

A Diagonal Optimal Algorithm to Solve Interval Integer Transportation Problem

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Abstract

In this article we applied Diagonal optimal algorithm to solve Interval Integer Transportation problem. An example is solved to illustrate the Diagonal optimal algorithm and results are validated with the existing methods.

Keywords: Interval Fuzzy number, Interval arithmetic operations, Interval ranking techniques, Interval Integer Transportation problems, Diagonal Optimal method, Optimal solution.

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INTRODUCTION

Transportation problems play an important role in industry and other applications. In a transportation problem goods should be transported from various sources to various sinks with the minimum cost. Various algorithms were proposed to solve transportation problems with certain parameters. In real life applications it is difficult to determine those parameters in certain. To deal with these vagueness many researchers [1-3,5 -8,12] proposed many techniques like fuzzy and interval numbers. Even though many researchers discussed about interval ranking techniques, very few of them used it for transportation problems. Juman et al., [6] developed a heuristic solution technique for solving transportation problems with interval numbers. Das et al. [3] solved interval transportation problem by considering the right bound and the midpoint of the interval. Sengupta et al., [11] proposed a method to solve interval transportation problems by taking the midpoint and width of the interval in the objective function. Safi et al.,[10] solved a fixed charge transportation problems convert the interval fuzzy constraints into multi-objective fuzzy constraints,. Pandian et al.,[9] proposed separation method to solve fully interval integer transportation problems. In this article we proposed diagonal optimal algorithm to solve interval integer transportation problem. The midpoint ranking technique was used to rank the fuzzy numbers in this article. The algorithm is illustrated through an example. This algorithm may useful for the decision makers to solve interval integer transportation problems. The organization of this article is given as follows:

In this paper we introduce the basic concepts and definitions of fuzzy numbers and deals with the proposed new algorithm. To solve the procedure with suitable example is illustrated.

1. Definition:

An interval number A is defined as $A = [a_1, a_2] = \{x: a_1 \leq x \leq a_2, x \in \mathbb{R}\}$. Here $a_1, a_2 \in \mathbb{R}$ are the lower and upper bound of the intervals [4].

2. Definition:

Interval Numbers Arithmetic [4]

Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are two interval numbers.

Addition:

$$A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2].$$

Subtraction:

$$A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1].$$

Multiplication:

$A * B = [x, y]$ where $x = \min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$ and $y = \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$.

3. Definition:

Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are two interval numbers. Let $\alpha = \frac{a_1+a_2}{2}$ and $\beta = \frac{b_1+b_2}{2}$.

If $\alpha \leq \beta$ then $A \leq B$. If $\alpha \geq \beta$ then $A \geq B$.

4. Definition

Equivalent Interval number: Two interval numbers $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are said to be equivalent if their crisp values $[R(A) = R(B)]$ are equal.

5. Interval Integer Transportation problem

$$\text{Minimize } [z_1, z_2] = \sum_{i=1}^m \sum_{j=1}^n [C_{ij}, D_{ij}] * [x_{ij}, y_{ij}]$$

subject

$$\text{to } \sum_{i=1}^m [x_{ij}, y_{ij}] = [s_i, s_j], \sum_{j=1}^n [x_{ij}, y_{ij}] = [d_i, d_j], i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

$$[z_1, z_2] = \sum_{i=1}^m \sum_{j=1}^n [C_{ij}, D_{ij}] * \text{total interval transportation cost}$$

$[s_i, s_j]$ the total fuzzy availability of the product at i^{th} source

$[d_i, d_j]$ the total fuzzy demand of the product at j^{th} destination

$[C_{ij}, D_{ij}]$ unit fuzzy transportation cost from the i^{th} source to the j^{th} destination

$[x_{ij}, y_{ij}]$ the number of approximate units of the product that should be transported from the i^{th} source to j^{th} destination or fuzzy decision variables

The new algorithm is as follow:

- 1) Locate the two cells that have minimum cost and next to minimum cost in each row, then find their difference (Penalty) along the side of the table against the corresponding row.
- 2) Locate the two cells that have minimum cost and next to minimum cost in each column, then find their difference (Penalty) below the table against the corresponding column.
- 3) Locate the maximum penalty. If it is along the side of the table, make maximum assignment to the cell having minimum cost in that row. If it is below the table, make maximum assignment to the cell having minimum cost in that column. Continue in the same manner until all assignments are made.
- 4) If the penalties corresponding to two or more rows/columns are equal, find the element-wise difference between first and third minimum value. Identify the maximum among them and assign the minimum cost among them. This step gives the initial solution.

Remarks: For finding the optimal solution, we will follow step 5 and 6

- 5) Write these assigned costs on the top of the column of original assignment problem. Let $[C_{ij}, D_{ij}]$ be the assigned cost for column. Subtract $[C_{ij}, D_{ij}]$ from each entry of cost matrix the corresponding column of assignment matrix.

- 6) Construct a rectangle in such a way that one corner contains negative Fuzzy penalty and remaining two corners are allocated to the assigned cost values in corresponding row and column. Calculate the sum of extreme cells of unassigned diagonal, say r_{ij} . Locate $\forall r_{ij}$. Identify the most negative r_{ij} and exchange the assigned cell of diagonals. Continue the process until all negative penalties are resolved.

- 7) **Remarks:** If any $r_{ij} \approx 0$, then exchange the cells of diagonals at the end.

- 8) In each assignment cell allocate the least possible amount. For example if the assigned cell is (i, j) then $[x_{ij}, y_{ij}] \approx \min\{[s_i, s_j], [d_i, d_j]\}$ in the cell (i, j) and cross out the i^{th} row if $[s_i, s_j] < [d_i, d_j]$ or j^{th} column if $[s_i, s_j] > [d_i, d_j]$. Cross out both the row and column if $[s_i, s_j] \approx [d_i, d_j]$. Repeat this procedure until all the requirements are satisfied.

Example 1

Consider the following interval integer transportation problem

D_1	D_2	D_3	D_4	Supply	
S_1	[3,5]	[2,6]	[2,4]	[1,5]	[7,9]
S_2	[4,6]	[7,9]	[7,10]	[9,11]	[17,21]
S_3	[4,8]	[1,3]	[3,6]	[1,2]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Applying the proposed algorithm

$$\text{penalty} \begin{pmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \\ [0,0] & [0,0] & [0,0] & [0,0] \\ [3,5] \uparrow & [1,3] & [2,4] & [1,2] \end{pmatrix} \begin{matrix} \text{penalty} \\ [-2,4] \\ [1,5] \\ [-1,2] \\ [0,0] \end{matrix}$$

$$\text{penalty} \begin{pmatrix} [2,6] & [2,4] & [1,5] \\ [7,9] & [7,10] & [9,11] \\ [1,3] & [3,6] & [1,2] \\ [-1,5] \uparrow & [-1,4] & [-1,4] \end{pmatrix} \begin{matrix} \text{penalty} \\ [-2,4] \\ [-3,2] \\ [-1,1] \end{matrix}$$

$$\text{penalty} \begin{pmatrix} [2,4] & [1,5] \\ [7,10] & [9,11] \\ [3,8] & [4,10] \uparrow \end{pmatrix} \begin{matrix} \text{penalty} \\ [-3,3] \\ [-1,4] \end{matrix}$$

The initial solution is

$$\begin{pmatrix} [0,0] & [1,3] & [7,10] & [1,5] \\ [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \\ [0,0] & [0,0] & [0,0] & [0,0] \end{pmatrix}$$

$$\begin{pmatrix} [3,5] & [-1,5] & [-8,-3] & [-4,4] \\ [4,6] & [4,8] & [-3,3] & [4,10] \\ [4,8] & [-2,2] & [-7,-1] & [-4,1] \\ [0,0] & [-3,-1] & [-10,-7] & [-5,-1] \end{pmatrix}$$

$$r_{13} = \begin{pmatrix} [-8,-3] & [-4,4] \\ [-3,3] & [4,10] \end{pmatrix} = [-8,-3] + [4,10] = [-4,7] \geq 0$$

$$r_{33} = \begin{pmatrix} [4,8] & [-3,3] \\ [-2,2] & [-7,-1] \end{pmatrix} = [4,8] + [-7,-1] = [-3,7] \geq 0$$

$$r_{34} = \begin{pmatrix} [-1,5] & [-4,4] \\ [-2,2] & [-4,1] \end{pmatrix} = [-1,5] + [-4,1] = [-5,6] \geq 0$$

$$r_{42} = \begin{pmatrix} [4,8] & [-2,2] \\ [0,0] & [-3,-1] \end{pmatrix} = [4,8] + [-3,-1] = [1,7] \geq 0$$

$$r_{43} = \begin{pmatrix} [4,6] & [-3,3] \\ [0,0] & [-10,-7] \end{pmatrix} = [4,6] + [-10,-7] = [-6,-1] \leq 0$$

$$r_{44} = \begin{pmatrix} [3,5] & [-4,4] \\ [0,0] & [-5,-1] \end{pmatrix} = [3,5] + [-5,-1] = [-2,4] \geq 0$$

Out of all unassigned cells $r_{43} \leq 0$. We replace the allocation (2,3) in the third column to (4,3) and the allocation (4,1) in the first column to (2,1).

$$\begin{pmatrix} [4,6] & [1,3] & [0,0] & [1,5] \\ [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \\ [0,0] & [0,0] & [0,0] & [0,0] \end{pmatrix}$$

$$\begin{pmatrix} [-3,1] & [-1,5] & [2,4] & [-4,4] \\ [-2,2] & [4,8] & [7,10] & [4,10] \\ [-2,4] & [-2,2] & [3,6] & [-4,1] \\ [-6,4] & [-3,-1] & [0,0] & [-5,-1] \end{pmatrix}$$

Here $r_{34} \geq 0, r_{41} \geq 0, r_{42} \geq 0, r_{44} \geq 0$, therefore the optimal assignment are

$$\begin{pmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \\ [0,0] & [0,0] & [0,0] & [0,0] \end{pmatrix}$$

Do the allocations according to the algorithm, we get

The optimal solution is given by

	D_1	D_2	D_3	D_4	Supply
S_1	[3,5]	[2,6]	[2,4]	[1,5] ^[7,9]	[7,9]
S_2	[4,6] ^[10,12]	[7,9]	[7,10] ^[3,13]	[9,11]	[17,21]
S_3	[4,8]	[1,3] ^[2,4]	[3,6] ^[2,10]	[1,2] ^[6,10]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

The transportation cost is given by $[1,5] * [7,9] + [4,6] * [10,12] + [7,10] * [3,13] + [1,3] * [2,4] + [3,6] * [2,10] + [1,2] * [6,10] = [7,45] + [40,72] + [21,130] + [2,12] + [6,60] + [6,20] = [82,349]$

CONCLUSION

An algorithm is proposed for solving interval integer transportation problem by using diagonal optimal algorithm. This algorithm is effective and easy to understand. This method is effective even for solving any kind of interval integer transportation problems.

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