vehicle parameters. Variables α_f and α_r are the slip angles of the front and rear tires which are given by the following equations:

$$\alpha_f = \delta_f - \frac{l_f \dot{\psi}}{v} - \beta, \alpha_r = \delta_r + \frac{l_r \dot{\psi}}{v} - \beta$$

Using the above described rules, the overall cornering forces are written as follows:

$$\begin{aligned} F_{yf} &= \mu_{f1}(C_{f1} + \Delta C_{f1})\alpha_f + \mu_{f2}(C_{f2} + \Delta C_{f2})\alpha_f \\ F_{yr} &= \mu_{r1}(C_{r1} + \Delta C_{r1})\alpha_r + \mu_{r2}(C_{r2} + \Delta C_{r2})\alpha_r \end{aligned} \quad (2)$$

with

$$\mu_{fi} = \frac{1}{(1 + \left|\frac{|\alpha_f| - c_{fi}}{a_{fi}}\right|)^{2b_{fi}}}$$

and

$$\mu_{ri} = \frac{1}{(1 + \left|\frac{|\alpha_r| - c_{ri}}{a_{ri}}\right|)^{2b_{ri}}}$$ for i=1,2.

The T-S model is designed by considering a constant longitudinal velocity and the variation of lateral and roll forces. T-S representation is considered in order to simplify the mobilization part.

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{4} \mu_i (A_i + \Delta A_i) x(t) + (B_{fi} + \Delta B_{fi})\delta_f(t) + (B_{ri} + \Delta B_{ri})\delta_r(t) \\ &+ B_m M_x(t) \\ &y(t) = C_1 x(t) \\ &z(t) = C_2 x(t) \end{aligned} \quad (3)$$

Where $x(t) = [\beta \quad \dot{\psi} \quad \dot{\varphi}_v \quad \varphi_v]^T$ is the state vector and the state matrices are given in Appendix A.

Considering the inertia moment M_x and the rear angle δ_r as the controlled inputs, the steering angle δ_f as the brake angle, so, model (3) has the following T-S representation:

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{4} \mu_i ((A_i + \Delta A_i) x(t) + (B_i + \Delta B_i) u(t) \\ &+ (B_{fi} + \Delta B_{fi})\delta_f(t) \\ &y(t) = C_1 x(t) \\ &z(t) = C_2 x(t) \end{aligned} \quad (4)$$

With

$$\begin{aligned} \Delta A_i &= E_{1i} M(t) F_1 \\ \Delta B_i &= E_{2i} M(t) F_2 \\ \Delta B_{fi} &= E_{3i} M(t) F_3 \end{aligned}$$

Where E_{1i}, E_{2i}, E_{3i}, F_1, F_2, F_3 are given in Appendix B.

Static Output Feedback Control

The aim of this section is to design a robust T-S static output feedback controller with input saturation in order to improve vehicle stability under critical situation, to prevent vehicle rollover and to ensure its security.

1.1.1 Control system description

Using the PDC structure, the T-S static output feedback controller is described by the following equation:

$$u(t)=\sum_{i=1}^{4}\mu_i K_i y(t) \quad (5)$$

Where K_i, $i = 1 : 4$, are the gains of control law to be determined.

The control input is subject to actuator saturation:

$$sat(u_i(t)) = sign(u_i(t))\min(u_{isat}, |u_i(t)|) \quad (6)$$

Model (4) will be written as a saturated system, given by:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{4}\mu_i(A_i+\Delta A_i)x(t)+(B_{f_i}+\Delta B_{f_i})\delta_f(t)\\&+(B_i+\Delta B_i)sat(u(t))\\y(t)&=C_1x(t)\\z(t)&=C_2x(t)\end{aligned} \quad (7)$$

Define the following subsets of R^n

$$\varepsilon(P,\rho)=\left\{x\in\Re^n, x^TPx\le\rho\right\} \quad (8)$$

$$L(K)=\left\{x\in\Re^n \mid \left|K_lCx\right|\le 1, l=1,...,m\right\} \quad (9)$$

With $P\in R^4$ a positive definite matrix, m the number of inputs, $\rho>0$ and K_l the l^{th} row of the matrix $K \in R^{n*m}$.

Thus, $\varepsilon(P,\rho)$ is an ellipsoid while $L(K)$ is a polyhedral region consisting of states for which the saturation does not occur.

Lemma 2.1. [15] For all $u \in R^m$ and $w \in R^m$ that $|w_l| \prec 1$, $l \in[1,m]$

$$sat(u)\in co\left\{E_su+\bar{E}_sv, s\in\left[1,2^m\right]\right\} \quad (10)$$

where co denotes the convex hull. Consequently, there exist $\varsigma_1 \ge 0, ... \varsigma_n \ge 0$ *with* $\sum_{s=1}^{2^m}\varsigma_s = 1$

such that:

$$sat(u)=\sum_{s=1}^{2^m}\varsigma_s(E_su+\bar{E}_sv) \quad (11)$$

where E_s, $\forall s = 1, ...m$ *is an* m *by* m diagonal matrix with elements either 0 *or* 1 *and*

$\bar{E}_S = I - E_s$. There are 2^m possible matrices of this type.

Assume that $x(t) \in L(H_i)$, using Lemma 2.1, the saturated output feedback control (5)

can be written as:

$$sat(u(t))=\sum_{i=1}^{n}\sum_{p=1}^{m}l_i\varsigma_p(E_pK_iC_1+\bar{E}_pH_i)x(t) \quad (12)$$

$$\varsigma_1\ge 0,\sum_{s=1}^{2^m}\varsigma_s=1 \quad (13)$$

Combining (5), (12) and (7), the closed-loop saturated T-S system can be expressed as follows:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{m}\mu_i\mu_j\varsigma_p\\&*((A_i+\Delta A_i)+(B_i+\Delta B_i)(E_pK_jC_1+\bar{E}_pH_j))x(t)\\&+(B_{fi}+\Delta B_{fi})\delta_f(t)\\y(t)&=C_1x(t)\\z(t)&=C_2x(t)\end{aligned} \quad (14)$$

For a prescribed scalar $\gamma > 0$, the performance index J is defined as:

$$J=\int_0^\infty(x^T(s)x(s)-\gamma^2\delta_f^T(t)\delta_f(t))ds$$

The aim of this part is to develop two control laws that ensure not only the stability but also disturbance rejection. The first law combines Lyapunov stability and H_∞ approach such that the following requirements are satisfied:

- The closed-loop T-S system (14) with $\delta_f(t) = 0$ is asymptotically stable.
- Under the zero initial condition, system (14) satisfies $\left\|z(t)\right\|_\infty \prec \gamma\left\|\delta_f(t)\right\|_2$ for any non-zero $\delta_f(t)$.

The second one uses Lyaponuv approach and H2- H∞ criteria such that the following lemma is satisfied:

Lemma 2.2. [16] Consider a continuous linear system:

$$\begin{aligned}\dot{\bar{x}}(t)&=\bar{A}\bar{x}(t)+\bar{B}\bar{w}(t))\\\bar{z}(t)&=\bar{C}\bar{x}(t)\end{aligned} \quad (15)$$

It is supposed that the norm-2 external disturbance is delimited. Then, there exists an energy-to-peak performance index γ such that $\left\|z(t)\right\|_\infty \prec \gamma\left\|\delta_f(t)\right\|_2$ if and only if the following constraints are satisfied:

$$\begin{bmatrix}\bar{A}\bar{P}+\bar{P}\bar{A}^T & \bar{B}\\ * & -I\end{bmatrix}<0 \quad (16)$$

$$\begin{bmatrix}-\gamma^2I & \bar{C}\bar{P}\\ * & -\bar{P}\end{bmatrix}<0 \quad (17)$$

Where $\bar{P}=\bar{P}^T>0$

We mention the following lemmas which are used to obtain our main results.

Lemma *2.3.* [17] Given a positive scalar γ and two matrices X and Y , it holds the following inequality:

$$Y^T X + X^T Y \leq \epsilon Y^T Y + \frac{1}{\epsilon} X^T X \quad (18)$$

Lemma *2.3.* For a positive definite matrix $R \in R^{n\times n}$, a square matrix $X \in R^{n\times n}$, and a scalar α, the following inequality holds:

$$-X^T R^{-1} X \leq \alpha^2 R \pm \alpha X \pm \alpha X^T$$

Proof: For any α, the inequality $(\alpha R \pm X)^T R^{-1}(\alpha R \pm X) \geq 0$ is always verified.

Lemma 2.5. [18] Given matrices X and Y with compatible dimensions. Then, the following inequality holds for any matrix $R \in R^{n\times n}$:

$$X^T Y + Y^T X \leq X^T R X + Y^T R^{-1} Y$$

MAIN RESULTS

Free control of vehicle dynamics:

Nominal case :

In this case, we will not take into account uncertainties, so, the considered system is given by:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i \left(A_i x(t) + B_{fi} \delta_f(t) \right) \quad (19)$$

The study of the stability of the open loop system can be given by Theorem 3.1:

Theorem 3.1:

System (19) is asymptotically stable and achieves a disturbance rejection level γ, if there exist P > 0 and Q solutions of the following LMI problem:

min γ

P,Q

Such that:

$$\begin{bmatrix} \Psi_i & B_{fi} \\ B_{fi}^T & -\gamma^2 I \end{bmatrix} < 0 \quad (20)$$

Where

$$\Psi_i = A_i^T P + P A_i + Q$$

Proof: We choose the Lyapunov function as follows:

$$V(x(t)) = x^T(t) P x(t) \quad (21)$$

The time derivative of (21) gives as follows:

$$\dot{V}(x(t)) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \quad (22)$$

The stability and disturbance rejection are studied if the following inequality is verified:

$$\dot{V}(x(t)) + x^T(t) Q x(t) - \gamma^2 \delta^T(t) \delta(t) < 0 \quad (23)$$

Therefore, we have :

$$\dot{V}(x(t)) + x^T(t) Q x(t) - \gamma^2 \delta^T(t) \delta(t)$$
$$= \sum_{i=1}^{4} \mu_i \left(A_i x(t) + B_{fi} \delta(t) \right)^T P x(t) + x^T(t) P \left(A_i x(t) + B_{fi} \delta(t) \right)$$
$$+ x^T(t) Q x(t) - \gamma^2 \delta^T(t) \delta(t)$$
$$= \sum_{i=1}^{4} \mu_i \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Psi_i & B_{fi}^T P \\ P B_{fi} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \quad (24)$$

With

$$\Psi_i = P A_i + A_i^T P + Q$$

If $\begin{bmatrix} \Psi_i & B_{fi}^T P \\ P B_{fi} & -\gamma^2 I \end{bmatrix} \leq 0$ holds, then

$\dot{V}(x(t)) + x^T(t) Q x(t) - \gamma^2 \delta^T(t) \delta(t) \leq 0$ for any $\begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \neq 0$

when $\delta(t) = 0$, (23) means $\dot{V}(x(t) < 0$, therefore system (19) is asymptotically stable in the case of $\delta(t) = 0$. When $\delta(t) \neq 0$, integrating both sides of (23) from 0 to t yields:

$$V(x(t)) - V(x(0)) + \int_0^t x^T(s) Q x(s) - \gamma^2 \delta^T(s) \delta(s) ds \leq 0 \quad (25)$$

Under zero initial conditions x(0) = 0 and letting t → ∞, we can obtain from (25):

$$\int_0^\infty x^T(s) Q x(s) ds \leq \int_0^\infty \gamma^2 \delta^T(s) \delta(s) ds \quad (26)$$

That is $\|x(t)\|_\infty \leq \|\delta(t)\|_\infty$ therefore, $J < 0$. Thus, the proof is completed.

Uncertain case:

In this case, parameter uncertainties are taken into account, so, system (14) is rewritten as:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i (\bar{A}_i x(t) + \bar{B}_{fi} \delta_f(t)) \quad (27)$$

With

$$\bar{A}_i = A_i + \Delta A_i, \bar{B}_{fi} = B_{fi} + \Delta B_{fi}$$

Theorem 3.2. System (27) is asymptotically stable and achieves a disturbance rejection level γ, if there exist some matrices P > 0 and Q solutions of the following LMI problem:

$$\min_{P,Q} \gamma$$

Such that

$$\begin{bmatrix} \Theta_i & PE_{1i} & B_{fi}^T P & 0 \\ E_{1i}^T P & -\varepsilon_{1i} I & 0 & 0 \\ 0 & 0 & -\gamma^2 I & PE_{3i} \\ 0 & 0 & E_{3i}^T P & -\varepsilon_{3i} I \end{bmatrix} < 0 \qquad (28)$$

Where

$$\Theta_i = PA_i + A_i^T P + Q + \varepsilon_{1i} F_1^T F_1 + \varepsilon_{3i} F_3^T F_3$$

Proof 2. We choose the Lyapunov function as follows:

$$V(x(t)) = x^T(t) P x(t) \qquad (29)$$

The time derivative of (29) gives as follows:

$$\dot{V}(x(t)) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \qquad (30)$$

The stability is guaranteed if the following inequality is verified:

$$\dot{V}(x(t)) + x^T(t) Q x(t) - \gamma^2 \delta^T(t)\delta(t) < 0 \qquad (31)$$

Therefore, we have:

$$\begin{aligned} &\dot{V}(x(t)) + x^T(t)Qx(t) - \gamma^2\delta^T(t)\delta(t) \\ &= \sum_{i=1}^{4} \mu_i \left(\bar{A}_i x(t) + \bar{B}_{fi}\delta(t)\right)^T P x(t) + x^T(t) P \left(\bar{A}_i x(t) + \bar{B}_{fi}\delta(t)\right) \\ &+ x^T(t)Qx(t) - \gamma^2\delta^T(t)\delta(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \Im_i \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \end{aligned} \qquad (32)$$

With

$$\Im_i = \begin{bmatrix} \Psi_i & \bar{B}_{fi}^T P \\ P\bar{B}_{fi} & -\gamma^2 I \end{bmatrix}$$

$$\Psi_i = P\bar{A}_i + \bar{A}_i^T P + Q$$

If $\begin{bmatrix} \Psi_i & \bar{B}_{fi}^T P \\ P\bar{B}_{fi} & -\gamma^2 I \end{bmatrix} \le 0$ holds, then $\dot{V}(x(t)) + x^T(t)Qx(t) - \gamma^2\delta^T(t)\delta(t) \le 0$ for any $\begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \ne 0$.

when $\delta(t) = 0$, (31) means $\dot{V}(x(t)) < 0$, therefore system (19) is asymptotically stable in the case of $\delta(t) = 0$. When $\delta(t) \ne 0$, integrating both sides of (31) from 0 to t yields:

$$V(x(t)) - V(x(0)) + \int_0^t x^T(s)Qx(s) - \gamma^2\delta^T(s)\delta(s)ds \le 0 \qquad (33)$$

Under zero initial conditions x(0) = 0 and letting t → ∞, we can obtain from (33):

$$\int_0^\infty x^T(s)Qx(s)ds \le \int_0^\infty \gamma^2\delta^T(s)\delta(s)ds \qquad (34)$$

That is $\|x(t)\|_\infty \le \|\delta(t)\|_\infty$, therefore, J < 0.

Decomposing matrix $\Im_i$ we obtain:

$$\begin{bmatrix} PA_i + A_i^T P + Q & B_{fi}^T P \\ PB_{fi} & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} P\Delta A_i + \Delta A_i^T P & \Delta B_{fi}^T P \\ P\Delta B_{fi} & -\gamma^2 I \end{bmatrix} \le 0 \qquad (35)$$

Using Lemma 2.3, we obtain:

$$\begin{bmatrix} P\Delta A_i + \Delta A_i^T P & \Delta B_{fi}^T P \\ P\Delta B_{fi} & -\gamma^2 I \end{bmatrix} \le \begin{bmatrix} \varepsilon_{1i}^{-1} PE_{1i}E_{1i}^T P + \varepsilon_{1i} F_1^T F_1 + \varepsilon_{3i} F_3^T F_3 & 0 \\ 0 & \varepsilon_{3i}^{-1} PE_{3i}E_{3i}^T P \end{bmatrix} \qquad (36)$$

Replacing (36) in (35) and using Schur complement, we obtain LMI (28) in Theorem 3.2.

In the rest of the work, we will consider the designed controller (12) and our main objective is the stabilization of the considered system under input constraints and ensuring the attenuation of the steering angle effect.

Vehicle dynamics control

Nominal case:

In this part of work, parameters uncertainties are not considered, so, model (14) is rewritten as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{m} \mu_i \mu_j \varsigma_p \bar{A}_{ijp} x(t) + B_{fi}\delta_f(t) \\ y(t) &= Cx(t) \end{aligned} \qquad (37)$$

Where

$$\bar{A}_{ijp} = A_i + B_i(E_p K_j C + \bar{E}_p H_j)$$

When infinity norm of the controlled output is required to be constrained under a certain level, the robust H_2 -H_∞ strategy is an excellent choice in robust control.

Theorem 3.3. Considering the closed loop system in (37), H2 – H∞ performance is guaranteed under input saturation constraints (9) subject to the saturated control input (12) with the output feedback control law (5) if there exist matrices Z, Fi, Li, M and S solutions to the following LMI problem:

$$\min_{P,Q} \gamma$$

Such that

$$\begin{bmatrix} \Upsilon_i & \Theta_i & B_{fi} \\ * & -\Pi_i & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} -\gamma^2 I & C_2 \\ C_2^T & -Z \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} u_{isat}^2 & L_i^T \\ L_i & Z \end{bmatrix} > 0 \tag{40}$$

Where

$$\Upsilon_i = ZA_i^T + A_i Z + C_1^T N_j^T E_p^T B_i^T + B_i E_p N_j C_1 + J_j^T \bar{E}_p^T B_i^T + B_i \bar{E}_p J_j \tag{41}$$

$$\Theta_i = \begin{bmatrix} B_i E_p N_j & (C_1 Z - \tilde{M} C_1)^T \end{bmatrix} \tag{42}$$

$$\Pi_i = \begin{bmatrix} R & 0 \\ 0 & \alpha\tilde{M} + \alpha\tilde{M}^T - \alpha^2 R \end{bmatrix} \tag{43}$$

Moreover, the gains K_i and H_i can be calculated by:

$$K_i = N_i \tilde{M}^{-1} \tag{44}$$

$$H_i = J_i Z^{-1} \tag{45}$$

Proof 3. Using Lemma 2.2, system (37) satisfies the following inequalities:

$$\begin{bmatrix} \bar{A}_{ijp} Z + Z\bar{A}_{ijp}^T & B_{fi} \\ * & -I \end{bmatrix} < 0 \tag{46}$$

And

$$\begin{bmatrix} -\gamma^2 I & C_2 Z \\ * & -Z \end{bmatrix} < 0 \tag{47}$$

Beginning by calculating (46),

$$\begin{aligned} &\begin{bmatrix} \bar{A}_{ijp} Z + Z\bar{A}_{ijp}^T & B_{fi} \\ * & -I \end{bmatrix} \\ &= (A_i + B_i(E_p K_j C_1 + \bar{E}_p H_j))Z \\ &+ Z(A_i + B_i(E_p K_j C_1 + \bar{E}_p H_j))^T + B_{fi} B_{fi}^T \\ &= A_i Z + B_i E_p K_j C_1 Z + + B_i \bar{E}_p H_j Z + ZA_i^T \\ &+ ZC_1^T K_j^T E_p^T B_i^T + ZH_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \\ &= A_i Z + B_i E_p K_j C_1 + B_i \bar{E}_p J_j + ZA_i^T \\ &+ C_1^T K_j^T E_p^T B_i^T + J_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \le 0 \end{aligned} \tag{48}$$

where $J_i = H_i Z$.

The addition and the substraction of the following terms $B_i E_p N_j C_1$ and $C_1^T N_j^T E_p^T B_i^T$ to expression (48) with

$N_j = K_j M$, we obtain:

$$\begin{aligned} &A_i Z + B_i E_p K_j C_1 Z + B_i \bar{E}_p J_j + ZA_i^T \\ &+ ZC_1^T K_j^T E_p^T B_i^T + J_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \\ &\Rightarrow A_i Z + B_i E_p K_j C_1 Z + B_i \bar{E}_p J_j + ZA_i^T + \\ &ZC_1^T K_j^T E_p^T B_i^T + J_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \\ &+ B_i E_p N_j C_1 - B_i E_p N_j C_1 + C_1^T N_j^T E_p^T B_i^T \\ &- C_1^T N_j^T E_p^T B_i^T \le 0 \end{aligned} \tag{49}$$

Then, we put in factor $K_j^T E_p^T B_i^T$ and $B_i E_p K_j$, we obtain:

$$\begin{aligned} &A_i Z + B_i E_p N_j C_1 + B_i \bar{E}_p J_j + ZA_i^T + C_1^T N_j^T E_p^T B_i^T \\ &+ J_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \\ &+ B_i E_p K_j (C_1 Z - MC_1) \\ &+ (ZC_1^T - C_1^T M^T) K_j^T E_p^T B_i^T \le 0 \end{aligned} \tag{50}$$

Using Lemma 2.5, equation (50) is equivalent to the following expression:

$$\begin{aligned} &A_i Z + B_i E_p N_j C_1 + B_i \bar{E}_p J_j + ZA_i^T \\ &+ C_1^T N_j^T E_p^T B_i^T + J_j^T \bar{E}_p^T B_i^T + B_{fi} B_{fi}^T \\ &+ (ZC_1^T - C_1^T M^T)(Q)^{-1}(C_1 Z - MC_1) \\ &+ B_i E_p K_j (Q) K_j^T E_p^T B_i^T \le 0 \end{aligned} \tag{51}$$

With Q = M R−1M T where Q and R are positive definite matrices. Replacing expression of matrix Q, (51) is equivalent to:

$$\begin{bmatrix} \Upsilon_i & B_i E_p N_j & (C_1 Z - MC_1)^T & B_{fi} \\ * & -R & 0 & 0 \\ * & * & -MR^{-1}M^T & 0 \\ * & * & * & -I \end{bmatrix} \leq 0 \quad (53)$$

With Υ_i is given in .

Using Lemma 2.4, we obtain inequality (38) in Theorem 3.3. The inclusion condition $\epsilon(P,\rho) \in L(H_i)$

$\forall i = 1,...,r$ holds if

$$1/\rho - \frac{u_{isat}^2}{\rho} - H_{il} P H_{il}^T > 0, \forall l \in [1, m]$$

Which is equivalent to $\frac{u_{isat}^2}{\rho} - (H_i P)_l P^{-1} (H_i P)_l^T > 0$

Then, using (45), we obtain $\frac{u_{isat}^2}{\rho} - (J_i)_l Z (J_i)_l^T > 0$

And by using schur Complement, LMI (40) is obtained.

In the first part of this paper, a nominal case of the cornering stiffness coefficients is studied. However, in real application, the vehicle is subject to model uncertainties. Thus, in the second part of this manuscript, this case will be considered.

Uncertain case :

This part focuses on the vehicle model and takes into account the uncertainties of the front and rear cornering stiffness coefficients due to the dependence on the road friction and the vehicle parameters. System (14) can be rewritten as:

$$\dot{x}(t) = A_{ijp} x(t) + \bar{B}_{fi} \delta_f(t) \quad (54)$$

Where $A_{ijp} = A_i + \Delta A_i + (B_i + \Delta B_i)(E_p K_j C_1 + \bar{E}_p H_j)$ and $\bar{B}_{fi} = B_{fi} + \Delta B_{fi}$

Theorem 3.4. The reduced uncertain T-S model (54) is stable by the saturated output feedback controller (12) and with the robust $H_2 - H_\infty$ index γ such that $\|z(t)\|_\infty \prec \gamma \|\delta_f(t)\|_2$ if there exist matrices Z, N_i, L_i, R and $\tilde{M}$ such that the following conditions are satisfied:

$$\begin{bmatrix} \Omega_{ijp} & B_{fi} & ZF_1^T & (C^T N_j^T E_p^T + L_j^T \bar{E}_p^T) F_2^T & 0 & \Theta_i \\ * & -I & 0 & 0 & F_3^T & 0 \\ * & * & -\varepsilon_{1i} I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{2i} I & 0 & 0 \\ * & * & * & * & -\varepsilon_{3i} I & 0 \\ * & * & * & * & * & -\Pi_i \end{bmatrix} < 0 \quad (55)$$

$$\begin{bmatrix} -\gamma^2 I & C_2 Z \\ Z C_2^T & -Z \end{bmatrix} < 0 \quad (56)$$

$$\begin{bmatrix} u_{isat}^2 & h_i^T \\ h_i & Z \end{bmatrix} > 0 \quad (57)$$

Where

$$\Omega_{ijp} = A_i Z + Z A_i^T + B_i E_p N_j C + B_i \bar{E}_p L_j$$
$$+ C^T N_j^T E_p^T B_i^T + L_j^T \bar{E}_p^T B_i^T + \epsilon_{1i} E_{1i} E_{1i}^T + \epsilon_{2i} E_{2i} E_{2i}^T + \epsilon_{3i} E_{3i} E_{3i}^T$$

$$\Theta_i = \begin{bmatrix} B_i \bar{E}_p N_j & (C_1 Z - \tilde{M} C_1)^T \end{bmatrix}$$

$$\Pi_i = \begin{bmatrix} R & 0 \\ 0 & \alpha \tilde{M} + \alpha \tilde{M}^T - \alpha^2 R \end{bmatrix}$$

And the gains K_i and H_i are given by the following expressions:

$K_i = N_i Z^{-1}$, $H_i = L_i Z^{-1}$

Proof 4. According to Lemma 2.2, the H_2 -H_∞ performance is satisfied if and only if the conditions in equation (56), and the following condition are satisfied:

$$\begin{bmatrix} A_{ijp} Z + Z A_{ijp}^T & \bar{B}_{fi} \\ * & -I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} -\gamma^2 I & C_2 Z \\ * & -Z \end{bmatrix} < 0 \quad (58)$$

Replacing expressions of A_{ijp} and $\bar{B}_{fi}$ and the uncertainty matrices ΔA_i, ΔB_i *and*

ΔB_{fi}, inequality (58) can be rewritten in the form of:

$$\psi_{ijp} + E_{123} \tilde{D}(t) F_{123} + F_{123}^T \tilde{D}^T(t) E_{123}^T \leq 0$$

Where

$$\psi_{ijp} = \begin{bmatrix} \Omega_{ijp} & B_{fi} \\ * & -I \end{bmatrix}$$

$$\Omega_{ijp} = A_i Z + Z A_i^T + B_i E_p K_j C Z + B_i \bar{E}_p H_j Z$$
$$+ Z^T C^T K_j^T E_p^T B_i^T + Z H_j^T \bar{E}_p^T B_i^T$$

$$E_{123i} = \begin{bmatrix} E_{1i} & E_{2i} & E_{3i} \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{123} = \begin{bmatrix} F_1 Z & 0 \\ F_2(E_p N_j C + \bar{E}_p L_j & 0 \\ 0 & F_3 \end{bmatrix}$$

$$\tilde{D}(t)=\begin{bmatrix} D(t) & 0 & 0 \\ 0 & D(t) & 0 \\ 0 & 0 & D(t) \end{bmatrix}$$

According to Lemma 2.3, inequality (59) is equivalent to:

$$\psi_{ijp}+\varepsilon_i E_{123}E_{123}^T+\varepsilon_i^{-1}F_{123}^T F_{123}<0 \qquad (59)$$

Using Schur complement and based on the demonstration given in the Proof of Theorem 3.3, we can easily obtain LMI (55) in Theorem 3.4 and LMI (57) has already been proved in the previous section.

SIMULATION RESULTS:

Nominal case:

In this section, the vehicle dynamics model is studied through a nonlinear bicycle system which is used to validate the effectiveness of the proposed controller. The parameters used for the vehicle model are listed in Table 1.

The output feedback controller is designed.

We first check the vehicle lateral dynamic performance in nominal case. Both vehicle dynamics with and without controller are checked to show the effects of the proposed controller with a variable longitudinal velocity given in Figure 2 and an input constraint as u_{1sat} = 0.01rad, u_{2sat} = 20N.m respectively for the rear steering angle $\delta_r(t)$ and the inertia moment $M_z(t)$.

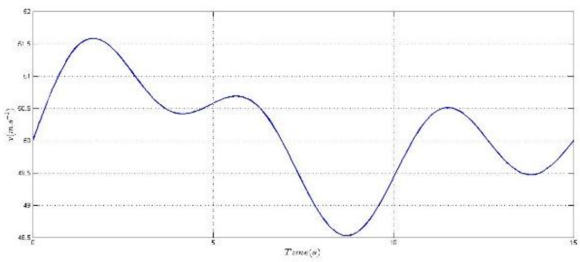

Figure 2: A speed longitudinal profile

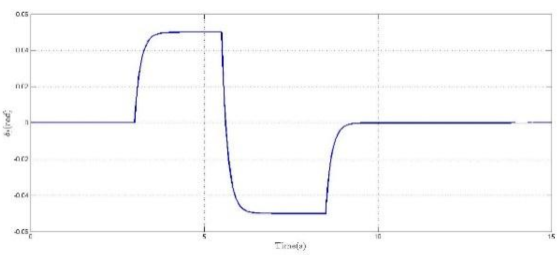

Figure 3: A chicane type maneuver

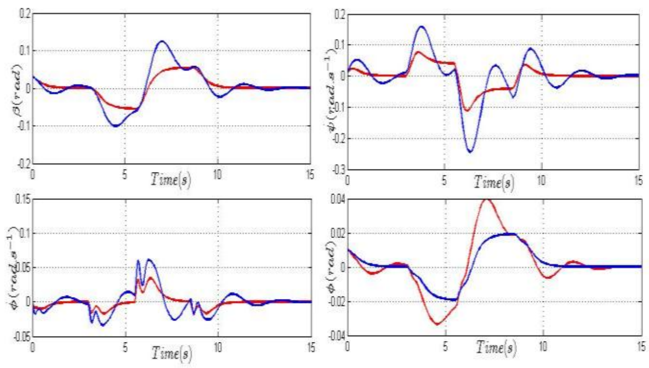

Figure 4: States evolution in open (blue) and closed (red) loop

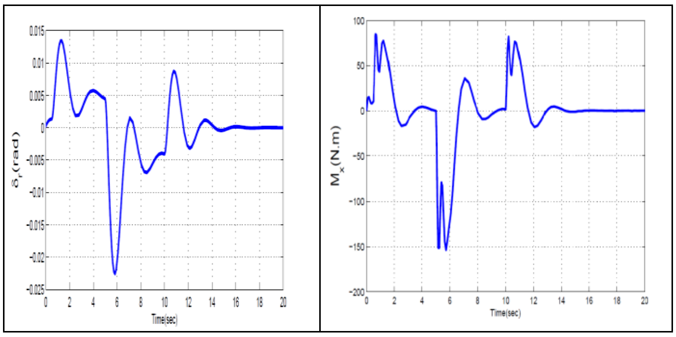

Figure 5 Control inputs

Figure 4 shows the evolution of the states and it is clearly that the system keep its stability under given constraints. Figure 5 shows the evolution of the controlled inputs.

Uncertain case

We now check the vehicle lateral dynamic performance in terms of uncertain cornering stiffness by a margin of 20%, beginning by a chicane type brake angle as it is given in Figure 3. Figures 6-7 show the simulation results of the states and control inputs evolution under the effect of a double chicane steering angle.

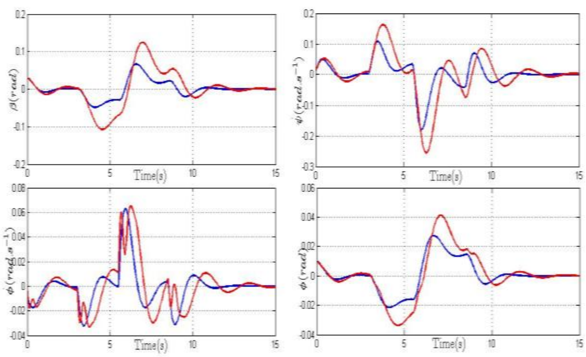

Figure 6: Comparison of States evolution in open (red) and closed (blue) loop

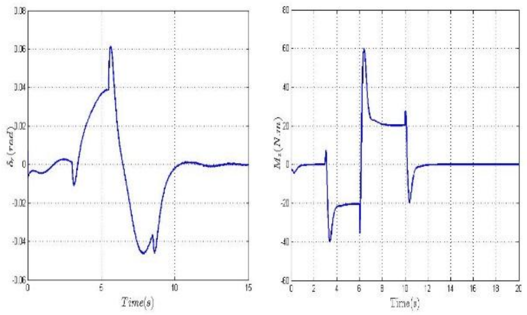

Figure 7: Control Inputs

Another test has been done: the steering angle has a sinusoidal variation as it is given in Figure 8. States evolution presented in Figure 9 and controlled inputs are given in Figure 10 show the performance of the proposed approach.

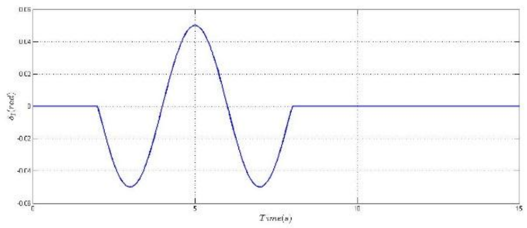

Figure 8: A sinusoidal steering angle

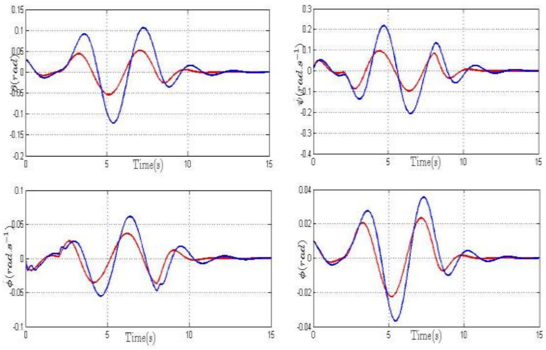

Figure 9: States evolution in case of sinusoidal steering angle

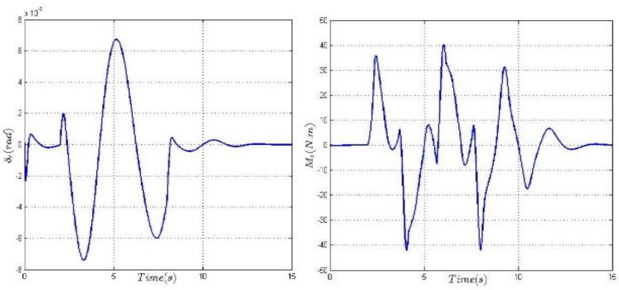

Figure 10: Control inputs evolution

We can conclude from these tests of simulation that the proposed output feedback controller has been successfully solved the stabilization and H_∞ problem under parametric uncertainties and different steering angle maneuvers.

CONCLUSION

A lateral and roll dynamics vehicle are considered when the front steer angle is the disturbance .The active roll torque and the rear steer angle are the control inputs. An input constraints are taken into account in order to reduce the degradation of the closed-loop system performance. To this, a robust output feedback controller with in- put saturation has been considered.

Then, the stiffness coefficients are not constant. For this, an uncertain T-S and dis- turbed model has been studied, to which, a robust H_2-H_∞ output feedback controller has been established to solve that problem.

The proposed works have been developed in terms of LMI formulation and designed by numerical tools (such Yalmip toolbox added to Matlab). From the obtained simulation results, we may conclude that the desired performance has been obtained.

APPENDICES

Appendix A :

$$A_1=\begin{bmatrix} -\frac{\sigma_1 I_{x1}}{m I_{x2} v} & -\frac{\rho_1 I_{x1}}{m I_{x2} v^2}-1 & -\frac{m_s h C_\varphi}{m I_{x2} v} & -\frac{m_s h(m_s g h-k_\varphi)}{m I_{x2} v} \\ \frac{\rho_1}{I_z} & -\frac{\tau_1}{I_z v} & 0 & 0 \\ -\frac{m_s h \sigma_1}{m I_{x2}} & \frac{m_s h \rho_1}{m I_{x2} v} & -\frac{C_\varphi}{I_{x2}} & \frac{mgh-k_\varphi}{I_{x2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_2=\begin{bmatrix} -\frac{\sigma_2 I_{x1}}{m I_{x2} v} & -\frac{\rho_2 I_{x1}}{m I_{x2} v^2}-1 & -\frac{m_s h C_\varphi}{m I_{x2} v} & -\frac{m_s h(m_s g h-k_\varphi)}{m I_{x2} v} \\ \frac{\rho_2}{I_z} & -\frac{\tau_2}{I_z v} & 0 & 0 \\ -\frac{m_s h \sigma_2}{m I_{x2}} & \frac{m_s h \rho_2}{m I_{x2} v} & -\frac{C_\varphi}{I_{x2}} & \frac{mgh-k_\varphi}{I_{x2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_3=\begin{bmatrix} -\frac{\sigma_3 I_{x1}}{m I_{x2} v} & -\frac{\rho_3 I_{x1}}{m I_{x2} v^2}-1 & -\frac{m_s h C_\varphi}{m I_{x2} v} & -\frac{m_s h(m_s g h-k_\varphi)}{m I_{x2} v} \\ \frac{\rho_3}{I_z} & -\frac{\tau_3}{I_z v} & 0 & 0 \\ -\frac{m_s h \sigma_3}{m I_{x2}} & \frac{m_s h \rho_3}{m I_{x2} v} & -\frac{C_\varphi}{I_{x2}} & \frac{mgh-k_\varphi}{I_{x2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_4=\begin{bmatrix} -\frac{\sigma_4 I_{x1}}{m I_{x2} v} & -\frac{\rho_4 I_{x1}}{m I_{x2} v^2}-1 & -\frac{m_s h C_\varphi}{m I_{x2} v} & -\frac{m_s h(m_s g h-k_\varphi)}{m I_{x2} v} \\ \frac{\rho_4}{I_z} & -\frac{\tau_4}{I_z v} & 0 & 0 \\ -\frac{m_s h \sigma_4}{m I_{x2}} & \frac{m_s h \rho_4}{m I_{x2} v} & -\frac{C_\varphi}{I_{x2}} & \frac{mgh-k_\varphi}{I_{x2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_m = \begin{bmatrix} \frac{m_s h}{m I_{x2} v} & 0 & \frac{1}{I_{x2}} & 0 \end{bmatrix}^T$$

$$B_{f1,2} = \begin{bmatrix} \frac{2C_{f1}I_{x1}}{mI_{x2}v} & \frac{2C_{f1}l_f}{I_z} & \frac{2m_s hC_{f1}}{mI_{x2}} & 0 \end{bmatrix}^T$$

$$B_{f3,4} = \begin{bmatrix} \frac{2C_{f2}I_{x1}}{mI_{x2}v} & \frac{2C_{f2}l_f}{I_z} & \frac{2m_s hC_{f2}}{mI_{x2}} & 0 \end{bmatrix}^T$$

$$B_{r1,3} = \begin{bmatrix} \frac{2C_{r1}I_{x1}}{mI_{x2}v} & -\frac{2C_{r1}l_r}{I_z} & \frac{2m_s hC_{r1}}{mI_{x2}} & 0 \end{bmatrix}^T$$

$$B_{r2,4} = \begin{bmatrix} \frac{2C_{r2}I_{x1}}{mI_{x2}v} & -\frac{2C_{r2}l_r}{I_z} & \frac{2m_s hC_{r2}}{mI_{x2}} & 0 \end{bmatrix}^T$$

Where

$$\sigma_1 = 2(C_{r1} + C_{f1}), \sigma_2 = 2(C_{r2} + C_{f1})$$
$$\sigma_3 = 2(C_{r1} + C_{f2}), \sigma_4 = 2(C_{r2} + C_{f2})$$
$$\rho_1 = 2(l_r C_{r1} - l_f C_{f1}), \rho_2 = 2(l_r C_{r2} - l_f C_{f1})$$
$$\rho_3 = 2(l_r C_{r1} - l_f C_{f2}), \rho_4 = 2(l_r C_{r2} - l_f C_{f2})$$
$$\tau_1 = 2(l_r^2 C_{r1} + l_f^2 C_{f1}), \tau_2 = 2(l_r^2 C_{r2} + l_f^2 C_{f1})$$
$$\tau_3 = 2(l_r^2 C_{r1} + l_f^2 C_{f2}), \tau_4 = 2(l_r^2 C_{r2} + l_f^2 C_{f2})$$

Appendix B:

$$E_{11} = \begin{bmatrix} -\frac{d\sigma_1 I_{x1}}{mI_{x2}v} & -\frac{d\rho_1 I_{x1}}{m_s I_{x2}v^2} & 0 & 0 \\ \frac{d\rho_1}{I_z} & -\frac{d\tau_1}{I_z v} & 0 & 0 \\ -\frac{m_s hd\sigma_1}{mI_{x2}} & \frac{m_s hd\rho_1}{m_s I_{x2}v} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_{12} = \begin{bmatrix} -\frac{d\sigma_2 I_{x1}}{mI_{x2}v} & -\frac{d\rho_2 I_{x1}}{m_s I_{x2}v^2} & 0 & 0 \\ \frac{d\rho_2}{I_z} & -\frac{d\tau_2}{I_z v} & 0 & 0 \\ -\frac{m_s hd\sigma_2}{mI_{x2}} & \frac{m_s hd\rho_2}{m_s I_{x2}v} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_{13} = \begin{bmatrix} -\frac{d\sigma_3 I_{x1}}{mI_{x2}v} & -\frac{d\rho_3 I_{x1}}{m_s I_{x2}v^2} & 0 & 0 \\ \frac{d\rho_3}{I_z} & -\frac{d\tau_3}{I_z v} & 0 & 0 \\ -\frac{m_s hd\sigma_3}{mI_{x2}} & \frac{m_s hd\rho_3}{m_s I_{x2}v} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_{14} = \begin{bmatrix} -\frac{d\sigma_4 I_{x1}}{mI_{x2}v} & -\frac{d\rho_4 I_{x1}}{m_s I_{x2}v^2} & 0 & 0 \\ \frac{d\rho_4}{I_z} & -\frac{d\tau_4}{I_z v} & 0 & 0 \\ -\frac{m_s hd\sigma_4}{mI_{x2}} & \frac{m_s hd\rho_4}{m_s I_{x2}v} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

F_1=eye(5)

$$E_{21} = \begin{bmatrix} \frac{2dC_{r1}I_{x1}}{mI_{x2}v} & 0 \\ \frac{2dC_{r1}l_r}{I_z} & 0 \\ \frac{2m_s hdC_{r1}}{mI_{x2}} & 0 \\ 0 & 0 \end{bmatrix}, E_{23} = E_{21}$$

$$E_{22} = \begin{bmatrix} \frac{2dC_{r2}I_{x1}}{mI_{x2}v} & 0 \\ \frac{2m_s hdC_{r2}}{mI_{x2}} & 0 \\ \frac{2m_s hdC_{r2}}{mI_{x2}} & 0 \\ 0 & 0 \end{bmatrix}, E_{24} = E_{22}$$

F_2=eye(2)

$$E_{31} = \begin{bmatrix} \frac{2dC_{f1}I_{x1}}{mI_{x2}v} & \frac{2dC_{f1}l_f}{I_z} & \frac{2m_s hdC_{f1}}{mI_{x2}I_z} & 0 \end{bmatrix}, E_{32} = E_{31}$$

$$E_{33} = \begin{bmatrix} \frac{2dC_{f2}I_{x1}}{mI_{x2}v} & \frac{2dC_{f2}l_f}{I_z} & \frac{2m_s hdC_{f2}}{mI_{x2}I_z} & 0 \end{bmatrix}, E_{34} = E_{33}$$

$$F_3 = 1$$

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