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Statistical Analysis of Type-II Progressively Censored Competing Risks Data form Chen Model

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Abstract

In reliability analysis, an investigator is often interested in the assessment of a specific risk in the presence of other risk factors. It is well known as the competing risks problem in statistical literature. So in this paper, we consider the competing risks model when the data is progressively Type-II censored with two parameter distribution with the bathtub shape or increasing failure rate function proposed by Chen [1]. The maximum likelihood procedure is used to get the point estimations and asymptotic confidence intervals of the unknown parameters, also the two bootstrap confidence intervals are also proposed. The analysis of a real data set to assess the performance of all these estimators. The different methods are compared through a simulation study.

Keywords: Chen model, competing risks model, Maximum likelihood estimation, Bayesian estimation, Markov Chan Montokarlo method.

INTRODUCTION

In the analysis of reliability data, the failure of individuals or items may be attributable to more than one cause or factor. These "risk factors" in some sense compete for the failure of the experimental unit. There are numerous examples in reliability experiments, where items may fail due to one of several causes. In traditional analyses of these datasets, the researcher is primarily interested in the distribution of lifetimes under one specific cause of failure, the effects of the other competing risks may play an important role in survival studies on slowly progressing diseases such as prostate cancer, say cancer, and all other causes are combined and treated as censored data. In engineering applications, the causes or risks may signify either multiple modes of failure for a complex unit or multiple components or subsystems which comprise an entire system. Occurrence of a system failure is caused by the earliest onset of any of these component failures. In this respect, the framework is that of a system with components connected in a series. Several studies have been carried out under this assumption and the risks follow different lifetime distributions, namely the exponential, lomax, lognormal, Weibull, modified Weibull, generalized exponential or exponentiated Weibull; see for example Moeschbergeret al. [2], Pascual [3], Cramer and Schmiedt [4], Sarhan et al. [5], Sarhan [6], Alwasel [7], Kundu and Basu [8] and Kundu and Sarhan [9].

Censoring occurs when exact lifetimes are known only for a portion of the individuals or units under study, while for the remainder of the lifetimes information on them is partial. There are several types of censored tests. The most common censoring schemes are Type-I (time) censoring, where the life testing experiment will be terminated at a prescribed time T, and Type-II (failure) censoring, where the life testing experiment will be terminated upon the r-th (r is pre-fixed) failure. However, the conventional Type-I and Type-II censoring schemes do not have the flexibility of allowing removal of units at points other than the terminal point of the experiment. A generalization of Type II censoring is the progressive Type II censoring. In this paper we consider competing risk data under progressive Type-II censoring. The censoring scheme is defined as follows:

Consider individuals in a study and assume that there are causes of failure which are known. At the time of each failure, one or more surviving units may be removed from the study at random. The data from a progressively Type-II censored sample is as follows:

$$(X_{1;m,n,\delta_1}, R_1) < (X_{2;m,n,\delta_1}, R_2) < \ldots < (X_{m;m,n,\delta_m}, R_m)$$

where $X_{1;m,n} < X_{2;m,n,} < \ \ldots \ < X_{m;m,n}$ denote the m observed failure times δ_1 , δ_2 , ..., δ_m are denote the causes of failure, and $\mathbf{R} = \{R_1, R_2, \ldots, R_m\}$ denote the number of units removed from the study at the failure times $X_{1;m,n} < X_{2;m,n,} < \ \ldots \ < X_{m;m,n}$. Note that the complete and Type-II right censored samples are special cases of the

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above scheme when $R_1 = R_2 = ... = R_n = 0$, and $R_1 = R_2 = ... = R_{m-1} = 0$ respectively. For an exhaustive list of references and further details on progressive censoring, the reader may refer to the book by Balakrishnan and Aggarwala [10].

Without loss of generality, we assume that there are only two independent causes of failure. All the methods presented in this paper may be easily extended to the case of $\delta > 2$. For a given censoring scheme, the likelihood function of the observed data $\mathbf{R} = \{R_1, R_2, ..., R_m\}$, the likelihood function of the observed data

$$L(\alpha,\beta_1,\beta_2) = C\prod_{i=1}^{m}\left\{h_1(x_{i;m,n})\right\}^{I(\delta_i=1)} \left\{h_2(x_{i;m,n})\right\}^{I(\delta_i=2)}\left\{S_1(x_{i;m,n})S_2(x_{i;m,n})\right\}^{I(R_i+1)}, \quad (1)$$

where

$$0 < X_{1;m,n} < X_{2;m,n,} < ... < X_{m;m,n} < \infty,$$

and

$$C = n(n-R_1-1)(n-R_1-R_2-1)...(n-R_1-R_2-R_{m-1}-m+1). \quad (2)$$

The main aim of this article is to assessment of a specific risk in the presence of other risk factors under the life time model with the bathtub shape or increasing failure rate function. Also, we present the explicit forms of the likelihood function with competing risks data in Sec. 3. in addation solving the mathematical model of the likelihood equations with different iterative procedure. Sec. 4 investigates the evolution of Fisher information matrix based on maximum likelihood estimation for the parameters under consideration. The MCMC method is addopted in the case of Bayes estimators. Analysis of real data sets for illustrative the results of the project. Making a simulation study for comparison between different methods of estimation.

MODEL FORMULATION AND NOTATION

Before proceeding any further, we assume that there are only two causes of failure. We describe different notations we are going to use in this paper.

Lifetime of the $j-th$ unit.

Lifetime of the $j-th$ individual under cause $j = 1,2.$

Cumulative distribution function (cdf) of $X_{j;}$.

Probability density function (pdf) of $F(.)$.

cdf of $X_{jj;}$.

pdf of $F(.)$.

Survival function of $X_{jj;}$

Indicator variable denoting the cause of failure of the j -th individual.

To simplify the notation we will use henceforth $X_{j;}$ instead of $X_{j;m,n}$, $j = 1,2,...,m$. The model studied in the paper satisfies the following assumptions

- The lifetime of unit is denoted as $X_{j;}$, $j = 1,2,...,m$. The time at which the unit fails due to cause is,. That is,.
- The distribution of the random variable $X_{ij;}$ is distribution with the Chen distribution with parameters α_i and β_j, $j = 1,2$ and $i = 1,2,...,m$. That is, the (pdf) and (cdf) of $X_{ij;}$ for each $i = 1,2,...,m$ are

$$f_j(x) = \alpha_j \beta x^{\beta-1} \exp\left\{x^\beta - \alpha\left(\exp\left(x^\beta\right)-1\right)\right\},$$
$$x > 0, \quad (\alpha_j > 0, \ \beta > 0), \quad (3)$$

$$F_j(x) = 1 - \exp\left\{1 - \alpha_j\left(\exp\left(x^\beta\right)-1\right)\right\}$$

The corresponding reliability and failure rate functions of this distribution at some , are given, respectively by

$$S_j(t) = \exp\left\{-\alpha_j\left(\exp\left(t^\beta\right)-1\right)\right\}, \quad t > 0,$$
$$H_j(t) = {}_\alpha \ j\beta x^{\beta-1}\exp\left(t^\beta\right), \quad t > 0. \quad (4)$$

Models with bathtub shaped or increasing failure rate function are useful in reliability analysis and particularly in reliability related decision making and cost analysis. There are a number of papers dealing with models for bathtub shaped failure rate. For example, Xie and Lai [11], Xie et al. [12] and Soliman et al. [13]. In this article, we focus on a two-parameter distribution with the bathtub shape or increasing FRF proposed by Chen [1]. This lifetime distribution has bathtub-shaped FRF if $\alpha \geq 1$; increasing FRF if $\beta < 1$ and this distribution becomes the exponential power distribution if $\alpha = 1$. Wu et al. [14] proposed the optimal estimation of the parameters of this lifetime distribution based on the doubly Type-II censored sample.

MAXIMUM LIKELIHOOD ESTIMATES

Let X_{ij} 's are CD random variables with parameters $(\alpha,\ \beta_j)$ for $i=1,\ 2,\ \ldots,\ m$ and $j=1,\ 2.$ For the distribution $F_j(.)$ of X_{ij} has the form (3), $\mathbf{R}=\{R_1, R_2, ..., R_m\}$ and the m observations $\{\ x^{\mathbf{R}}_{ij;m,n,k},\ j\in I_1\ \}$ and $\{\ x^{R}_{ij;m,n,k},\ j\in I_2\ \}$ the likelihood function (1), without the additive constant given by,

$$L(\beta,\alpha_1,\alpha_2 \mid \underline{x}) = \beta^m \alpha_1^{m_1}\alpha_2^{m_2} \exp\left\{\beta\sum_{i=1}^{m}\left(\log(x_i)+x_i\right) - (\alpha_1+\alpha_2)\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)\right\}, \tag{5}$$

Also the log-likelihood function written as follows,

$$\ell(\beta,\alpha_1,\alpha_2 \mid \underline{x}) = m\log\beta + m_1\log\alpha_1 + m_2\log\alpha_2 + \beta\sum_{i=1}^{m}\left(\log(x_i)+x_i\right) - (\alpha_1+\alpha_2)\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1) \tag{6}$$

The first partial derivatives of (6) can be calculated with respect to $\alpha_1,\ \alpha_1,\beta$ and equating each to zero, to get the likelihood equations as follows

$$\frac{\partial\ell(\beta,\alpha_1,\alpha_2 \mid \underline{x})}{\partial\alpha_1} = \frac{m_1}{\alpha_1} - \sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1) = 0, \tag{7}$$

$$\frac{\partial\ell(\beta,\alpha_1,\alpha_2 \mid \underline{x})}{\partial\alpha_2} = \frac{m_2}{\alpha_2} - \sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1) = 0. \tag{8}$$

Hence the ML estimate of α_1 and β_2 given as a function of β as follows

$$\hat{\beta}_1(\lambda) = \frac{m_1}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}. \tag{9}$$

and

$$\hat{\beta}_2(\lambda) = \frac{m_2}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}. \tag{10}$$

The profile likelihood function for β presented from (9) and (10), as

$$h(\beta) = m\log\beta + m_1\log\left[\frac{m_1}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}\right] + m_2\log\left[\frac{m_2}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}\right] + \beta\sum_{i=1}^{m}\left(\log(x_i)+x_i\right) - \left(\frac{m_1+m_2}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}\right)\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1). \tag{11}$$

by calculating the first partial derivatives of (11) with respect to $\beta,$ we obtain

$$\beta = \frac{m}{\left(\frac{m_1+m_2}{\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)}\right)\sum_{i=1}^{m}(R_i+1)x_i\exp(\beta x_i) - \sum_{i=1}^{m}\left(\log(x_i)+x_i\right)}. \tag{12}$$

Since (12) cannot be solved analytically for β, we can used numerical methods such fixed point or Newton ' s methods with initial value of the β obtained from the plot of profile likelihood function in (11).

Observed Fisher Information

By consideration the eq. (6), we have

$$\frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1^2} = -\frac{m_1}{\alpha_1^2}, \tag{13}$$

$$\frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2^2} = -\frac{m_2}{\alpha_2^2}, \tag{14}$$

$$\frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta^2} = -\frac{m}{\beta^2} - (\alpha_1+\alpha_2)\sum_{i=1}^{m}(R_i+1)x_i^2\exp(\beta x_i), \tag{15}$$

$$\frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1\partial\alpha_2} = \frac{\partial\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2\partial\alpha_1} = 0 \tag{16}$$

$$\frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1\partial\beta} = \frac{\partial\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2\partial\beta} = \frac{\partial^2\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta\partial\alpha_1} = \frac{\partial\ell(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta\partial\alpha_2} = -\sum_{i=1}^{m}(R_i+1)x_i\exp(\beta x_i) \tag{17}$$

The minus eq. (13-17) presence the Fisher information matrix $I(\alpha_1,\alpha_2,\beta)$. Under some mild regularity conditions, $(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta})$ is approximately bivariate normal with mean

$\left(\alpha_1,\alpha_2,\beta\right)$ and covariance matrix $I^{-1}\left(\alpha_1,\alpha_2,\beta\right)$. In practice, we usually estimate $I^{-1}\left(\alpha_1,\alpha_2,\beta\right)$ by $I^{-1}(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta})$. A simpler and equally valued procedure is to use the approximation

$$(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta})?\, N\big((\alpha_1,\alpha_2,\beta), I_0^{-1}(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta})\big), \qquad (18)$$

where $I_0\left(\alpha_1,\alpha_2,\beta\right)$ is observed information matrix

$$I_0^{-1}(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta}) = \begin{bmatrix} -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1^2} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1\partial\alpha_2} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_1\partial\beta} \\ -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2\partial\beta_1} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2^2} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\alpha_2\partial\beta} \\ -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta\partial\alpha_1} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta\partial\alpha_2} & -\frac{\partial^2(\beta,\alpha_1,\alpha_2|\underline{x})}{\partial\beta^2} \end{bmatrix}^{-1}_{(\hat{\beta}_1,\hat{\beta}_2,\hat{\lambda})} . \qquad (19)$$

Approximate confidence intervals for α_1,α_2 and β can be found by to be bivariate normal distributed with mean $\left(\alpha_1,\alpha_2,\beta\right)$ and covariance matrix $I_0^{-1}(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta})$. Thus, the $100(1-\alpha)\%$ approximate confidence intervals (ACI) for α_1,α_2 and $\acute{\epsilon}$ are

$$\hat{\alpha}_1 \mp z_{\frac{\alpha}{2}}\sqrt{v_{11}},\ \hat{\alpha}_1 \mp z_{\frac{\alpha}{2}}\sqrt{v_{22}} \text{ and } \hat{\beta} \mp z_{\frac{\alpha}{2}}\sqrt{v_{33}} \qquad (20)$$

respectively, where v_{11}, v_{22} and v_{33} are the elements on the main diagonal of the covariance matrix $I_0^{-1}(\hat{\beta}_1,\hat{\beta}_2,\hat{\lambda})$ and $z_{\frac{\alpha}{2}}$ is the percentile of the standard normal distribution with right-tail probability $\frac{\alpha}{2}$.

BAYES ESTIMATIONS USING MCMC

Under the squared error loss function the Bayes estimate of parameters $(\alpha_1,\alpha_2,\beta)$ and the corresponding credible interval by the Gibbs sampling technique are considered. It is assumed that (α_1,α_2) have gamma prior and uniform prior density for β. So that the joint prior density of α_1,α_2 and β can be written as $\pi(\alpha_1,\alpha_2,\beta)=\pi_1(\alpha_1)\pi_2(\alpha_2)$, where

$$\pi_1(\alpha_1)=\frac{b^a}{\Gamma(a)}\alpha_1^{a-1}\exp(-b\alpha_1),\ \alpha_1>0,\ a,\ b>0, \qquad (21)$$

and

$$\pi_2(\alpha_2)=\frac{d^c}{\Gamma(c)}\alpha_2^{c-1}\exp(-d\alpha_2),\ \alpha_1>0,\ c,\ d>0, \qquad (22)$$

Multiplying $\pi_1(\alpha_1)$, $\pi_2(\alpha_2)$ we obtain the joint prior density of α_1,α_2 and β as

$$\pi(\alpha_1,\alpha_2,\beta)=\frac{b^a d^c \alpha_1^{a-1}\alpha_2^{c-1}}{\Gamma(a)\Gamma(c)}\exp\left(-\left(b\alpha_1+d\alpha_2\right)\right),\ (\alpha_1,\alpha_2,\ \beta>0). \qquad (23)$$

Using the joint prior distribution of α_1,α_2 and β, the joint posterior density function given the data, denoted by $\pi^*(\alpha_1,\alpha_2,\beta\mid\underline{x})$, can be written as

$$\pi^*(\alpha_1,\alpha_2,\beta\mid\underline{x}) = \frac{L(\alpha_1,\alpha_2,\beta\mid\underline{x})\times\pi(\alpha_1,\alpha_2,\beta)}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha_1,\alpha_2,\beta\mid\underline{x})\times\pi(\alpha_1,\alpha_2,\beta)d\alpha_1 d\alpha_2 d\beta}, \qquad (24)$$

therefore, the Bayes estimate of any function of α_1,α_2 and β say $g(\alpha_1,\alpha_2,\beta)$, under squared error loss (SEL) function is

$$\hat{g}(\alpha_1,\alpha_2,\beta)=E_{\alpha_1,\alpha_2,\beta|\underline{x}}(g(\alpha_1,\alpha_2,\beta)) = \frac{\int_0^\infty\int_0^\infty\int_0^\infty g(\alpha_1,\alpha_2,\beta)L(\alpha_1,\alpha_2,\beta\mid\underline{x})\times\pi(\alpha_1,\alpha_2,\beta)d\alpha_1 d\alpha_2 d\beta}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha_1,\alpha_2,\beta\mid\underline{x})\times\pi(\alpha_1,\alpha_2,\beta)d\alpha_1 d\alpha_2 d\beta}. \qquad (25)$$

Generally, the ratio of two integrals given by (25) cannot be obtained in a closed form. In this case, we use the MCMC method to generate samples from the posterior distributions and then compute the Bayes estimator of $g(\alpha_1,\alpha_2,\beta)$ under the SEL function.

MCMC approach

A wide variety of MCMC schemes are available, and it can be difficult to choose among them. An important sub-class of MCMC methods are Gibbs sampling and more general Metropolis-within-Gibbs samplers. The conditional posterior pdf's of β_1, β_2 and λ are as follows

$$L\left(\beta,\alpha_1,\alpha_2\mid\underline{x}\right)=\beta^m\alpha_1^{m_1}\alpha_2^{m_2}\exp\left\{\beta\sum_{i=1}^{m}\left(\log\left(x_i\right)+x_i\right) -\left(\alpha_1+\alpha_2\right)\sum_{i=1}^{m}(R_i+1)(\exp(\beta x_i)-1)\right\}, \qquad (26)$$

$$\pi_1^*(\beta \mid \alpha_1, \alpha_2, \underline{x}) \propto \beta^m \exp\left\{\beta \sum_{i=1}^{m} (\log(x_i) + x_i) - (\alpha_1 + \alpha_2) \sum_{i=1}^{m} (R_i + 1)(\exp(\beta x_i) - 1)\right\} \quad (27)$$

$$\pi_2^*(\alpha_1 \mid \alpha_2, \beta, \underline{x}) \sim \text{Gamma}(m_1 + a, b_1 + \sum_{i=1}^{m} (R_i + 1)(\exp(\beta x_i) - 1)), \quad (28)$$

and

$$\pi_3^*(\alpha_2 \mid \alpha_1, \beta, \underline{x}) \sim \text{Gamma}(m_2 + c, b_2 + \sum_{i=1}^{m} (R_i + 1)(\exp(\beta x_i) - 1))), \quad (29)$$

The log likelihood function of (26) is log-concave for $\frac{\partial(\beta|\alpha_1,\alpha_2,\underline{x})}{\partial\beta^2} < 0$. Therefore to generate from these distributions, we use the devroy or Metropolis—Hastings (MH) methods (Metropolis et al.[Metropolis et al.(1953)]) with normal proposal distribution. The algorithm of Gibbs sampling is as follows.

Algorithm:

- $\beta_0 = \hat{\beta}$, $M =$ n-burn.
- Generate α_1 from gamma distribution $\pi_2^*(\alpha_1 \mid \alpha_2, \beta, \underline{x})$ given by (27).
- Generate α_2 from gamma distribution $\pi_3^*(\alpha_2 \mid \alpha_1, \beta, \underline{x})$ given by (28).
- Generate β from $\pi_1^*(\beta \mid \alpha_1, \alpha_2, \underline{x})$ given by (26).
- Repeat steps 2-4 N times we obtain $\Phi_1^l, \Phi_2^l, \ldots, \Phi_N^l$, $l = 1, 2, 3$, ($\Phi^1 = \alpha_1$, $\Phi^2 = \alpha_2$, $\Phi^3 = \beta$).
- Obtain the Bayes estimate of Φ^l with respect to the SEL function as

$$\hat{E}(\Phi^l \mid \underline{x}) = \frac{1}{N - M} \sum_{i=M+1}^{N} \Phi_i^l. \quad (30)$$

- To compute the credible intervals of Φ^l , order Φ_{M+1}^l, Φ_{M+2}^l, . . . , Φ_N^l as $\Phi_{(1)}^l, \Phi_{(2)}^l$, . . . , $\Phi_{(N-M)}^l$. Then the $100(1-\alpha)\%$ symmetric credible intervals

$$\left(\Phi^l_{((N-M)\alpha/2)},\ \Phi^l_{((N-M)(1-\alpha/2))}\right). \quad (31)$$

SIMULATION STUDY

In this section we primarily perform some simulation experiments to observe the behavior of the different methods. Monte Carlo simulations were performed utilizing 1000 progressively first-failure-censored competing risks samples for each simulations. The samples were generated by using the algorithm described in Balakrishnan and Sandhu [14] using different choices of n, and m. Now, we describe choosing the true values of parameters α_1, α_2 and β with known prior. For given values (a, b, c, d), generate according the last α_1, α_2 , from gamma distribution (23) . The prior parameters are selected to satisfy $E(\alpha_1) = \frac{a}{b} \cong \alpha_1$, and $E(\alpha_2) = \frac{c}{d} \cong \alpha_2$ is approximately the mean of gamma distribution. We used different sample sizes n , different effective sample sizes m and the following set of parameters { α_1 = 1.01, β_2 = 1.02, β =1.0} . In both case, we considered the different sampling schemes.

For each data point with probability $\frac{\alpha_1}{\alpha_1+\alpha_2}$ and $\frac{\alpha_2}{\alpha_1+\alpha_2}$, we assigned the cause of failure as 1 or 2 respectively. In each case, we calculate the MLEs and Bayes MCMCs as well as the asymptotic confidence intervals based on the approximate Fisher information matrix and the credible intervals. This process is repeated 1000 times and compute the average, standard errors of the different estimates. We also compute 95% confidence intervals using the observed Fisher information matrix, replacing the parameters by MLEs and Bayes MCMCs. We report the average confidence lengths and the coverage percentages over 1000 replications. The results of simulation study are reported in Tables 4 and 5.

Table1: The average estimates and their mean squared errors (within brackets) for $\alpha_1 = 1.01$, $\alpha_2 = 1.02$ and $\beta = 0.9$

n	m	Scheme	Metod	α_1	α_2	β
30	15	$(15,0^{14})$	MLE	1.0081(0.1000)	0.0091(0.0078)	0.0190(0.0183)
			B:MCMC	1.0710(0.1200)	0.0254(0.0068)	0.0105(0.0015)
		$(0^{14},15)$	MLE	1.0088(0.1066)	0.0183(0.0097)	0.0327(0.0516)
			B:MCMC	1.0123(0.1062)	0.0254(0.0083)	0.0107(0.0020)
		$(0^7,15,0^7)$	MLE	1.0105(0.1055)	0.0191(0.0079)	0.0197(0.0214)
			B:MCMC	1.0135(0.1052)	0.0250(0.0078)	0.0108(0.0018)
		(1^{15})	MLE	1.0094(0.1056)	0.0201(0.0088)	0.0256(0.0343)
			B:MCMC	1.0131(0.1051)	0.0265(0.0081)	0.0105(0.0017)
	25	$(5,0^{24})$	MLE	1.0092(0.1041)	0.0195(0.0076)	0.0144(0.0113)
			B:MCMC	1.0112(0.1037)	0.0232(0.0067)	0.0100(0.0010)
		$(0^{24},5)$	MLE	1.0101(0.1047)	0.0199(0.0078)	0.0163(0.0191)
			B:MCMC	1.0120(0.1042)	0.0237(0.0064)	0.0100(0.0012)
50	25	$(25,0^{24})$	MLE	1.0088(0.1037)	0.0181(0.0075)	0.0173(0.0130)
			B:MCMC	1.0114(0.1035)	0.0231(0.0066)	0.0101(0.0007)
		$(0^{24},25)$	MLE	1.0100(0.1046)	0.0184(0.0070)	0.0252(0.0361)
			B:MCMC	1.0122(0.1043)	0.0230(0.0061)	0.0101(0.0008)
		$(0^{12},25,0^{12})$	MLE	1.0096(0.1040)	0.0199(0.0067)	0.0168(0.0139)
			B:MCMC	1.0117(0.1028)	0.024(0.0072)	0.0101(0.0009)
		(1^{25})	MLE	1.0096(0.1039)	0.0184(0.0057)	0.0186(0.0205)
			B:MCMC	1.0115(0.1210)	0.0224(0.0056)	0.0100(0.0009)

Table2: The average 95% confidence lengths and the corresponding coverage percentages (within brackets) for $\alpha_1 = 1.01$, $\alpha_2 = 1.02$ and $\beta = 0.9$

n	m	Scheme	Metod	β_1	β_2	λ
30	15	$(15,0^{14})$	MLE	0.0199(0.89)	0.0340(0.91)	0.0566(0.94)
			B:MCMC	0.0163(0.91)	0.0239(0.92)	0.0032(0.97)
		$(0^{14},15)$	MLE	0.0219(0.89)	0.0382 (0.88)	0.0838(0.94)
			B:MCMC	0.0165(0.92)	0.0237(0.90)	0.0033(0.98)
		$(0^7,15,0^7)$	MLE	0.0211(0.92)	0.0314(0.90)	0.0621(0.95)
			B:MCMC	0.0174(0.90)	0.0239(0.91)	0.0032(0.96)
		(1^{15})	MLE	0.0208(0.89)	0.0354(0.91)	0.1009(0.92)
			B:MCMC	0.0172(0.93)	0.0248(0.90)	0.0033(0.98)
	25	$(5,0^{24})$	MLE	0.0160(0.92)	0.0271(0.90)	0.0402(0.94)
			B:MCMC	0.0120(0.94)	0.0175(0.92)	0.0032(0.99)
		$(0^{24},5)$	MLE	0.0180(0.92)	0.0298(0.90)	0.0464(0.93)
			B:MCMC	0.0127(0.92)	0.0180(0.92)	0.0032(0.97)
50	25	$(25,0^{24})$	MLE	0.0152(0.93)	0.0253(0.90)	0.0413(0.91)
			B:MCMC	0.0122(0.93)	0.0175(0.92)	0.0031(0.99)
		$(0^{24},25)$	MLE	0.0186(0.92)	0.0299(0.9)	0.0409(0.95)
			B:MCMC	0.0128(0.94)	0.0176(0.91)	0.0032(1.00)
		$(0^{12},25,0^{12})$	MLE	0.0152(0.92)	0.0248(0.94)	0.0455(0.95)
			B:MCMC	0.0125(0.93)	0.0181(0.94)	0.0032(0.98)
		(1^{25})	MLE	0.0158(0.92)	0.0250(0.93)	0.0678(0.95)
			B:MCMC	0.0121(0.93)	0.0170(0.90)	0.0032(0.96)

CONCLUSIONS

The latent failure times under the competing risks follow independent Chen distributions with common one of the shape parameters under general censoring scheme called progressive type censored scheme. We compared different methods and the performance of the unknown parameters based on MLE, Bayes in this setting. We have then conducted a simulation study to assess the performance of all these procedures and a numerical example has been presented to illustrate all the methods of inference developed in this paper. A simulation study was conducted to examine and compare the performance of the proposed methods for different sample sizes and different censoring schemes. From the results, we observe the following.

1) All of the results obtained in this article can be specialized to: the complete sample, the first-failure censoring sample and the Type-II censoring.

2) Tables 1 and 2 show that the Bayes estimators perform better than the MLEs.

3) Tables 1 and 2, as the effective sample proportion $\frac{m}{n}$ increases, the MSEs of the estimators, reduce significantly. For fixed n, m, we can determine the censoring scheme which is most efficient, for example, from the tables, we observe that the censoring scheme, corresponding to the case of withdraw a length first stage of the test, seems to provide the smallest MSE for the estimate of parameters.

4) From Tables 1 and 2, we see that the coverage probabilities of the approximate confidence intervals and credible intervals are close to the desired level.

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