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Unified Advanced Model of Effective Moment of Inertia of Reinforced Concrete Members

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Abstract

A new modification of Branson's model was proposed to consider the variation of some parameters such as the type of concrete (normal, high strength), loading conditions (three-point loading, four-point loading, and uniform load), shear deformation (thickness to length ratio), and type of reinforcement (steel and fiber reinforced polymer (FRP) reinforcement). The high values of concrete modulus of elasticity and low ones of FRP bars were considered. A comparison was carried out with the experimental studies and other models for both normal and high strength concrete beams with steel/FRP reinforcement. The comparisons were extended for deep beams with FRP reinforcement. The results of the proposed model showed a better agreement with the experimental studies compared to ACI equation and other models from literature. The maximum difference in deflection results from the proposed model in this study ranged between 1% to 10% of those obtained by experimental work.

Keywords: Deflection, Effective moment of inertia, High strength concrete, FRP, RC beam, Shear deformation.

INTRODUCTION

Beams deflection is one of the most important parameters in the structural design, of which, it restricts the span to depth ratio of beams. In the past, all reinforced concrete (RC) members have been constructed using mild steel as the main reinforcement and many studies have been conducted on these types of members. Recently, the use of fiber reinforced polymer (FRP) bar as the main reinforcement in concrete beams has been increased widely because of its superior properties. The low elastic modulus of FRP reduces the stiffness of concrete members, especially after cracking stage, which increases the deflection, therefore it is a governing parameter in the design of such members. The well-known equations for finding the deflection of RC members depend on calculating the effective moment of inertia of the cracked section. Branson's equation was adopted by ACI-318 and it was appeared in the 1971 publication edition and remained as a major equation for the finding of the effective moment of inertia in the calculations of the deflection of RC beams. This equation has been modified by many researchers to make it more appropriate for concrete beams with steel/FRP reinforcement as shown in Table 1. Since that period, many arguments about this equation have been raised for different reasons, but almost they focused on the accuracy of this model. An argument by designers showed that the cumbersome computation of cracked moment of inertia I_{cr} is complex and time-consuming, especially for flanged sections. Researchers showed that in some cases, a 100% error occurred by using Branson's model. These arguments encouraged the researchers to examine Branson's equation validity for such structures.

Al-Zaid et al. (1991) compared the computational moment of inertia to that obtained from the experimental tests under different load conditions for rectangular RC beams. They found that the values of experimental moment of inertia for beams under three-point loading was greater by 12% than that one under four-point loading and by 20% than the one under uniform load. The experimental results proved that Branson's model is not appropriate for different load condition. Fikry & Thomas (1998) proposed a new effective moment of inertia model that focused on the variation in reinforcement ratio as well as load conditions. Ammash & Muhaisin (2009) introduced a new advanced model that considers the effect of shear deformation. Their model showed that there was a reduction of 27% in the effective moment of inertia for span/depth ratio of (5.0 to 20.0) because of the effect of shear deformation. Rafi & Nadjai (2009) compared the theoretical deflection of concrete beams reinforced with FRP bars with the experimental data. The theoretical prediction was based on the Eurocode-2 (2004) deflection model, which is used for normal RC structures with steel bars. Results of deflections for 75 simply supported beams and slabs were compared with the Eurocode-2 (2004) method, including authors tested beams. The proposed expression considered the effects of reinforcement ratio proportional to the balanced state, also elastic modulus ratio of FRP to steel bars. A good agreement was found between the results of deflections of the suggested model and those of FRP bars type.

Benmokrane et al. (1996) suggested modifications to ACI Code formula to well predict the flexural behavior of concrete beams with FRP reinforcement. Since the deformation of FRP bars is larger than the steel bar, they introduced two new factors in the ACI formula to accommodate this issue and the results showed a good agreement with the experimental results. Faza & GangaRao (1992) introduced a new expression of concrete beams reinforced with FRP bar under four-point load bending test to estimate the deflection. The neutral axis depth and crack pattern nature for this type of beams were considered in the new model. This expression is valid for beams reinforced with sand-coated FRP bars.

Bischoff and Gross (2010) studied the benefits of deflection computation with an alternative moment of inertia. Their concept was derived from the integration of curvature to consider the variations in the stiffness of the member along the span length. Results were examined for FRP and steel reinforcement of RC members with different loading configurations and support conditions. They found that their method for finding the moment of inertia can improve the deflection's prediction.

High values of compressive strength and span to depth ratio (L/d) are not considered in the previous equations. In the present study, the effect of compressive strength (normal strength (NC) and high strength concrete (HSC)), span to depth ratio (L/d), loading conditions (three-point loading, four-point loading, and uniform load), and type of reinforcement (steel and FRP) were considered in calculation of deflection. A comparative study using previous test results from prior researches was carried out to evaluate the efficiency of the proposed equation.

Table 1. Summery of Previous Equations

Source	Equation
ACI-318	$\mathrm{I_e} = (\frac{\mathrm{M_{cr}}}{\mathrm{M_a}})^3 \mathrm{I_g} + \left(1 - \left(\frac{\mathrm{M_{cr}}}{\mathrm{M_a}}\right)^3\right) \mathrm{I_{cr}} < \mathrm{I_g}$
Al-Zaid et al. (1991)	$I_{eff} = (\frac{M_{cr}}{M_a})^m I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^m\right) I_{cr} < I_g$
Fikry and Thomas (1998)	$\mathrm{I_{cr}} = (\alpha + \beta\eta\rho)\left(\frac{\mathrm{bd}^3}{12}\right) < \mathrm{I_g}$
Ammash and Muhaisin (2009)	$\mathrm{I_{eff}} = \left(\frac{\mathrm{M_{cr}}}{\mathrm{M_a}}\right)^{(\zeta+\gamma)} \mathrm{I_g} + \left((\zeta+\gamma) - \left(\frac{\mathrm{M_{cr}}}{\mathrm{M_a}}\right)^{(\zeta+\gamma)}\right) \mathrm{I_{cr}}(\zeta+\gamma) < \mathrm{I_g}$
ACI-440.1R (2006)	$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 \beta d\, I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} < I_g$
Rafi and Nadjai (2009)	$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 \beta d\, I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) \frac{I_{cr}}{\gamma} < I_g$
Hall and Ghali (2000)	$I_{eff} = \frac{I_g I_{cr}}{[I_g + \beta_1\beta_2\left(\frac{M_{cr}}{M_a}\right)^2 (I_{cr} - I_g)]}$
Eurocode-2 (2004)	$\Delta = f_1\beta_1\beta_2\left(\frac{M_{cr}}{M_a}\right)^2 + f_2\left(1 - \beta_1\beta_2\left(\frac{M_{cr}}{M_a}\right)^2\right)$
Faza and GangaRao (1992)	$I_m = \frac{23 I_{cr} I_e}{8 I_{cr} + 15 I_e}$
(Benmokrane et al. (1996)	$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 \frac{I_g}{7} + 0.84\left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} < I_g$
Bischoff (2005)	$I_{eff} = \frac{I_{cr}}{1 - \left(1 - \frac{I_{cr}}{I_g}\right)\left(\frac{M_{cr}}{M_a}\right)^2} < I_g$

NEW PROPOSED MODEL

The effective moment of inertia for the deflection's calculation of concrete beams reinforced with either steel or with FRP bars is presented. This equation is based on Branson's equation to calculate the effective moment of inertia. A modifications factors were established to accommodate the effect of compressive strength, loading condition, and shear deformation effects. These factors were based on several experimental studies from previous researches. Also, the proposed model considers both effects of high values of elastic modulus of HSC calculated from ACI-318 (2014) equation ($4700\sqrt{f_c'}$), and the low value of elastic modulus of FRP bars. The proposed model for the calculation of deflection and moment of inertia of concrete sections with steel or FRP bar reinforcement is:

$$\Delta = \frac{C\,M_a L^2}{E_c I_{eff}} \zeta\psi$$

where; C is a constant for moment calculation depend on loading type, which:

- C = 5/48 for uniform load.
- C = 1/12 for three-point loading.
- C = 23/216 for four-point loading,

M_a is the applied moment, $E_c = 4700\sqrt{f_c'}$ is the modulus of elasticity for all types of concrete, ζ is a factor accommodate for the effect of loading type, where:

ζ = 0.9 for uniform load,

ζ = 1.15 for three-point loading.

ζ = 1.0 for four-point loading,

ψ is a factor accommodate the effect of compressive strength, modulus of elasticity and rupture, where:

- ψ = 1.00 for NC.
- ψ = 1.25 for HSC.

$$I_{eff} = (\frac{M_{cr}}{M_a})^{\alpha} I_g \chi + \left(\chi - \left(\frac{M_{cr}}{M_a}\right)^{\alpha}\right) I_{cr} \quad < I_g$$

$$\alpha = 5 - 0.03\,\lambda f_c' - \frac{h}{L}\ ;\ \chi = 1 - \frac{h}{L}$$

- λ = 1.0 for $f_c' \leq 40$ MPa.
- λ = 0.8 for $f_c' \geq 40$ MPa.

M_{cr} is the cracking moment for the section which is equal to $M_{cr} = \frac{f_r I_g}{y}$, $f_r = 0.62\sqrt{f_c'}$, h is the beam height, L is the supported length, I_g is the moment of inertia of the gross section, y is the distance from the neutral axis to the extreme compression fiber, I_{cr} is the cracked moment of inertia.

NUMERICAL EXAMPLES

To validate the proposed model, some cases recorded by prior researchers are used and compared with the proposed and other equations.

RC Beams with Steel Reinforcement

Three-point load flexural test (Al-Zaid et al. 1991)

The experimental results examined by Al-Zaid et al. (1991) under three-point load flexural were compared with the proposed model and previous models. The total length of the tested beams was 2500 mm with a square cross-section with a side length of 200 mm. The results shown in Figure 1 showed that the proposed model gives a better agreement with the experimental test in term of load-deflection behavior compared to those obtained from other models. The differences in the calculated deflection from the proposed model and ACI-318 equation with the experimental results were 6% and 7%, respectively.

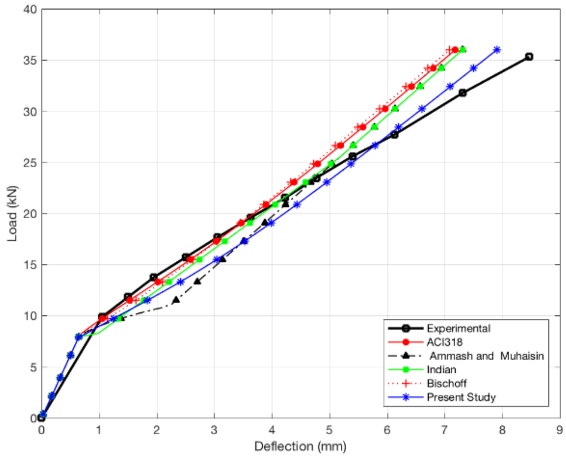


Figure 1. Results comparison for RC beam with steel bar under three-point flexural test

Four-point load flexural test (Ning 1998)

The experimental results from Ning (1998) under four-point load flexural for doubly RC beams were also compared with the proposed model and previous models. The clear span of the simply supported beams was 3000 mm with 300 × 400 mm cross-section. Different tensile reinforcements with different beams were considered in his study with 2φ12 for the compressive reinforcement. The beam with 3φ25 was considered here as a case study. Figure 2 compared the experimental results with the proposed model and some other models. This figure showed that there is a satisfactory agreement between the experiment and the proposed model for the post-cracking stage. The difference in term of deflection between the proposed model and the experimental results was 4%, while it was 19% between the ACI-318 equation and the experimental results.

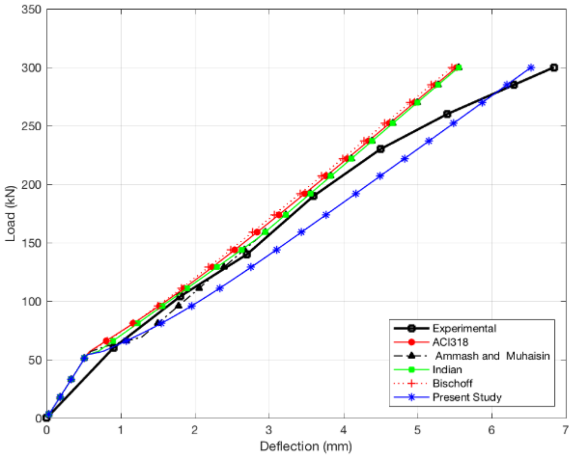


Figure 2. Results comparison for RC beam with steel bar under four-point flexural test

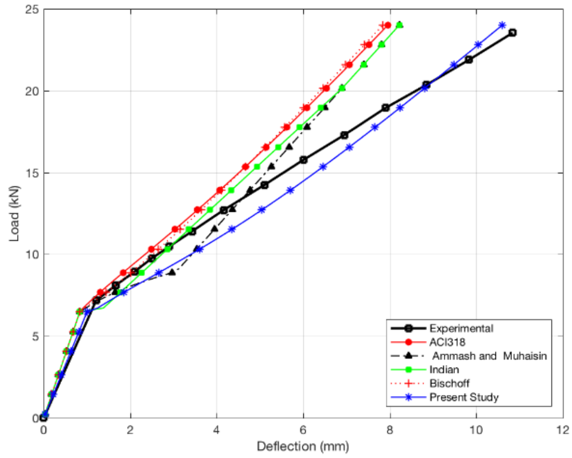


Figure 3. Results comparison for RC beam with steel bar under uniform load

Uniformly distributed load (Al-Zaid et al. 1991)

Experimental results of Al-Zaid et al. (1991) for a simply supported RC beam under uniform distributed load were compared with the proposed model and previous models. The total length of the tested beams was 2500 mm and has a square cross-section with a side length of 200 mm. The numerical and experimental results of load mid-span deflection are shown in Figure 3. It is clearly noticed that the proposed model deflection behavior shows a good agreement with the experimental results. The difference in deflection results between the proposed model and experimental data was about 2%, while it was about 25% for ACI-318 equation.

Concrete beams with FRP bars

Some experimental and numerical cases recorded by prior researchers for RC beam with both CFRP and GFRP bars are used and compared with proposed and other equations. Different examples were considered based on the parameters of the proposed model; type of concrete, loading conditions, and shear deformation effect.

NC beam with GFRP bar (Adam et al. 2015)

The experimental result studied by Adam et al. (2015) on simple GFRP RC beams with NC was compared with the proposed model and previous models. One beam was selected (A25-1) with a compressive strength of 24.5 MPa. The effective span was 2500 mm and the shear span was 1100 mm with 120 × 300 mm cross-section dimensions. It could be noticed from Figure 4 that the proposed model showed a good agreement with the experimental results in terms of maximum strength and trend compared to other models. The difference in deflection value between the proposed model and the experimental results was 1%, while it was 33% between the ACI-440 equation and the experimental results.

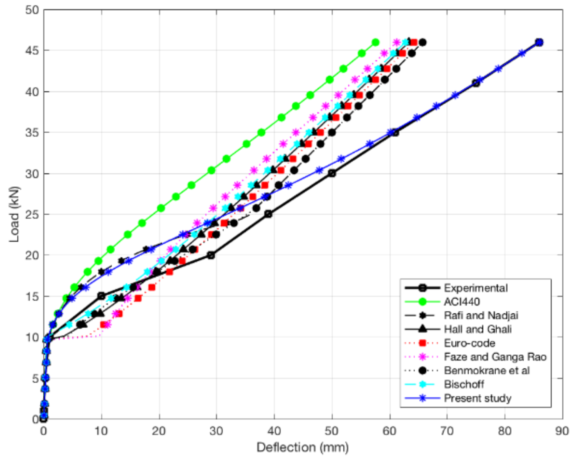
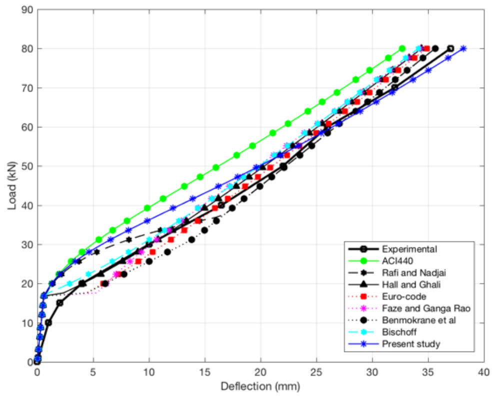


Figure 4. Results comparison for NC beam with GFRP bar

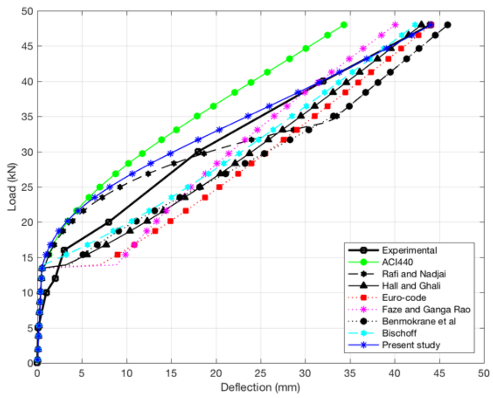
HSC beam with CFRP and GFRP bar bar (Al-Sunna et al. 2012)

The experimental results of simply supported RC beams reinforced with CFRP and GFRP bars tested by Al-Sunna et al. (2012) were compared with the proposed and available equations. The compressive strength for the GFRP beam was 47.7 MPa and for the CFRP beam was 55.4 MPa. The cross-section dimensions of the beams were 150 × 250 with 2300 mm clear span and 767 mm shear span. Figure 5 shows the comparison between the experimental and analytical results in term of load-displacement curves. It could be noticed from Figure. 5a and Figure. 5b that the proposed model gave a good agreement with both experimental results in term of maximum strength and deflection. Other models showed different behavior compared to the experimental results. The calculated deflections from the proposed have a difference of 3% and 4% from the experimental results for the CFRP and GFRP bar, respectively. On the other side, the differences were 11% and

25% between ACI-440 equation and experimental results for the CFRP and GFRP bar, respectively.



a- CFRP beam



b- GFRP beam

Figure 5. Results comparison for HSC beams with FRP bar

Deep beam with self-compacted concrete and CFRP bar(Heiza 2016)

The experimental results of RC deep beams made from self-compacted concrete and CFRP bars tested by Heiza (2016) were compared with the proposed model as well as the previous model to validate the accuracy of the proposed model. The deep beams cross-section was 100 × 500 mm with 1000 mm clear span. The modulus of elasticity of CFRP bar was 130 GPa and the concrete compressive strength was 40 MPa. The comparison of the experimental and theoretical results is shown in Figure 6 in terms of load-displacement curves. It is clearly noticed from this figure that the behavior and the deflection value of the proposed model approximately match the one from experiment results, while other models go so far from experimental results since the shear effect was neglected in these models. The difference between the proposed model and the experimental results was 1% in term of deflection, while it was 50% between the ACI-440 equation and the experimental results.

HSC deep beam with GFRP bar (Andermatt and Lubell 2013)

Twelve large-scale deep beams made from HSC and GFRP bars were tested by Andermatt and Lubell (2013) under four-point load configuration with different span to depth ratios. One beam was selected (B6H) to compare with the proposed model as well as the other models. The cross-section of the selected beam was (300×610) with 2580 mm clear span and 2.07 effective shear span to depth ratio. The GFRP modulus of elasticity was 37.9 GPa and the concrete compressive strength was 68.5 MPa. Figure 7 shows the experimental and theoretical load-displacement curve of this case. It can be clearly noticed from this curve that the proposed model has a good match with the experimental data. The other models showed significant differences with experimental results. The differences in the theoretical deflection from the proposed model and ACI-440 equation with the experimental results were 6.5% and 37%, respectively. This difference can be attributed to the absence of the effect of high compressive strength and shear effect in ACI-440 models.

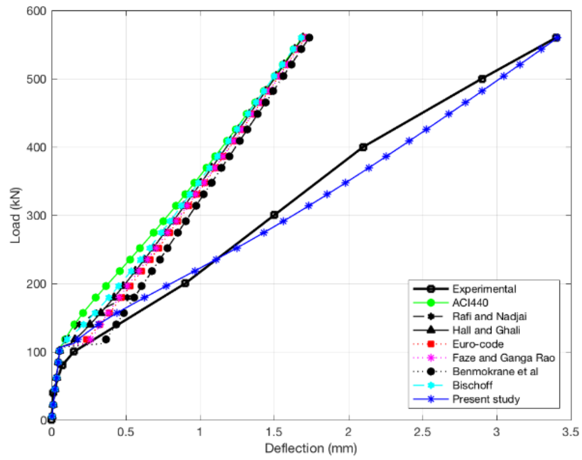


Figure 6. Results comparison for RC deep beam with self-compacted concrete

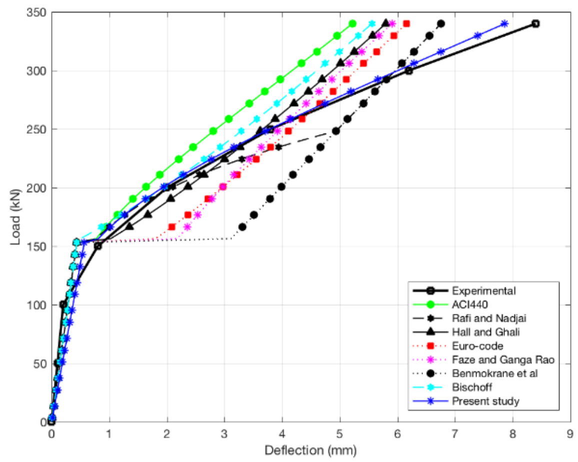


Figure 7. Results comparison for HSC deep beam with GFRP bar

RC beam with FRP bar beam under three-point load (Massam 2001)

The applicability the proposed model for concrete beam reinforced with FRP under three-point load configuration was verified in this section. Massam (2001) conducted a series of experimental tests to study the behavior of GFRP beams in shear. One beam was selected for the validation purposes. The total length of the selected beam was 4000 mm with 450 × 500 mm cross-section area. The modulus of elasticity of GFRP bar was 40.8 GPa and the concrete compressive strength was 35 MPa. The comparison of the experimental results with the proposed model as well as with the other models is shown in Figure 8. It can be shown from this figure that the proposed model well predicts the load-deflection response of RC beam with FRP bars under three-point load flexural test compared to other models. The difference in the deviation between the proposed model and the experimental results was 2%. On the other hand, it was 50% between the ACI-440 equation and the experimental results.

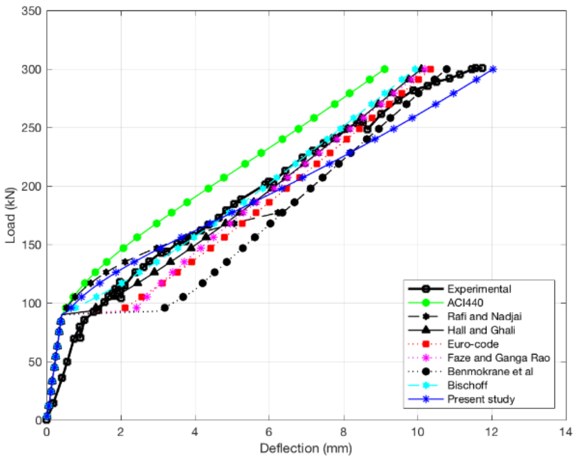


Figure 8. Results comparison for RC beam with GFRP bar under three-point flexural test

CONCLUSIONS

The present study introduced a new global form of the real moment of inertia by developing Branson's model to consider several factors effects such as a variation of compressive strength of concrete (normal strength and high strength), type of reinforcement (steel and FRP), type of loading (single concentrated load, two-point load, and distributed load), and shear deformation (shallow and deep beams). The results of the proposed model were compared with other studies (ACI model, experimental, and other models result). The proposed model's results gave a good agreement with experimental results than other studies such as ACI model. The following conclusions can be drawn from this study:

1. Good agreement was achieved between the experimental and numerical results in terms of the predicted deflection behavior of the present models compared with experimental behavior. In addition, the numerical results reflected the significance of the taken parameters in present model.
2. The type of reinforcement, depth of beams, the strength of concrete, reinforcement ratios, type of loading, and elastic modulus of FRP bars are significant variables for calculating the deflection of concrete beams. The effects of all previously mentioned variables are considered in the proposed equation. The deflection estimated using present model directed to have more accuracy than those adopted by many standards and regulations provisions.
3. The proposed model showed the significance of high compressive strength and shear effect in the calculation of the effective moment of inertia and theoretical deflection. By considering both the compressive strength and shear effect, the proposed model well predicted the deflection up to 94% to 99% of the experimental results.
4. Different loading conditions were successfully represented in the proposed model with different reinforcement and concrete types. The model can well estimate the total deflection with accuracy ranged between 93%- 99% of the experimental results.

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