

Epsilon Support Vector Regression based Controller

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Abstract

This paper explore epsilon support vector regression (ε -SVR) for control signal reconstruction. ε -SVR is employed to identify a dynamic system given an input set, constituted by the state variables, the error and the control past signal information, and an output set, constituted by the resulting control signal given a sample of the input set. The resulting trained function is used as a controller. Finally, the proposed approach is validated performing the simulation of the stabilization of an inverted pendulum. The control approach shows to be useful as a control methodology when the plant model is not available or cannot be identified, commonly known as black box models.

Keywords: Support vector regression, control signal reconstruction, ε -SVR learning, plant identification.

INTRODUCTION

For many years, the problem of controlling highly nonlinear systems has received much attention within the control community. As a result of the strong need to find easy and robust methods, many solutions have been proposed [1]. One of these approaches, is based on using machine learning algorithms. In this sense researchers have demonstrated, for example, the positive use of neural networks in learning system uncertainties [2] and in helping in the generation of control laws [3]–[5]. In this way, machine learning as a control methodology can be understood as, instead of mathematically computing the required feedforward compensation, learnt from the feedback control signal by using a function approximator. This may have distinct advantages, as dealing with unknown (non-linear) system properties, as it was already mentioned, or compensate disturbances or noise [6].

For computational reasons, the function approximator will typically involve a basis function expansion, e.g. using a B-spline neural network [6]. However, when the function to be approximated depends on several variables, the learning process will fail because of the curse of dimensionality [7]. Nonetheless, Support Vector Machines (SVM) have shown to be superior approximators for regression problems [8], [9] that does not suffer from the curse of dimensionality [10], due to the fact, they are learning machines implementing the structural risk minimization inductive principle to obtain good generalization on a limited number of learning patterns [11].

In this paper, it is presented a machine learning approach for control signal reconstruction using the ε -SVR algorithm. First section shows the basic concepts presented on [12] about this technique. Then, in section three, the proposed methodological control scheme is described. Finally simulation results and

conclusions are made for the stabilization of an inverted pendulum.

ε -SVR

Given a Training set $T = \{x_k, y_k\}_{k=1}^l$, where x_k is the k^{th} input data in the input space $\mathcal{X} \subseteq \mathbb{R}^n$, y_k is the corresponding output value in the output space $\mathcal{Y} \subseteq \mathbb{R}$ and l is the number of samples.

SVR model is given by:

$$y_k(x_k) = \langle w, x_k \rangle + b \quad (1)$$

For linear regression, where $\langle \cdot, \cdot \rangle$ denotes the dot product in the input space with $w \in \mathcal{X}$, $b \in \mathbb{R}$. Using de dual form and the kernel trick, $y_k(x_k)$ can be written as follows:

$$y_k(x_k) = \sum_{i=1}^l (\alpha_i - \alpha_i^*) k(x_i, x_k) + b \quad (2)$$

Where $k(\cdot, \cdot)$ is called a kernel function in a feature space \mathcal{F} such that, $k: \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ and $k(x, y) = \langle \Phi(x), \Phi(y) \rangle$, where Φ is a function such that $\Phi: \mathcal{X} \rightarrow \mathcal{F}$.

The regression problem is reduced to find $(\alpha_i - \alpha_i^*)$. This can be made using a dual optimization formulation given as:

$$\begin{aligned} & \text{maximize} \quad \begin{cases} \frac{1}{2} \sum_{i,j=1}^l (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) k(x_i, x_j) \\ -\varepsilon \sum_{i=1}^l (\alpha_i - \alpha_i^*) + \sum_{i=1}^l y_i (\alpha_i - \alpha_i^*) \end{cases} \\ & \text{subject to} \quad \begin{cases} \sum_{i=1}^l (\alpha_i - \alpha_i^*) = 0 \\ \alpha_i, \alpha_i^* \in [0, C] \end{cases} \end{aligned} \quad (3)$$

Where $C > 0$ determines the trade off between the flatness of $y_k(x_k)$ and the amount up to which deviations larger than ε are tolerated. A graphical framework on ε -SVR is shown below on Figure 1.

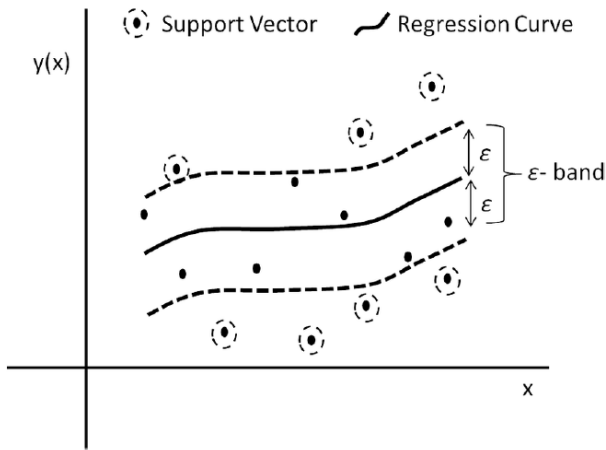


Figure 1. Visualization of support vector regression framework (taken from [13]).

ε-SVR BASED CONTROL

In this section proposed ε-SVR based control is described. The training set for learning is given in terms of the state space variables, the error and the past control signal values. Finally, the proposed control scheme is given.

ε-SVR training and Control

Given the training set $\mathcal{M} = \{\{S_k, T_k, U_k\}, u_k\}_{k=1}^N$ with $S_k = (x_{1k}, x_{1k-1}, x_{1k-2}, \dots, x_{nk}, x_{nk-1}, x_{nk-2})$ where $x_{ik}, x_{ik-1}, x_{ik-2}$ are the present state variable value, the sampled state variable with a unit delay and the sampled state variable with a double delay respectively. $T_k = (e_k, e_{k-1}, e_{k-2})$ where e_k, e_{k-1}, e_{k-2} are, analogously, the sampled errors. $U_k = (u_{k-1}, u_{k-2})$ where u_{k-1}, u_{k-2} are the past sampled control signal values. Finally, u_k is the output of the training set corresponding to the present control signal value. In this way, the regression process for u_k^* , the estimated control signal, can be expressed as follows:

$$u_k(\mathcal{M}_k) = \sum_{i=1}^l (\alpha_i - \alpha_i^*) k(\mathcal{M}_i, \mathcal{M}_k) + b \quad (4)$$

Where $\mathcal{M}_j = \{\{S_k, T_k, U_k\}, u_k\}_{k=j}$ is the j^{th} sample. The training scheme is showed below in Figure 2.

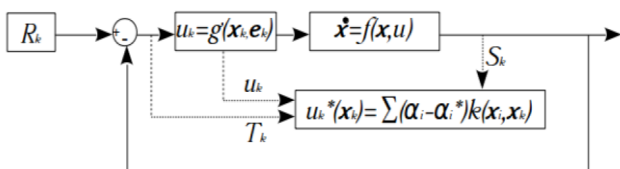


Figure 2. ε-SVR Control Training Scheme

For the training process, R_k is a random reference value and u_k is random control function in terms of the state variables and

the error. The control scheme is showed below in Figure 3.

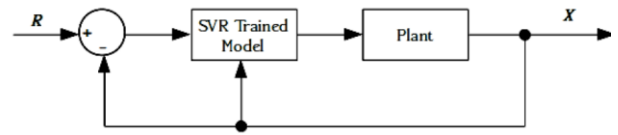


Figure 3. ε-SVR Control Scheme

SIMULATION RESULTS

This section validates the proposed algorithm by using numerical simulations.

The numerical simulation was performed on a cart-inverted pendulum model as the following:

$$(\theta_s + \mathcal{M}_1 l_s^2) \ddot{\phi} + C \dot{\phi} - \mathcal{M}_1 l_s (\ddot{r} \cos \phi - g \sin \phi) = 0 \quad (5)$$

$$(\mathcal{M}_0 + \mathcal{M}_1) \ddot{r} + F_r \dot{r} + \mathcal{M}_1 l_s (\dot{\phi}^2 \sin \phi - \ddot{\phi} \cos \phi) = F \quad (6)$$

Where ϕ is inverted pendulum angle, r is the cart displacement, C is the friction coefficient in the pivot, F_r is the viscous friction coefficient, l_s is the distance from the pivot to the center of mass of the pendulum, θ_s is the pendulum moment of inertia, $\mathcal{M}_0, \mathcal{M}_1$, are the cart mass and the pendulum mass respectively and F is the input force applied to the cart.

The parameters used for the numerical simulation are condensed in Table I.

Table 1. Cart-Pendulum Parameters

Parameter	Value
\mathcal{M}_1	3.200 Kg
\mathcal{M}_0	0.329 Kg
θ_s	0.008 Kg · m ²
F_r	6.200 Kg/s
C	0.009 Kg · m ² /s
l_s	0.440 m

According to section III, taking into account that the control objective is the pendulum stabilization, i.e. the desire angle ϕ is zero, the training sets were defined as follows:

$$S_k = (\phi_k, \phi_{k-1}, \phi_{k-2}, \dot{\phi}_k, \dot{\phi}_{k-1}, \dot{\phi}_{k-2}, r_k, r_{k-1}, r_{k-2}, \dot{r}_k, \dot{r}_{k-1}, \dot{r}_{k-2})$$

$$T_k = (-\phi_k, -\phi_{k-1}, -\phi_{k-2})$$

$$U_k = (u_{k-1}, u_{k-2})$$

And the kernel function used was:

$$k(x, y) = \exp(-\gamma \|x - y\|)$$

With $\gamma = 0.1$.

Results are shown in Figures 4, 5 and 6 for the pendulum stabilization in ideal conditions and with an input disturbance with a maximum power of 0.001 N.

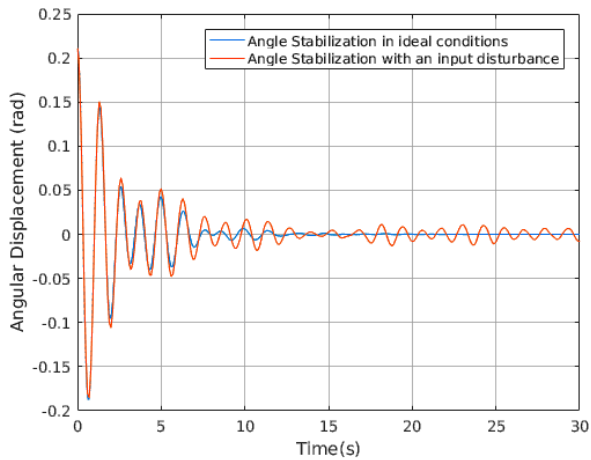


Figure 4. Pendulum Stabilization Angle

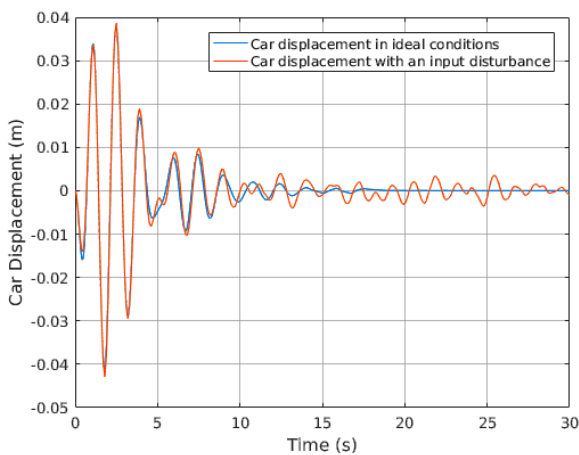


Figure 5. Car Displacement

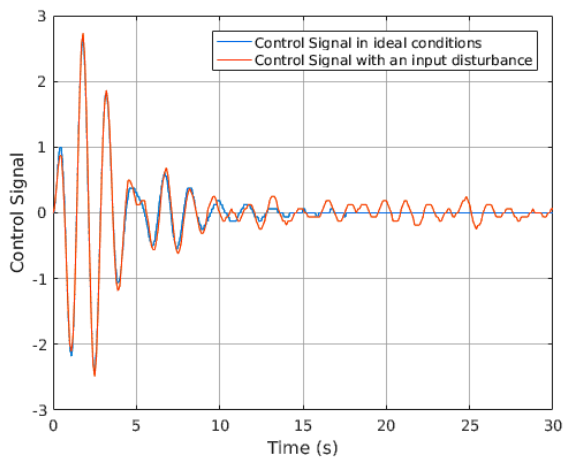


Figure 6. Stabilization Control Signal

CONCLUSIONS

This paper presented a methodology for using ε -SVR to reconstruct the control signal based on the system past information. Simulation results shows that the control approach is useful as control method, in this way, due to the way in which ε -SVR is formulated, it can be used when the plant model is not available or cannot be identified. It can also be observed, that the proposed methodology can deal with low input disturbances and non-linearities.

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