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A Feedback Queuing Model with Chances of Revisit of Customer at Most Twice to Any of the Three Servers

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Abstract

A queuing model has been developed for a system having three servers wherein a customer may revisit to any of the servers. The revisit of the customer is limited to maximum twice. The service is firstly done by the first server and then by the second or third server. However the customer after getting the service from second/third server may revisit to any of the servers or may leave the system depending upon his/her satisfaction. Whenever, a customer revisits, the probability of leaving the server does not remain same as that was for leaving that server on his/her previous visit. The steady-state equations have been derived for finding mean queue length using generating function technique and the graphical analysis of the model is done thereafter.

Keywords: Queuing model, Three servers, Feedback, Revisit of customer limited to twice, Expected Queue Length.

INTRODUCTION

Multi-server and multilevel queuing models have widely been discussed in the field of Queuing theory by a large number of researchers including Jianghua and Jinting (2006), Houdt et al. (2008), Zadeh (2015). Queuing models with feedback have also been studied by a good number of researchers such as Luo et al. (2009), Peng (2016), Kumar and Taneja (2017), Kotb and Akhdar (2018), Ayyappan and Udayageetha (2018). Situation of revisit of customer to a server may be observed in practical life such as in administration, hospitals, manufacturing etc. Kumar and Taneja (2017) discussed a feedback queuing model considering three servers wherein one is centrally linked with the other two servers and customers can revisit for service at most once. However, in practical situations, the customer may not be satisfied even after getting the service twice. Therefore s/he may revisit any server more than once also but at most twice and hence the present study.

The present paper deals with a queuing system with provision of revisit to any of the three servers. Initially, a customer arrives at first server from outside the system and then s/he may go to second or third server depending upon his/her satisfaction. If s/he is not satisfied by the service then s/he may revisit to the concerned servers but at most twice. The differential-difference equations in steady state have been derived to find the mean queue length using generating function technique. Numerical examples have also been given at the end to validate the results.

NOTATIONS

λ = Mean Arrival rate at 1st server (S_1)

μ_1 = mean service rate of 1st server (S_1)

μ_2 = Mean service rate of 2nd server (S_2)

μ_3= Mean service rate of 3rd server (S_3)

n_1, n_2, n_3 = be the no. of customers at 1st, 2nd and 3rd server at any time t.

a^i = probability of customer leaving 1st server ith time, where i = 1, 2, 3.

b^j = probability of customer leaving 2nd server jth time where j = 1, 2, 3.

c^k = probability of customer leaving 3rd server kth time, where k = 1, 2, 3.

p_{12}^i = probability of customer going from 1st server to 2nd server ith time, where i = 1, 2, 3.

p_{13}^i = probability of customer going from 1st server to 3rd server ith time, where i = 1, 2, 3.

p_2^j = probability of customer going outside the system from 2nd server jth time, where j = 1, 2, 3.

p_{23}^j = probability of customer going from second server to 3rd server jth time, where j = 1, 2, 3.

p_{21}^j = probability of customer going from second server to 1st server jth time, where j = 1, 2, 3.

p_3^k = probability of customer going outside the system from third server kth time, where k = 1, 2, 3.

p_{32}^{k} = probability of customer going from third server to second server kth time, where k = 1, 2, 3.

p_{31}^{k} = probability of customer going from third server to 1st server kth time, where k = 1, 2, 3.

$$A_3 = \sum_{i=1}^{3} a^i p_{12}^i,\; B_3 = \sum_{i=1}^{3} a^p p_{13}^i,\; C_3 = \sum_{j=1}^{2} b^j p_{21}^j,\; D_3 = \sum_{j=1}^{3} b^j p_2^j,$$

$$E_3 = \sum_{j=1}^{3} b^j p_{23}^j,\; L_3 = \sum_{k=1}^{3} c^k p_3^k,\; M_3 = \sum_{k=1}^{2} c^k p_{31}^k,\; N_3 = \sum_{k=1}^{3} c^k p_{32}^k$$

such that $A_3 + B_3 = 1$, $C_3 + D_3 + E_3 = 1$, and $L_3 + M_3 + N_3 = 1$.

FORMULATION OF THE PROBLEM

The queue network consists of three service channels 1st, 2nd and 3rd; whereas 2nd and 3rd are commonly linked with the 1st server. It is assumed that customer arrive at first server from outside the system with mean rate λ according to a poisson process and then goes to second and third servers for required services. Let we show the situation by the following diagram.

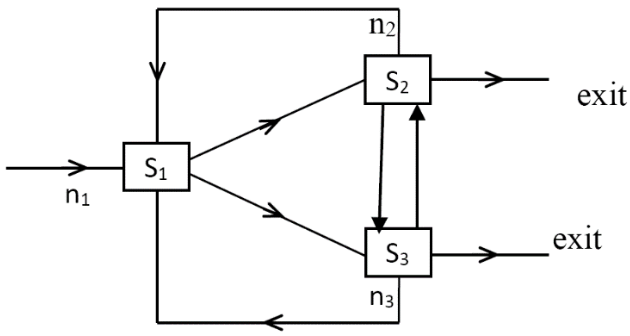
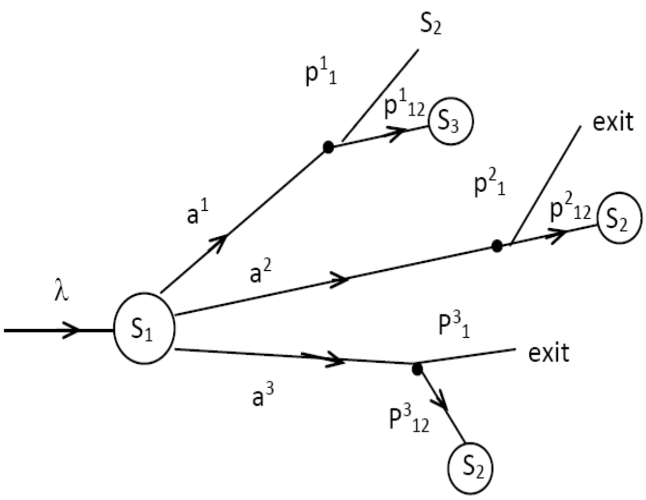


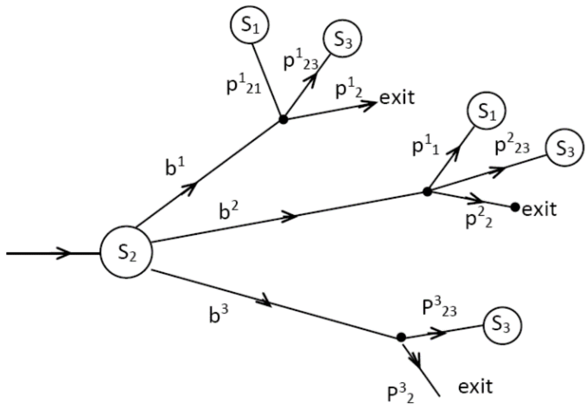
Figure 1. Diagram Representing General Feature of the Model

Let us now show all the possibilities occurring in the queuing system under consideration by the state transition diagram given as follows:

1. Possible states leaving S_1 server:



2. Possible states leaving S_2 server:



3. Possible States leaving S_3:

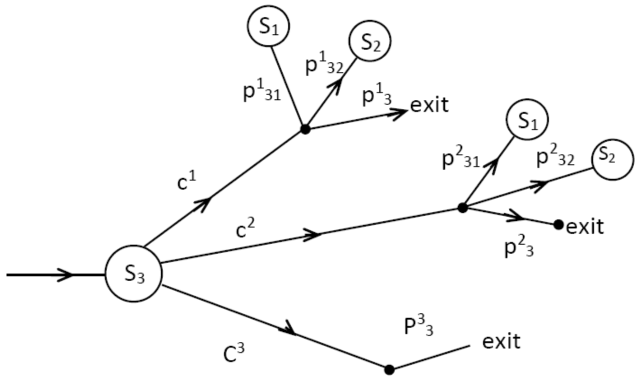


Figure 2. Transition diagram showing various states of the system

A customer after getting service at first server first time, either s/he goes to second server with probability p_{12}^{1} or to 3rd server with probability p_{13}^{1} such that $p_{12}^{1} + p_{13}^{1} = 1$. If the customer is not satisfied with the service then s/he may revisit the server atmost twice. So, $p_{12}^{i} + p_{13}^{i} = 1$ for i=1, 2, 3.

If the customer goes to second server after getting service first time from first server then, either s/he quits the system with probability p_2^1 or goes back to the first server with probability p_{21}^1 or to third server with probability p_{23}^1 such that $p_2^1 + p_{21}^1 + p_{23}^1 = 1$. If not satisfied with the service then s/he may revisit the server atmost twice. So, $p_2^j + p_{21}^j + p_{23}^j = 1$ for j=1, 2 and $p_2^j + p_{23}^j = 1$ for j=3.

Similarly if the customer goes to third server from first server first time then from third server either s/he exits the system with probability p_3^1 or goes to second server with probability p_{32}^1 or back to first server with probability p_{31}^1 and hence $p_3^1 + p_{32}^1 + p_{31}^1 = 1$. If not satisfied with the service then s/he may revisit the server atmost twice. So, $p_3^k + p_{31}^k + p_{32}^k = 1$, for k=1,2 and $p_3^k + p_{32}^k = 1$ for k=3.

Let $P_{n_1,n_2,n_3}(t)$ be the probability that there are n_1, n_2, n_3 customers on 1st, 2nd and 3rd at time t. The steady–state equations are given by:

$$(\lambda + \mu_1 + \mu_2 + \mu_3) P_{n_1,n_2,n_3} = \lambda P_{n_1-1,n_2,n_3} + A_3\mu_1 P_{n_1+1,n_2-1,n_3} + B_3\mu_1 P_{n_1+1,n_2,n_3-1} + C_3\mu_2 P_{n_1-1,n_2+1,n_3} + D_3\mu_2 P_{n_1,n_2+1,n_3} + E_3\mu_2 P_{n_1,n_2+1,n_3-1} + L_3\mu_3 P_{n_1,n_2,n_3+1} + M_3\mu_3 P_{n_1-1,n_2,n_3+1} + N_3\mu_3 P_{n_1,n_2-1,n_3+1} \quad \ldots(1)$$

for $n_1, n_2, n_3 > 0$.

$$(\lambda + \mu_2 + \mu_3) P_{0,n_2,n_3} = A_3\mu_1 P_{1,n_2-1,n_3} + B_3\mu_1 P_{1,n_2,n_3-1} + E_3\mu_2 P_{0,n_2+1,n_3-1} + L_3\mu_3 P_{0,n_2,n_3+1} + N_3\mu_3 P_{0,n_2-1,n_3+1} + D_3\mu_3 P_{0,n_2+1,n_3} \quad \ldots(2)$$

for $n_1 = 0$, $n_2, n_3 > 0$.

$$(\lambda + \mu_1 + \mu_2) P_{n_1,n_2,0} = \lambda P_{n_1-1,n_2,0} + A_3\mu_1 P_{n_1+1,n_2-1,0} + C_3\mu_2 P_{n_1-1,n_2+1,0} + N_3\mu_3 P_{n_1,n_2-1,1} + D_3\mu_2 P_{n_1,n_2+1,0} + L_3\mu_3 P_{n_1,n_2,1} + M_3\mu_3 P_{n_1-1,n_2,1} \quad \ldots(3)$$

for $n_3 = 0$ and $n_1, n_2 > 0$.

$$(\lambda + \mu_1 + \mu_2) P_{n_1,n_2,0} = \lambda P_{n_1-1,n_2,0} + A_3\mu_1 P_{n_1+1,n_2-1,0} + C_3\mu_2 P_{n_1-1,n_2+1,0} + N_3\mu_3 P_{n_1,n_2-1,1} + D_3\mu_2 P_{n_1,n_2+1,0} + L_3\mu_3 P_{n_1,n_2,1} + M_3\mu_3 P_{n_1-1,n_2,1} \quad \ldots(4)$$

for $n_3 = 0$ and $n_1, n_2 > 0$.

$$(\lambda + \mu_3) P_{0,0,n_3} = B_3\mu_1 P_{1,0,n_3-1} + D_3\mu_2 P_{0,1,n_3} + E_3\mu_2 P_{0,1,n_3-1} + L_3\mu_3 P_{0,0,n_3+1} \quad \ldots(5)$$

for $n_1, n_2 = 0$ and $n_3 > 0$.

$$(\lambda + \mu_1) P_{n_1,0,0} = \lambda P_{n_1-1,0,0} + D_3\mu_2 P_{n_1,1,0} + L_3\mu_3 P_{n_1,0,1} + M_3\mu_3 P_{n_1-1,1} + N_3\mu_3 P_{n_1,n_2-1,n_3+1} \quad \ldots(6)$$

for $n_2, n_3 = 0$ and $n_1 > 0$.

$$(\lambda + \mu_2) P_{0,n_2,0} = A_3\mu_1 P_{1,n_2-1,0} + D_3\mu_2 P_{0,n_2+1,0} + L_3\mu_3 P_{0,n_2,1} + N_3\mu_3 P_{0,n_2-1,1} \quad \ldots(7)$$

for $n_1, n_3 = 0$ and $n_2 > 0$.

$$\lambda P_{0,0,0} = D_3\mu_2 P_{0,1,0} + \mu_3 L_3 P_{0,0,1} \quad \ldots(8)$$

for $n_1, n_2, n_3 = 0$.

To find steady–state solution of the model

Let us define g.f. to solve the steady-state equations from (1) to (8)

$$F(x,y,z)=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}P_{n_1,n_2,n_3}(t)x^{n_1}y^{n_2}z^{n_3} \quad \ldots (9)$$

In order to solve the expression, we use the following partial generating functions:

$$F_{n_2,n_3}(x)=\sum_{n_1=0}^{\infty}P_{n_1,n_2,n_3}(t).x^{n_1}\ ,G_{n_1,n_3}(y)=\sum_{n_2=0}^{\infty}P_{n_1,n_2,n_3}(t).y^{n_2} \quad \ldots(10)$$

and

$$H_{n_1,n_2}(z)=\sum_{n_3=0}^{\infty}P_{n_1,n_2,n_3}(t).z^{n_3}$$

$$I_{n_3}(x,y)=\sum_{n_2=0}^{\infty}F_{n_2,n_3}(x).y^{n_2}=\sum_{n_1=0}^{\infty}F_{n_1,n_3}(y).x^{n_1}$$

$$J_{n_1}(y,z)=\sum_{n_3=0}^{\infty}G_{n_1,n_3}(y).z^{n_3}=\sum_{n_2=0}^{\infty}H_{n_1,n_2}(z).y^{n_2} \quad \ldots(11)$$

$$K_{n_2}(x,z)=\sum_{n_3=0}^{\infty}F_{n_2,n_3}(x).z^{n_3}=\sum_{n_1=0}^{\infty}H_{n_1,n_2}(z).x^{n_1}$$

Multiplying (1) by x^{n_1}, summing over n_1 from 0 to ∞, using (2) and (10), we obtained:

$$(\lambda+\mu_2+\mu_3)F_{n_2,n_3}(x)+\mu_1F_{n_2,n_3}(x)-\mu_1P_{0,n_2,n_3}=\lambda.x.F_{n_2,n_3}(x)+\frac{A_3\mu_1}{x}[F_{n_2-1}(x)-P_{0,n_2-1,n_3}]$$
$$+\frac{B_3\mu_1}{x}[F_{n_2,n_3-1}(x)-P_{0,n_2,n_3-1}]+x.C_3\mu_2.F_{n_2+1,n_3}(x)+D_3\mu_2F_{n_2+1,n_3}(x)+E_3\mu_2.F_{n_2+1,n_3-1}(x)$$
$$+L_3\mu_3F_{n_2,n_3+1}(x)+M_3\mu_3xF_{n_2,n_3+1}(x)+N_3\mu_3F_{n_2-1,n_3+1}(x) \quad \ldots(12)$$

for $n_1 \geq 0$ and $n_2, n_3 > 0$.

Similarly from (3), (4), and (6) by using (5), (7) and (8) respectively we obtained following:

$$(\lambda+\mu_1+\mu_3)F_{0,n_3}(x)-\mu_1P_{0,0,n_3}=\lambda xF_{0,n_3}(x)+\frac{B_3\mu_1}{x}[F_{0,n_3-1}-P_{0,0,n_3-1}]$$
$$+C_3\mu_2.x.F_{1,n_3}+D_3\mu_2F_{1,n_3}+E_3\mu_2F_{1,n_3-1}$$
$$+L_3\mu_3.F_{0,n_3+1}+M_3\mu_3xF_{0,n_3+1} \quad \ldots(13)$$

for $n_1 \geq 0$ and $n_2, n_3 > 0$.

$$(\lambda+\mu_1+\mu_2)F_{n_2,0}(x)-\mu_1P_{0,n_2,0}=\lambda xF_{n_2,0}(x)+\frac{A_3\mu_1}{x}[F_{n_2-1,0},P_{0,n_2-1,0}]$$
$$+C_3\mu_2xF_{n_2+1,0}+N_3\mu_3F_{n_2-1,1}+\mu_2D_3F_{n_2+1,0}$$
$$+L_3\mu_3F_{n_2,1}+M_3\mu_3.x.F_{n_2,1} \quad \ldots(14)$$

for $n_1, n_3 \geq$ and $n_2 > 0$.

$$(\mu_1+\lambda)F_{0,0}-\mu_1P_{0,0,0}=\lambda xF_{0,0}+C_3\mu_2xF_{1,0}+D_3\mu_2F_{1,0}+L_3\mu_3F_{0,1}+M_3\mu_3xF_{0,1} \quad \ldots (15)$$

for $n_1 \geq 0$ and $n_2, n_3 = 0$.

Multiplying (12) by y^{n_2}, taking sum over n_2 from 0 to ∞, using (13) and def. in (10) and (11), we have :

$$(\lambda+\mu_1+\mu_2+\mu_3)I_{n_3}(x,y)-\mu_2F_{0,n_3}(x)-\mu_1G_{0,n_3}(y)=\lambda xI_{n_3}(x,y)+\frac{A_3y.\mu_1}{x}[I_{n_3}(x,y)-G_{0,n_3}(y)]$$
$$+\frac{D_3\mu_2}{y}[I_{n_3}(x,y)-F_{0,n_3}(x)]+\frac{\mu 2xC_3}{y}[I_{n_3}(x,y)-F_{0,n_3}(x)]+\frac{B_3\mu_1}{x}[I_{n_3-1}(x,y)-G_{0,n_3-1}(y)]$$
$$+\frac{\mu_2E_3}{y}[I_{n_3-1}(x,y)-F_{0,n_3-1}(x)]+\mu_3L_3I_{n_3+1}(x,y)+M_3x\mu_3I_{n_3+1}(x,y)+N_3\mu_3yI_{n_3+1}(x,y) \quad \ldots(16)$$

for $n_1, n_2 \geq 0$ and $n_3 > 0$.
Similarly from (14) by using (15) we have:

$$(\lambda+\mu_1+\mu_2)I_0(x,y)-\mu_1 G_{0,0}(y)=\lambda.x.I_0(x,y) + \frac{A_3\mu_1 y}{x}[I_0-G_{0,0}(y)] + \frac{C_3\mu_2 x}{y}[I_0(x,y)-F_{0,0}(x)]+E_3\mu_3 y.I_1(x,y) + \frac{D_3\mu_2}{y}[I_0(x,y)-F_{0,0}]+\mu_3 L_3 I_1(x,y)+M_3\mu_3 x I_1(x,y) \quad \ldots(17)$$

Multiplying (16) by z^{n_3}, taking sum over n_3 from 0 to ∞, using (17) and def. of generating function, we have :

$$F(x,y,z)=\left[\frac{\mu_3\left[1-\frac{1}{z}\{L_3+M_3x+N_3y\}\right]I_0(x,y)+\mu_2\left[1-\frac{1}{y}\{D_3+C_3x+E_3z\}\right]K_0(x,z) +\mu_1\left[1-\frac{1}{x}\{A_3y+B_3z\}\right]J_0(y,z)}{\lambda(1-x)+\mu\left[1-\frac{1}{x}\{A_3y+B_3z\}\right]+\mu_2\left[1-\frac{1}{y}\{D_3+C_3x+E_3z\}\right] +\mu_3\left[1-\frac{1}{z}\{L_3+M_3x+N_3y\}\right]}\right] \quad \ldots(18)$$

From (18) we have following:

$$-\mu_3 M_3 I_0(1,1)-\mu_2 C_3 K_0(1,1)+\mu_1 J_0(1,1)=-\lambda+\mu_1-\mu_2 C_3-M_3\mu_3 \quad \ldots(19)$$

$$-\mu_3 N_3 I_0(1,1)+\mu_2 K_0(1,1)-\mu_1 A_3 J_0(1,1)=-\mu_1 A_3+\mu_2-\mu_3 N_3 \quad \ldots(20)$$

$$\mu_3 I_0(1,1)-\mu_2 E_3 K_0(1,1)-\mu_1 B_3 J_0(1,1)=-\mu_1 B_3-\mu_2 E_3+\mu_3 \quad \ldots(21)$$

From (19), (20) and (21), we have:

$$K_0(1,1)=1+\frac{\lambda(A_3+B_3N_3)}{\mu_2[A_3M_3E_3+E_3N_3+B_3M_3-1+A_3C_3+B_3N_3C_3]} \quad \ldots(22)$$

$$I_0(1,1)=1+\frac{\lambda(A_3E_3+B_3)}{\mu_3[A_3M_3E_3+E_3N_3+B_3M_3+A_3C_3-1+B_3N_3C_3]} \quad \ldots(23)$$

$$J_0(1,1)=1+\frac{\lambda(1-E_3N_3)}{\mu_1[A_3M_3E_3+E_3N_3+B_3M_3-1+A_3C_3+B_3N_3C_3]} \quad \ldots(24)$$

Let us denote $F(x, y, z) = \frac{f(x,y,z)}{g(x,y,z)}$

where

$$f(x,y,z)=\mu_3\left[1-\frac{1}{z}\{L_3+M_3x+N_3y\}\right]I_0(x,y)+\mu_2\left[1-\frac{1}{y}\{D_3+C_3x+E_3z\}\right]K_0(x,z) +\mu_1\left[1-\frac{1}{x}\{A_3y+B_3z\}\right]J_0(y,z) \quad \ldots(25)$$

$$g(x,y,z)=\lambda(1-K)+\mu_1\left[1-\frac{1}{x}\{A_3y+B_3z\}\right]+\mu_2\left[1-\frac{1}{y}\{D_3+C_3x+E_3z\}\right] +\mu_3\left[1-\frac{1}{z}\{L_3+M_3x+N_3y\}\right] \quad \ldots(26)$$

Let Lq_1 = Marginal mean queue length in front of 1st server

$$= \frac{-\left(\frac{\partial f}{\partial x}\right)_{(1,1,1)}\left(\frac{\partial^2 g}{\partial x^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial x}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial x^2}\right)_{(1,1,1)}}{2\left(\frac{\partial g}{\partial x}\right)^2_{(1,1,1)}}$$

$$= \frac{\lambda(1-E_3N_3)}{(-\lambda+\mu_1-C_3\mu_2-M_3\mu_3)(1-A_3M_3E_3-E_3N_3-B_3M_3-A_3C_3-B_3N_3C_3)} \quad \ldots(27)$$

Lq_2 = Marginal Mean queue length of 2nd server:

$$= \frac{-\left(\frac{\partial f}{\partial x}\right)_{(1,1,1)}\cdot\left(\frac{\partial^2 g}{\partial y^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial y}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial y^2}\right)_{(1,1,1)}}{2\left(\frac{\partial g}{\partial y}\right)^2_{(1,1,1)}}$$

$$= \frac{\lambda(A_3+B_3N_3)}{(1-A_3M_3E_3-E_3N_3-B_3M_3-A_3C_3-B_3N_3C_3)(\mu_2-A_3\mu_1-N_3\mu_3)} \quad \ldots(28)$$

Lq_3 = Marginal mean queue length in front of 3rd server:

$$= \frac{-\left(\frac{\partial f}{\partial z}\right)_{(1,1,1)}\left(\frac{\partial^2 g}{\partial z^2}\right)_{(1,1,1)} + \left(\frac{\partial g}{\partial z}\right)_{(1,1,1)}\left(\frac{\partial^2 f}{\partial z^2}\right)_{(1,1,1)}}{2\left(\frac{\partial g}{\partial z}\right)^2_{(1,1,1)}}$$

$$= \frac{\lambda(A_3E_3+B_3)}{(-\mu_1B_3-E_3\mu_2+\mu_3)(1-A_3M_3E_3-E_3N_3-B_3M_3-A_3C_3-B_3N_3C_3)} \quad \ldots(29)$$

Lq =Mean Queue Length of the System

$$= Lq_1 + Lq_2 + Lq_3$$

$$= \frac{\lambda}{(1-A_3M_3E_3-E_3N_3-B_3M_3-A_3C_3-B_3N_3C_3)}\left[\frac{(1-E_3N_3)}{(-\lambda+\mu_1-C_3\mu_2-M_3\mu_3)} + \frac{(A_3+B_3N_3)}{(-A_3\mu_1+\mu_2-N_3\mu_3)} + \frac{(A_3E_3+B_3)}{(-\mu_1B_3-E_3\mu_2+\mu_3)}\right] \quad \ldots(30)$$

RESULTS AND DISCUSSION

We discuss particular cases varying the values of n, i.e. number of times the service by the same server is allowed.

Case 1: when n=2

$$Lq = \frac{\lambda}{A'}\left[\frac{A}{(-\lambda+\mu_1-bp_{21}\mu_2-cp_{31}\mu_3)} + \frac{\left(ap_{12}+a'p'_{12}\right)+\left(ap_{13}+a'p'_{13}\right)\left(cp_{32}+c'p'_{32}\right)}{\left[\mu_2-\mu_1\left(ap_{12}+a'p'_{12}\right)-\mu_3\left(cp_{32}+cp_{32}'\right)\right]} + \frac{\left(ap_{12}+a'p'_{12}\right)\left(bp_{23}+b'p'_{23}\right)+\left(ap_{13}+a'p'_{13}\right)}{\left[\mu_3-\mu_2\left(bp_{23}+b'p'_{23}\right)-\mu_1\left(ap_{13}+a'p'_{13}\right)\right]}\right]$$

Where $A = \left\{1-\left(bp_{23}+b'p'_{23}\right)\left(cp_{32}+c'p'_{32}\right)\right\}$

and

$$A' = \{1-(bp_{23}+b'p'_{23})(cp_{32}+c'p'_{32})-cp_{31}(ap_{12}+a'p'_{12})(bp_{23}+b'p'_{23})-bp_{21}(ap_{12}+a'p'_{12})$$
$$-cp_{31}(ap_{13}+a'p'_{13})-bp_{21}(ap_{13}+a'p'_{13})(cp_{32}+c'p'_{32})\}$$

This result is same as obtained by Kumar and Taneja (2017) wherein the authors discussed the case of revisiting the servers atmost once for service .

Case 2: When n=3

1. Behaviour of the mean queue length of the entire system with respect to arrival rate (λ) is depicted in **Table 1**. The probability of leaving the first server first time (a^1) has been varied whereas the value of other parameters has been kept fixed.

Table 1: Mean Queue Length with respect to λ and a^1.

$\mu_1=24,\mu_2=33,\mu_3=45,p_{12}^1=0.45, ,p_{12}^2=0.3,p_{12}^3=0.6,p_{13}^1=0.7, p_{13}^2=0.55, p_{13}^3=0.4,b^1=0.6,b^2=0.25,b^3=0.15,p_{21}^1=0.2, p_{21}^2=0.15, p_{21}^3=0.1,p_2^1=0.5, p_2^2=0.6, p_2^3=0.7,p_{23}^1=0.3, p_{23}^2=0.25, p_{23}^3=0.2,c^1=0.7,c^2=0.2,c^3=0.1,p_3^1=0.65, p_3^2=0.6,p_3^3=0.5,p_{31}^1=0.2,p_{31}^2=0.15,p_{31}^3=0.1,p_{32}^1=0.2,p_{32}^2=0.25,p_{32}^3=0.4$

	$a^1=0.55, a^2=0.3, a^3=0.15$				$a^1=0.65, a^2=0.25, a^3=0.1$				$a^1=0.75, a^2=0.2, a^3=0.05$			
λ	Lq_1	Lq_2	Lq_3	Lq	Lq_1	Lq_2	Lq_3	Lq	Lq_1	Lq_2	Lq_3	Lq
1	0.142	0.054	0.047	0.244	0.141	0.053	0.042	0.236	0.141	0.053	0.042	0.236
2	0.32	0.108	0.094	0.522	0.316	0.106	0.085	0.507	0.316	0.106	0.085	0.507
3	0.546	0.163	0.141	0.85	0.54	0.159	0.127	0.826	0.54	0.159	0.127	0.826
4	0.845	0.217	0.188	1.251	0.836	0.212	0.17	1.217	0.836	0.212	0.17	1.217
5	1.259	0.271	0.236	1.766	1.246	0.264	0.212	1.722	1.246	0.264	0.212	1.722
6	1.871	0.325	0.283	2.478	1.85	0.317	0.254	2.422	1.85	0.317	0.254	2.422
7	2.863	0.379	0.33	3.572	2.831	0.37	0.297	3.498	2.831	0.37	0.297	3.498
8	4.754	0.434	0.377	5.564	4.702	0.423	0.339	5.464	4.702	0.423	0.339	5.464
9	9.777	0.488	0.424	10.69	9.67	0.476	0.382	10.53	9.67	0.476	0.382	10.53
9.5	17.61	0.515	0.447	18.58	17.42	0.503	0.403	18.33	17.42	0.503	0.403	18.33
10	63.22	0.542	0.471	64.23	62.52	0.529	0.424	63.48	62.52	0.529	0.424	63.48

Graph for Mean Queue Length of the System (Lq) has been plotted with respect to arrival rate λ for different values of a^1 as shown in the Fig. 3.

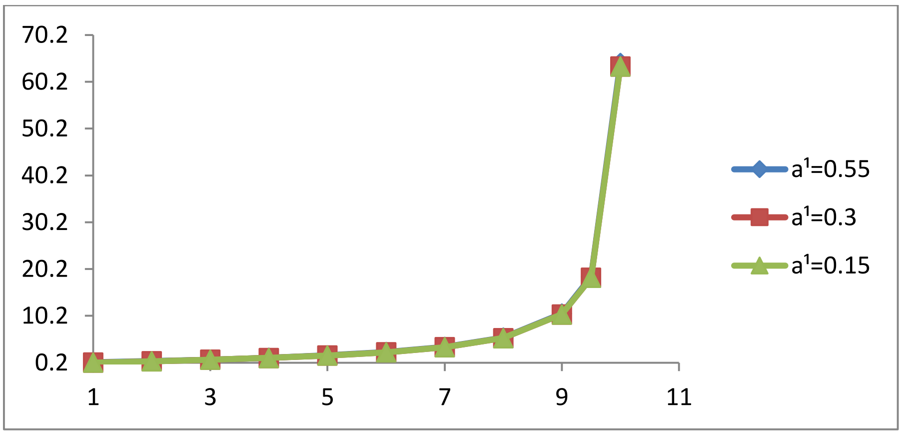


Figure 3. Mean queue length Vs Arrival rate

Following can be interpreted from the **Table 1** and **Fig. 3**

Mean queue length (Lq) of the system increases with the increase in mean arrival rate (λ) whereas it decreases with the increase in probability (a^1). However there is no significant difference between mean queue lengths with respect to (a^1).

2. Behavior of mean queue length at each server and the entire system with respect to service rate of first server (μ_1) is depicted in Table 2. The service rate of second server (μ_2) has also been varied whereas the values of other parameters have been kept fixed.

Table 2. Mean queue lengths with respect to μ_1 for different μ_2.

$\mu_3=21,\lambda=5$, $a^1=0.65$, $a^2=0.3$, $a^3=0.05$, $p_{12}{}^1=0.4$, , $p_{12}{}^2=0.55$, $p_{12}{}^3=0.6$, $p_{13}{}^1=0.6$, $p_{13}{}^2=0.45$, $p_{13}{}^3=0.4$, $b^1=0.5$, $b^2=0.35$, $b^3=0.15$, $p_{21}{}^1=0.2$, $p_{21}{}^2=0.15$, $p_{21}{}^3=0.1$, $p_2{}^1=0.5$, $p_2{}^2=0.6$, $p_2{}^3=0.7$, $p_{23}{}^1=0.3$, $p_{23}{}^2=0.25$, $p_{23}{}^3=0.2$, $c^1=0.7$, $c^2=0.2$, $c^3=0.1$, $p_3{}^1=0.65$, $p_3{}^2=0.6$, $p_3{}^3=0.5$, $p_{31}{}^1=0.25$, $p_{31}{}^2=0.2$, $p_{31}{}^3=0.1$, $p_{32}{}^1=0.1$, $p_{32}{}^2=0.2$, $p_{32}{}^3=0.4$												
	$\mu_2=15$				$\mu_2=16$				$\mu_2=17$			
μ_1	Lq_1	Lq_2	Lq_3	Lq	Lq_1	Lq_2	Lq_3	Lq	Lq_1	Lq_2	Lq_3	All
20	0.859	1.356	0.761	2.975	0.878	0.994	0.796	2.667	0.897	0.785	0.834	2.516
20.5	0.807	1.478	0.796	3.081	0.823	1.058	0.835	2.716	0.841	0.824	0.877	2.542
21	0.761	1.624	0.835	3.22	0.776	1.131	0.878	2.785	0.791	0.868	0.925	2.584
21.5	0.72	1.803	0.879	3.401	0.733	1.215	0.926	2.874	0.747	0.916	0.978	2.641
22	0.683	2.026	0.927	3.635	0.695	1.313	0.979	2.986	0.707	0.971	1.038	2.715
22.5	0.65	2.312	0.98	3.941	0.66	1.427	1.039	3.126	0.671	1.032	1.105	2.808
23	0.619	2.691	1.04	4.351	0.629	1.563	1.106	3.299	0.639	1.101	1.182	2.922
23.5	0.592	3.22	1.108	4.92	0.601	1.728	1.183	3.512	0.61	1.181	1.27	3.061
24	0.567	4.008	1.185	5.76	0.575	1.931	1.272	3.778	0.583	1.272	1.373	3.229
24.5	0.544	5.306	1.274	7.124	0.551	2.19	1.375	4.116	0.559	1.379	1.494	3.432
25	0.522	7.848	1.377	9.747	0.529	2.527	1.496	4.553	0.536	1.506	1.638	3.68

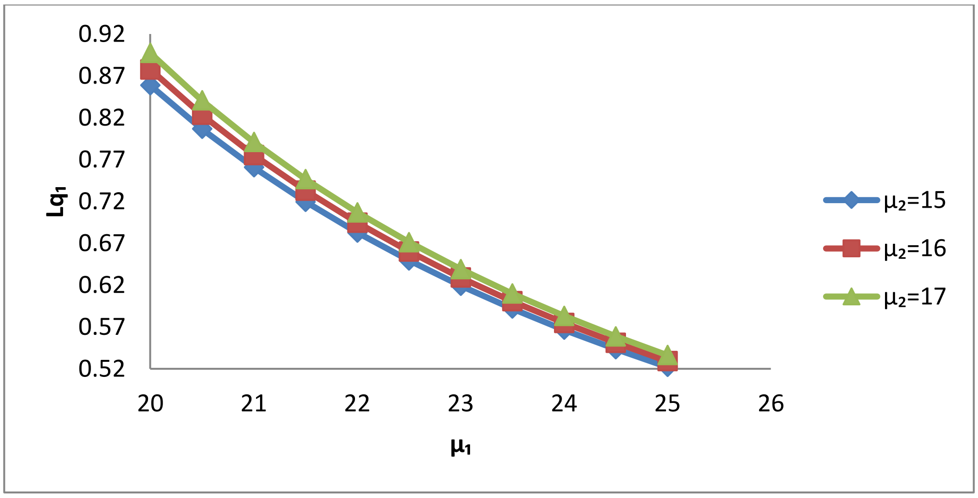


Figure 4. Marginal mean queue length at First Server Vs Service rate of first server

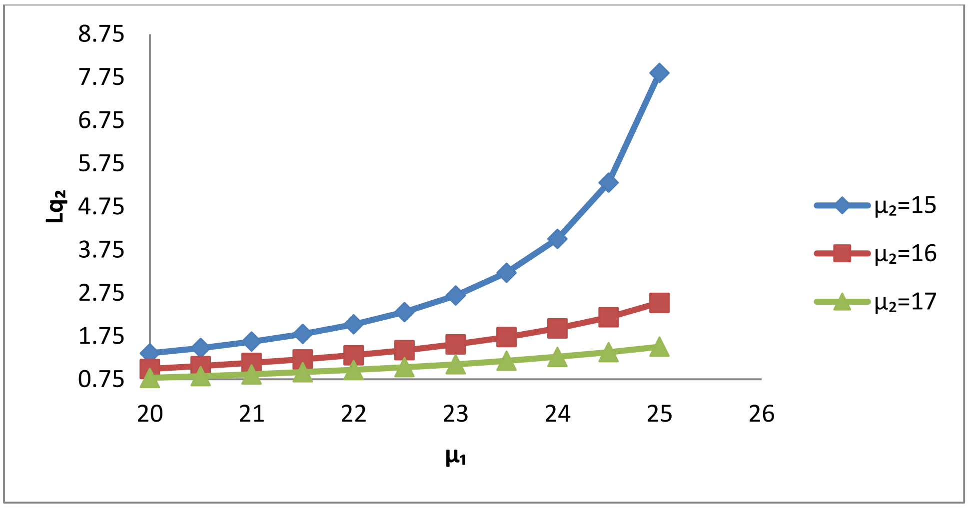


Figure 5. Marginal mean queue length at second server Vs Service rate of first server

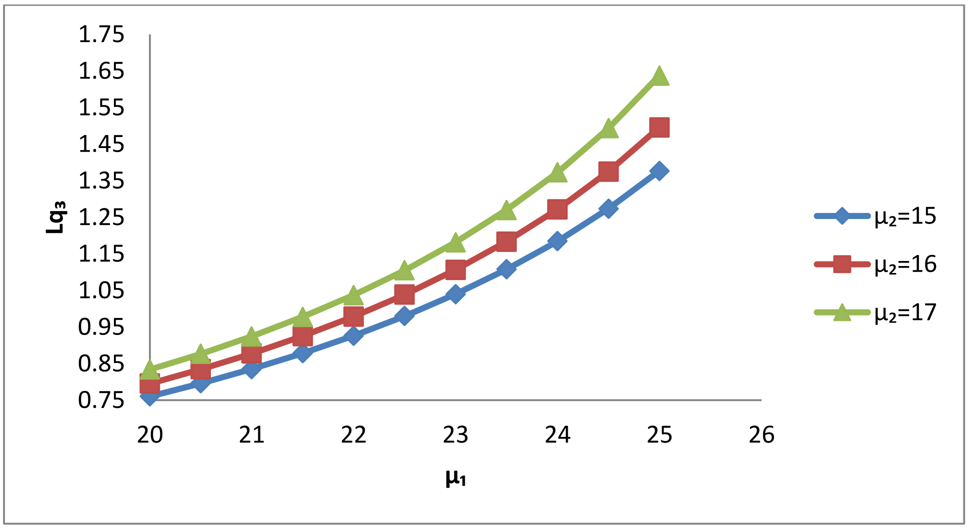


Figure 6. Marginal mean queue length at third server Vs Service rate of first server

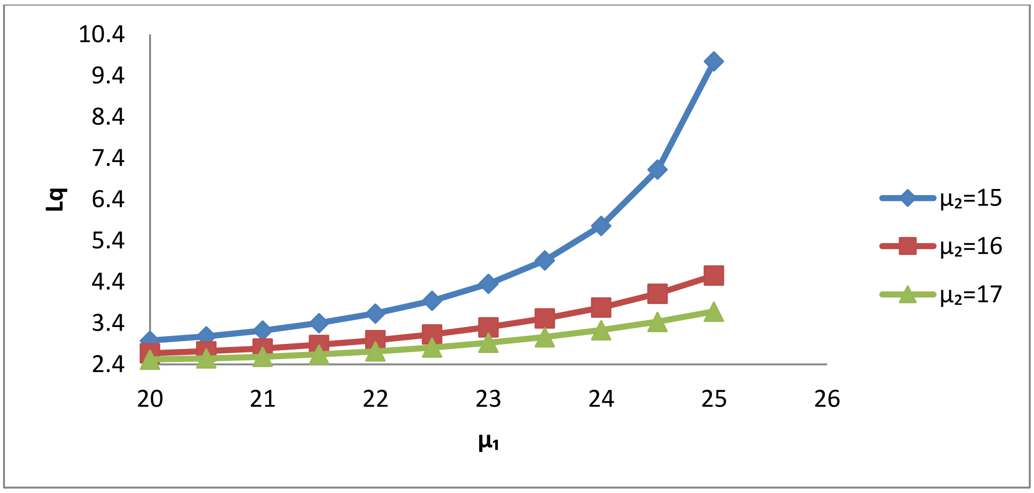


Figure 7. Mean queue length of entire system Vs Service rate of first server

Following can be interpreted from above tables and graphs:

I. Marginal mean queue length at first server (Lq_1) decreases with the increase in service rate of first server (μ_1) but has higher values for higher values of service rate of second server (μ_2). However, there is no significant difference between marginal mean queue lengths with respect to μ_2.

II. Marginal mean queue length at second server (Lq_2) increases with the increase in service rate of first server (μ_1) but has lower values for higher values of service rate of second server (μ_2).

III. Marginal mean queue length at third server (Lq_3) increases with the increase in the value of service rate of first server (μ_1) as well as increase in service rate of second server (μ_2). However, there is no significant difference between mean queue lengths with respect to μ_2.

IV. Mean queue length of the system (Lq) increases with the increase in the value of service rate of first server (μ_1) as well as increase in service rate of second server (μ_2). However, there is no significant difference between mean queue lengths with respect to μ_2.

CONCLUSION

A queuing model has been developed on a practical situation which may be observed in various fields of life such as hospitals, administration and manufacturing. A customer/ product is served/ dealt by three servers to whom the customer can revisit but at most twice. Expected queue length has been obtained for the model and its nature has been depicted through numerical results and graphs with respect to arrival/service rates taking particular cases and interesting interpretations have been given.

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