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Effect of Varying Magnetic Field on the Linear Stability of Parallel Stratified Shear Flows

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Abstract

This paper deals with the stability of an inviscid incompressible stratified shear flow confined between two infinite plates in the presence of a variable horizontal magnetic field. The flow is characterized by an arbitrary basic velocity profile and a varying magnetic field is assumed to be aligned in the horizontal direction. Analytical expressions to calculate the growth rate of the disturbances are found by employing the method of small oscillations. The analysis is restricted to long waves. Numerical computations of stability characteristics are carried out for linear velocity and magnetic profiles.

Keywords: growth rate, varying magnetic field, small oscillations, shear flow, stratification.

INTRODUCTION

The stability of stratified parallel shear flows under the action of buoyancy forces is of much interest and importance in astrophysical and meteorological phenomena. A detailed account on theoretical and experimental results on the onset of thermal instability of an incompressible viscous fluid under varying assumptions on hydrodynamics and hydomagnetics has been given in the celebrated monograph by Chandrasekar [3].

Gupta [6] investigated the stability of a horizontal layer of a perfectly conducting fluid, with continuous density and viscous stratification in the presence of a horizontal magnetic field. Kent [9] has studied the stability of parallel flows due to a horizontal magnetic field which varies in the vertical direction. It was shown that the system is unstable when $U_0'' > \left(\frac{A_0'}{U_0'}\right) A_0''$, while in the absence of the magnetic field the system is stable. Howard [8] found eigen bounds for unstable waves in a plane parallel flow of an inviscid incompressible stratified fluid.

The stability of the parallel shear flows of an inviscid incompressible fluid to infinitesimal two dimensional disturbances by linear analysis was investigated by Blumen [2]. Small disturbances of parallel shear flow in an inviscid incompressible fluid of variable density were considered by Miles [11]. Farrell and Ioannou [5] studied the transient development of perturbations in the inviscid stratified shear flow.

Drazin [4] examined some general aspects of the stabilizing influence of a parallel magnetic field on a plane parallel flow, considering only two-dimensional disturbances. Lock [10] discussed the stability of a hydromagnetic flow between two parallel plates in the presence of a uniform magnetic field. Agarwal and Agarwal [1] analyzed the stability of a heterogeneous shear flow in the presence of a parallel magnetic field. The linear stability of a stratified shear flow of a perfectly conducting bounded fluid in the presence of a magnetic field aligned with the flow under the action of buoyancy forces has been studied by Parthi and Nath [13].

The stability of a stratified flow varying in two directions of an incompressible conducting fluid permeated by a uniform magnetic field was investigated by Gupta [7]. It was found that a strong magnetic field can completely stabilize flows with unstable density stratification. Padmini and Subbiah [12] studied the problem of linear stability of inviscid, incompressible non-parallel stratified shear flows to normal mode disturbances. Stability analysis of a simple shear flow of an incompressible fluid with a piecewise linear velocity profile in the presence of a magnetic field has been carried out by Ruderman and Brevdo [14]. The stability of incompressible, inviscid density stratified fluid in sea straits of arbitrary cross section was studied by Sridevi and Ganesh [15].

The existing literature is dominated primarily by works on the effect of uniform magnetic field on shear flow instabilities. In the present work, we examine the effect of variable magnetic field on the linear stability of an inviscid incompressible parallel stratified shear flow using normal mode analysis.

MATHEMATICAL FORMULATION

We consider an unsteady flow of an inviscid, incompressible Boussinesq electrically conducting fluid confined between two infinite horizontal rigid plates at $y = \pm L$. The fluid is characterized by a shear layer with an arbitrary velocity profile. The basic flow is taken as (U(y), 0, 0). We have introduced a Cartesian coordinate system in such a way that x axis taken in the direction of the flow, y axis is taken in the perpendicular direction. We have introduced varying magnetic field in the x direction given by $\vec{B} = \mu_m(H(y), 0, 0)$. Under the above mentioned assumptions the geometry of the flow configuration is shown in figure.

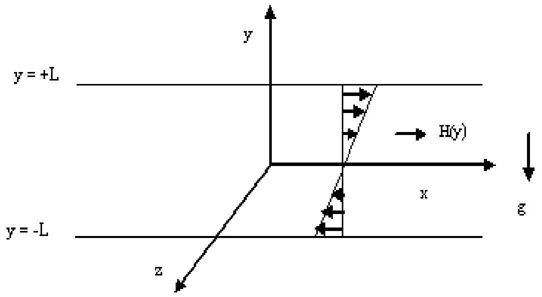

Figure 1: Flow configuration

The equations governing the fluid flow are

The equation of continuity

$$\nabla.\vec{q} \qquad = 0 \qquad (1)$$

Momentum equation

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = \frac{-\nabla p}{\rho_0} - \frac{\rho g \hat{y}}{\rho_0} + \mu_m(\nabla \times \vec{H}) \times \vec{H} \qquad (2)$$

It is known that the density of the fluid particle moving with the fluid remains unchanged. Hence, the condition for incompressibility is

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0 \qquad (3)$$

The magnetic induction equation

$$\frac{\partial \vec{H}}{\partial t} \qquad = \eta \nabla^2 \vec{H} + \nabla \times \left(\vec{q} \times \vec{H}\right) \qquad (4)$$

The solenoidal condition for the magnetic field

$$\nabla.\vec{H} \qquad = 0 \qquad (5)$$

where $\vec{q}$, ρ, p, g, $\vec{H}$, η, μ_m denote respectively the velocity vector, density, pressure, gravitational acceleration, magnetic field vector, magnetic resistivity, magnetic permeability and $\hat{y}$ is the unit vector in the vertical direction. Due to no slip condition velocity vanishes at the plates y = ± L.

At equilibrium, we have

$$p_e' \qquad = -\rho_e g - \mu_m H(y)H'(y) \qquad (6)$$

where a prime denotes differentiation with respect to y and U(y), $\rho_e(y)$, $p_e(y)$ and H(y) are continuously differentiable functions of y in the flow domain.

Introducing the non-dimensional quantities for time, pressure, density, magnetic field and length as

$t = \frac{Lt^*}{U_0}$, $p = \rho_0 U_0^2 p^*$, $\rho = \frac{\rho_0 U_0^2 N_0^2}{Lg}\rho^*$, $\vec{H} = H_0\vec{H}^*$ and (x, y, z) = L(x*, y*, z*)

where $N^2 = -\frac{g}{\rho_0}\left(\frac{d\rho}{dz}\right)$ is the Brunt-Vaisala frequency which is assumed to be positive for static stability and N_0^2 is a typical value of Brunt-Vaisala frequency in the flow domain, L is the characteristic length and U_0 is the characteristic velocity.

Substitute the above dimensionless quantities in the governing equations, Eqs (1) - (5) reduce to (on removing asterisks)

$$\nabla.\vec{q} \qquad = 0 \qquad (7)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\nabla p - Ri\,\rho\,\hat{y} + S(\nabla \times \vec{H}) \times \vec{H} \qquad (8)$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0 \qquad (9)$$

$$\frac{\partial \vec{H}}{\partial t} \qquad = \frac{1}{Rm}\nabla^2 \vec{H} + \nabla \times (\vec{q} \times \vec{H}) \qquad (10)$$

$$\nabla.\vec{H} \qquad = 0 \qquad (11)$$

where $S = \frac{\mu_m H_0^2}{\rho U_0^2}$, Magnetic pressure number

$Rm = \frac{LU_0}{\eta}$, Magnetic Reynolds number

$Ri = \frac{g\beta L^2}{\rho_0 U_0^2}$, Richardson number

The boundary condition in non-dimensional form is

$$\vec{q} = 0 \qquad \text{on } y = \pm 1 \qquad (12)$$

In a perturbed state the velocity, density, pressure and the magnetic field are taken as $(U(y) + u, v, w)$, $\rho_e(y) + \rho$, $p_e(y) + p$ and $(H(y) + h_x,\ h_y,\ h_z)$.

Hence, the linearized perturbation equations for infinitesimal normal modes of the form $e^{i(kx+klz-k\sigma t)}$ are obtained as

$$ik(u + lw) + \frac{\partial v}{\partial y} = 0$$

$$ik(U - \sigma)u + v.\frac{\partial U(y)}{\partial y} = -ik\left(p + Sh_y\right)$$

$$ik(U - \sigma)v \qquad = -\frac{\partial p}{\partial y} - Ri\,\rho$$

$$+ S\left(H(y)\left(ikh_y - \frac{\partial h_x}{\partial y}\right)\right) - h_x$$

$$ik(U - \sigma)w \qquad = -ikl(p + SH(y)h_x)$$

$$ik(U - \sigma)\rho - \frac{N^2}{N_0^2}v = 0$$

$$ik(h_x + lh_z) + \frac{\partial h_y}{\partial y} = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1 + l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_x = \frac{1}{Rm}\frac{\partial^2\left(H(y)\right)}{\partial y^2}$$

$$+ \frac{\partial}{\partial y}\left(Uh_y\right) - \frac{\partial(v\,H(y))}{\partial y} - ikl(H(y)w - Uh_z)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1 + l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_y = ik\left(H(y)v - Uh_y\right)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1 + l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_z = ik(H(y)w - Uh_z) \quad (13)$$

The corresponding boundary conditions are

$$u = v = w = 0 \qquad \text{on } y = \pm 1 \tag{14}$$

EIGEN VALUES AND EIGEN FUNCTIONS FOR LONG WAVES

Here, we consider the analysis for long wave approximation (i.e) k is assumed to be small. The flow is assumed to be bounded between two plates $y = \pm 1$. To make the problem mathematically tractable, we consider the linear velocity profile as the basic flow U(y) = y. Without loss of generality, from the equilibrium condition, we obtain the magnetic profile as H(y) = 1 + y.

Hence Eq. (13) reduces to the form

$$iku + iklw + \frac{\partial v}{\partial y} = 0$$

$$ik(y-\sigma)u + v.\frac{\partial U(y)}{\partial y} = -ik(p + Sh_y)$$

$$ik(y-\sigma)v = -\frac{\partial p}{\partial y} - Ri\,\rho\, S\left((1+y)\left(ikh_y - \frac{\partial h_x}{\partial y}\right)\right) - h_x$$

$$ik(y-\sigma)w = -ikl(p + S(1+y)h_x)$$

$$ik(y-\sigma)\rho - \frac{N^2}{N_0^2}v = 0$$

$$ik(h_x + lh_z) + \frac{\partial h_y}{\partial y} = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_x = \frac{\partial}{\partial y}(y\, h_y) - \frac{\partial(v\,(1+y))}{\partial y} - ikl(Hw - yh_z)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_y = ik(Hv - yh_y)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_z = ik(Hw - yh_z) \tag{15}$$

We assume the series expansions with respect to the wave number k in the form

$$f = f_0 + kf_1 + k^2 f_2 + \cdots \tag{16}$$

where $f = (u, v, w, \sigma, \rho, h_x, h_y, h_z)$

Incorporating Eq. (16) into Eq. (15) and collecting the coefficients of like powers of k

The zero order equations are

$$iu_0 + ilw_0 + \frac{\partial v_0}{\partial y} = 0$$

$$iR(y)u_0 + v_0 = -ip_0$$

$$-\frac{\partial p_0}{\partial y} - Ri\,\rho_0 = 0$$

$$iR(y)w_0 = -ilp_0$$

$$iR(y)\rho_0 - \frac{N^2}{N_0^2}v_0 = 0 \tag{17}$$

$$ih_{x0} + ilh_{z0} + \frac{\partial h_{y0}}{\partial y} = 0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{x0}}{\partial y^2}\right) = -v_0 - (1+y)\frac{\partial v_0}{\partial y} - il\big((1+y)w_0\big)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y0}}{\partial y^2}\right) = i(1+y)v_0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{z0}}{\partial y^2}\right) = i(1+y)w_0 \tag{18}$$

where $R(y) = y - \sigma_0$

The first order equations are

$$iu_1 + ilw_1 + \frac{\partial v_1}{\partial y} = 0$$

$$-i\sigma_1 u_0 + iR(y)u_1 + v_1 = -ip_1 + Sh_{y0}$$

$$\frac{\partial p_1}{\partial y} + Ri\,\rho_1 = S\left((1+y)\frac{\partial h_{x0}}{\partial y} + Bh_{x0}\right)$$

$$-i\sigma_1 w_0 + iR(y)w_1 = -ilp_1 - Slh_{x0}(1+y)$$

$$iR(y)\rho_1 - i\sigma_1\rho_0 - \frac{N^2}{N_0^2}v_1 = 0 \tag{19}$$

$$ih_{x1} + ilh_{z1} + \frac{\partial h_{y1}}{\partial y} = 0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{x1}}{\partial y^2}\right) = i\sigma_0 h_{x0} + h_{y0} + y\frac{\partial h_{y0}}{\partial y} - v_1 - (1+y)\frac{\partial v_1}{\partial y} - l\big(w_1 + y(w_1 - h_{z0})\big)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y1}}{\partial y^2}\right) = i(1+y)v_1 - i(R(y))h_{y0}$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{z1}}{\partial y^2}\right) = i(1+y)w_1 - iR(y)h_{z0} \tag{20}$$

The boundary conditions Eq. (14) reduces to

$$u_0 = u_1 = 0, \quad v_0 = v_1 = 0, \quad w_0 = w_1 = 0 \tag{21}$$

Eliminating ρ_0, p_0, u_0, w_0 in favour of v_0 from Eq. (17) we obtain

$$R(y)^2\frac{\partial^2 v_0}{\partial y^2} + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)v_0 = 0 \tag{22}$$

Equation (19) is simplified to find v_1.

$$R(y)^2\frac{\partial^2 v_1}{\partial y^2} + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)v_1 = \sigma_1\left(R(y)^2\frac{\partial^2 v_0}{\partial y^2} - Ri\,(1+l^2)\,i\,\rho_0\right)$$

$$-S\,R(y)\left(\frac{\partial\, h_{y0}}{\partial y} - \left(l^2 - i\,(1+l^2)\right)\left((1+y)\frac{\partial h_{xo}}{\partial y} + h_{x0}\right)\right) \tag{23}$$

The solution of Eq. (22) is given as

$$v_0 = \begin{cases} C\,R(y)^{m_1} + D\,R(y)^{m_2}, & \lambda > 0 \\ R(y)^{\frac{1}{2}}(E + F\,R(y)), & \lambda = 0 \\ R(y)^{\frac{1}{2}}\begin{pmatrix} G\cos\left(klog(R(y))\right) \\ +H\sin\left(klog(R(y))\right)\end{pmatrix}, & \lambda < 0 \end{cases}$$

where $m_{1,2} = \frac{1 \pm \sqrt{\lambda}}{2}$, $\lambda = 1 - 4\,Ri\frac{N^2}{N_0^2}(1+l^2)$, k = - λ, C, D, E, F, G and H are arbitrary constants.

By applying the boundary condition that the velocity should vanish at the boundaries (i.e) $v_0 = 0 \ at \ y = \pm 1$, we obtain the value of σ_0 as

$$\sigma_0 = \begin{cases} \dfrac{1+e^{\frac{2n\pi i}{m_1-m_2}}}{1-e^{\frac{2n\pi i}{m_1-m_2}}}, & \lambda \geq 0 \\ \dfrac{1+e^{\frac{n\pi}{k}}}{1-e^{\frac{n\pi}{k}}}, & \lambda < 0 \end{cases} \tag{24}$$

The solution of Eqs. (17) and (18) are given by

$$u_0 = \begin{cases} C_5 R(y)^{m_1-1} + C_6 R(y)^{m_2-1}, & \lambda \geq 0 \\ iR(y)^{-\frac{1}{2}} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)\left(1 - \frac{kH-\frac{1}{2}}{1+l^2}\right) \\ -\sin\left(klog(R(y))\right)\left(\frac{k+\frac{H}{2}}{1+l^2} + H\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$v_0 = \begin{cases} R(y)^{m_1} + DR(y)^{m_2}, & \lambda \geq 0 \\ R(y)^{\frac{1}{2}}\left(\cos\left(klog\left(R(y)\right)\right) + F\sin\left(klog\left(R(y)\right)\right)\right), & \lambda < 0 \end{cases}$$

$$w_0 = \begin{cases} C_7 R(y)^{m_1-1} + C_8 R(y)^{m_2-1}, & \lambda \geq 0 \\ \frac{-lR(y)^{-\frac{1}{2}}}{i(1+l^2)} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)\left(kH - \frac{1}{2}\right) \\ +\sin\left(klog(R(y))\right)\left(k + \frac{H}{2}\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$p_0 = \begin{cases} C_3 R(y)^{m_1} + C_4 R(y)^{m_2}, & \lambda \geq 0 \\ \frac{R(y)^{\frac{1}{2}}}{i(1+l^2)} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)\left(kH - \frac{1}{2}\right) \\ +\sin\left(klog(R(y))\right)\left(k + \frac{H}{2}\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$\rho_0 = \begin{cases} C_1 R(y)^{m_1-1} + C_2 R(y)^{m_2-1}, & \lambda \geq 0 \\ R(y)^{-\frac{1}{2}} \frac{N^2}{iN_0^2} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right) \\ +H\sin\left(klog\left(R(y)\right)\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$h_{x0} = \begin{cases} Rm \begin{pmatrix} (A+By)(C_9\, R(y)^{m_1+1} + C_{10}\, R(y)^{m_2+1}) \\ +C_{11}\, R(y)^{m_1+2} + C_{12}\, R(y)^{m_2+2} \end{pmatrix}, & \lambda \geq 0 \\ Rm \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)\left(R(y)^{\frac{5}{2}} c_{56} + R(y)^{\frac{3}{2}} c_{57}\right) \\ +\sin\left(klog(R(y))\right)\left(R(y)^{\frac{5}{2}} c_{58} + R(y)^{\frac{3}{2}} c_{59}\right) \end{pmatrix} & \lambda < 0 \end{cases}$$

$$h_{y0} = \begin{cases} Rm \begin{pmatrix} (A+By)(C_{13}\, R(y)^{m_1+2} + C_{14}\, R(y)^{m_2+2}) \\ +C_{15}\, R(y)^{m_1+3} + C_{16}\, R(y)^{m_2+3}) \end{pmatrix}, & \lambda \geq 0 \\ Rm\, R(y)^{\frac{5}{2}} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)(c_{64}y + c_{65}) \\ +sin\left(klog\left(R(y)\right)\right)(c_{66}y + c_{67}) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$h_{z0} = \begin{cases} Rm \begin{pmatrix} (A+By)(C_{17}\, R(y)^{m_1+1} + C_{18}\, R(y)^{m_2+1}) \\ +C_{19}\, R(y)^{m_1+2} + C_{20}\, R(y)^{m_2+2} \end{pmatrix}, & \lambda \geq 0 \\ Rm\, R(y)^{\frac{3}{2}} \begin{pmatrix} \cos\left(klog\left(R(y)\right)\right)(c_{72}y + c_{73}) \\ +sin\left(klog(R(y))\right)(c_{74}y + c_{75}) \end{pmatrix}, & \lambda < 0 \end{cases} \tag{25}$$

By imposing the boundary condition that $v_1(\pm 1) = 0$ we get the second approximation of σ as

$$\sigma_1 = \begin{cases} \frac{S\,Rm\,C_{51}}{Ri\,C_{80} - C_{49}}, & \lambda \geq 0 \\ \frac{-S\,Rm\,C_{109}}{C_{108}}, & \lambda < 0 \end{cases} \quad (26)$$

For the sake of brevity the constants are given in *Appendix*.

RESULTS AND DISCUSSION

In order to obtain the physical insight of the problem, a comprehensive numerical computation is carried out for various values of parameters that describe the stability characteristics, and the results are reported in terms of Figs. 2 - 14. Imaginary part of growth rate σ as a function of wave number is plotted through Figs. 2 - 5 when > 0 .

From Figs. 2 and 3, it is observed that an increase in magnetic pressure number and Magnetic Reynolds number increases the growth rate. Figure 4 depicts the nature of longitudinal wave number. Increasing longitudinal wave number results in unstable growth rate.

From the analytical expression derived for σ_0, it can be noted that there exist an infinite number of modes, both stable and unstable corresponding to the values of n. This nature of n is shown through Fig. 5. Fig. 6 show the relationship between growth rate and Brunt-Vaisala frequency. It is understood that with the increase in Brunt-Vaisala frequency the frequency of disturbance increases upto certain level and then decreases thereby destabilize the flow when $\lambda > 0$.

The relation between growth rate and wave number for various parameters are shown through Figs 7 - 11 when $\lambda < 0$. Figs 7 and 8 exhibit the growth rate as a function of wave number. It is noted that increase in Magnetic Reynolds number and Magnetic pressure number enhances the region of instability. Fig. 9 depicts the growth rate as a function of the wave number with various n. It is known from the figure that there exists infinite number of normal modes with an increase in n.

Fig. 10 presents the relation between growth rate and longitudinal wave number. Increase in longitudinal wave number decreases the growth rate. The flow becomes stable. Fig. 11 shows the growth rate as a function of wave number for various Richardson number. It is known that increasing Richardson number destabilize the nature of the flow.

Growth rate as a function of Magnetic Reynolds number is exhibited in Fig. 12 when $\lambda < 0$. With increasing Richardson number growth rate increases and hence the flow becomes destabilized. The velocity profile for various nondimensional parameters is shown through Figs. 13 and 14. It is noted that the velocity decreases with the increase of various parameters like Magnetic Reynolds number and Magnetic pressure number.

CONCLUSION

The linear stability analysis of an inviscid, incompressible, parallel stratified shear fluid in the presence of varying magnetic field is analyzed. The governing equations of the flow coincide with those obtained by Padmini and Subbiah [12] when *Rm* = 0. The reduction of normal mode stability problem of non-parallel flows to the normal mode stability problem of parallel flows has been discussed in this paper. Sufficient condition for stability and an estimate for the growth rate of an unstable mode also obtained.

The stability of the flow is analyzed using the normal mode approach and the analysis is restricted to long wave approximation. The behavior of various nondimensional numbers like magnetic pressure number, magnetic Reynolds number, longitudinal wave number, transverse wave number, Brunt Vaisala frequency and Richardson number on the stability of parallel shear flow confined between the plates y = ± 1. From the results obtained, it is concluded that

- Richardson number plays a crucial role in the stability of parallel stratified shear flows.
- Increase in wave number increases the growth rate for varying magnetic pressure number and magnetic Reynolds number when $\lambda > 0$ thereby destabilizes the flow.
- Increase in transverse wave number destabilizes the fluid flow.
- The flow become unstable with the increase in Brunt Vaisala frequency when $\lambda > 0$.
- Growth rate increases for varying magnetic Pressure number and Magnetic Reynolds number thereby destabilizes the flow. In the case of an increase in transverse wave number the growth rate decreases and the flow becomes stable when $\lambda < 0$.
- Increase in Richardson number destabilizes the flow as Magnetic Reynolds number increases.

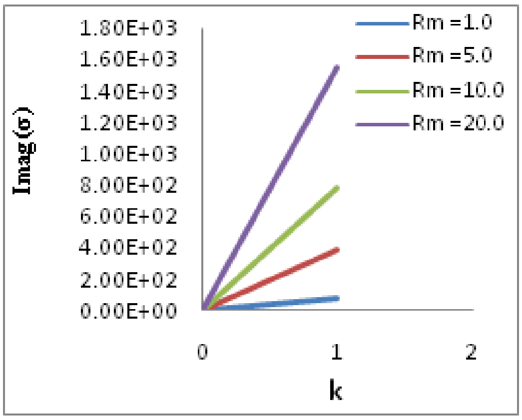

Figure 2. Growth rate vs wave number for various Rm (Ri =0.1, n = 1.0, S = 10.0, N^2 = 0.1, l = 1.0)

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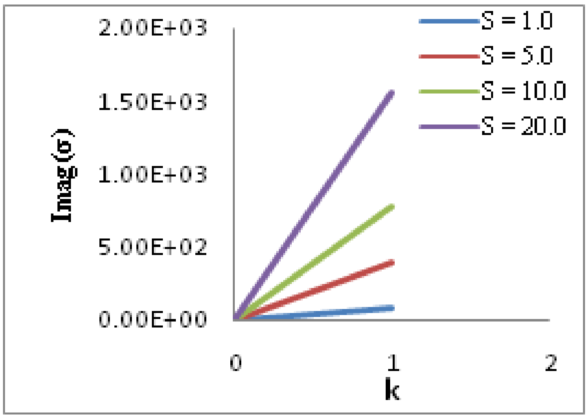

Figure 3. Growth rate vs wave number for various S (Ri =0.01, n = 1.0, Rm = 10.0, N^2 = 0.1, l = 1.0)

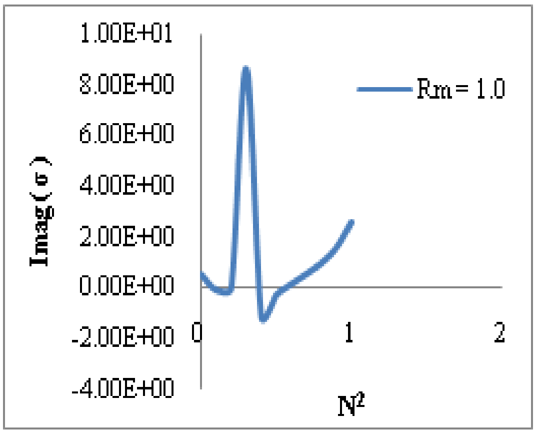

Figure 6. Growth rate vs N^2 (Ri = 0.1, n = 1.0, S = 10.0, l = 1.0)

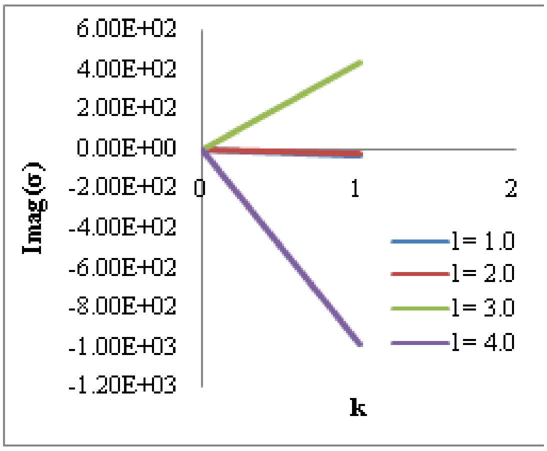

Figure 4. Growth rate vs wave number for various l (Rm =10.0, n = 1.0, S = 10.0, N^2 = 0.1, Ri = 0.01)

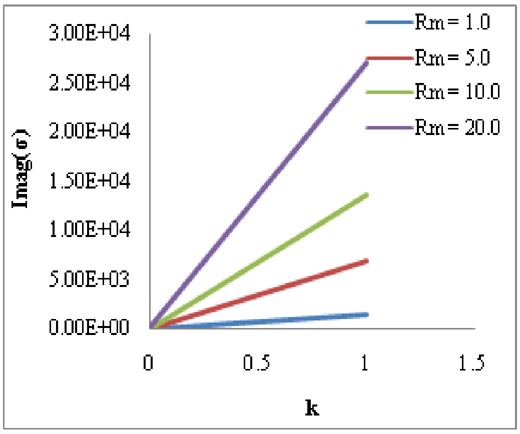

Figure 7. Growth rate vs wave number for various Rm **($\lambda < 0$)** (N^2 =10.0, l = 1.0, S = 10.0, Ri = 1.0, n = 1.0)

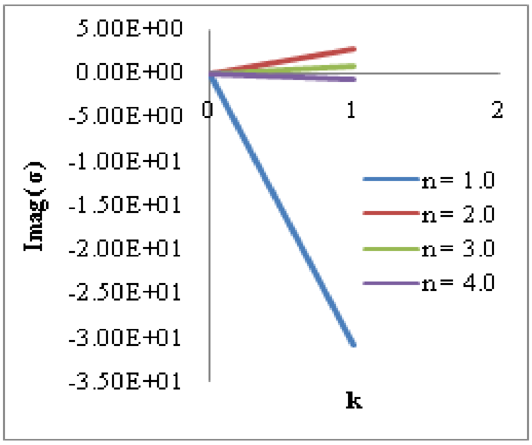

Figure 5. Growth rate vs wave number for various n (Rm =10.0, l = 1.0, S = 10.0, N^2 = 0.4, Ri = 0.1)

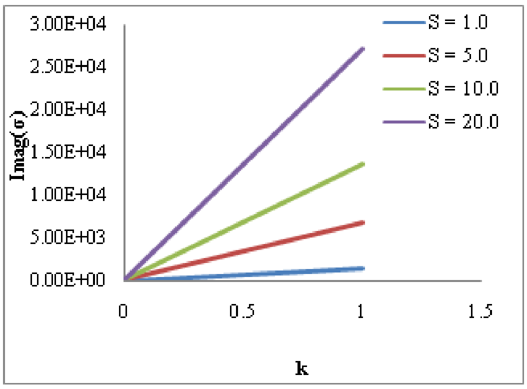

Figure 8. Growth rate vs wave number for various S **($\lambda < 0$)**(N^2 = 10.0, l = 1.0, Rm = 10.0, Ri = 1.0, n = 1.0)

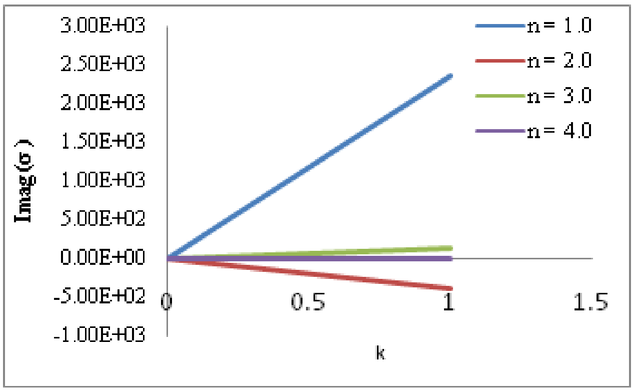

Figure 9. Growth rate vs wave number for various n **(λ < 0)**(N^2 = 10.0, l = 1.0, S = 10.0, Rm = 10.0, *Ri* = 1.0)

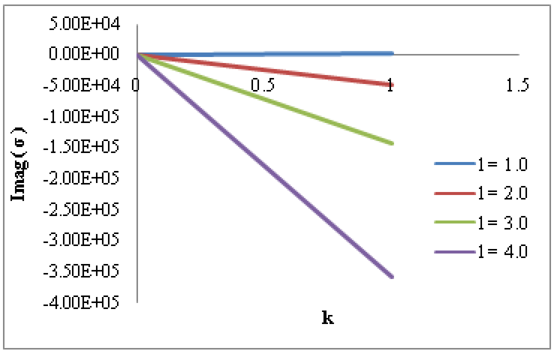

Figure 10. Growth rate vs wave number for various l **(λ < 0)**(N^2 = 10.0, l = 1.0, S = 10.0, k = 0.1, *Ri* = 1.0)

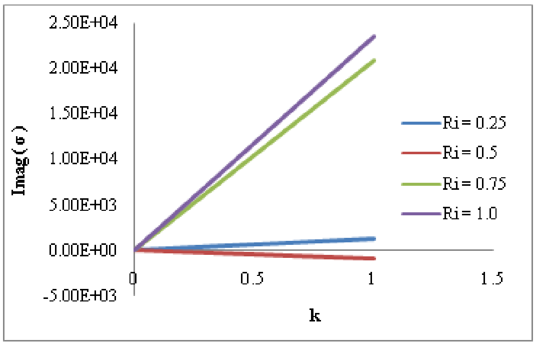

Figure 11. Growth rate vs wave number for various Ri **(λ < 0)**(N^2 = 10.0, l = 1.0, S = 10.0, n = 1.0, *Rm* = 10.0)

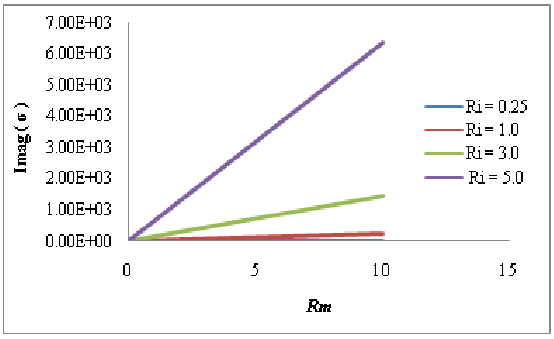

Figure 12. Growth rate vs *Rm* for various *Ri* (λ < 0) (N^2 = 10.0, l = 1.0, S = 10.0, k = 0.1, n = 1.0)

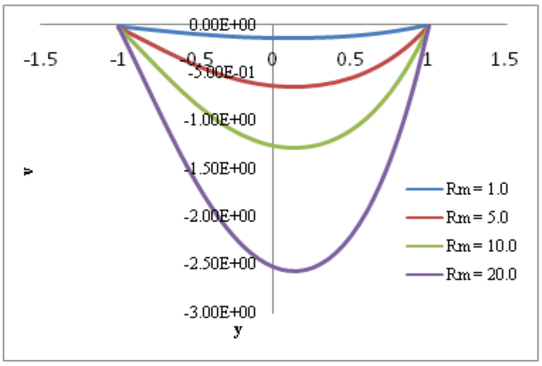

Figure 13. Velocity profile for various *Rm* (λ > 0) (l = 1.0, S = 10.0, k = 0.1, *Ri* = 0.1, N^2 = 0.1)

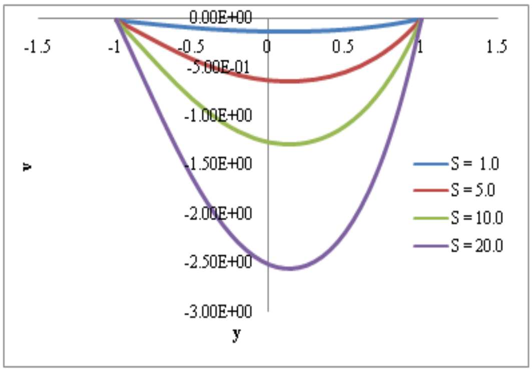

Figure 14. Velocity profile for various S (λ > 0) (l = 1.0, *Rm* = 10.0, k = 0.1, *Ri* = 0.1, N^2 = 0.4)

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APPENDIX

$D = -(1-\sigma_0)^{m_1-m_2}$

$C_1 = \frac{N^2}{i\,{N_0}^2}$

$C_2 = \frac{N^2}{i\,{N_0}^2}\,D$

$C_3 = \frac{(m_1-1)}{i\,(1+l^2)}$

$C_4 = \frac{(m_2-1)}{i\,(1+l^2)}D$

$C_5 = -C_3 + Ai$

$C_6 = -C_4 + Bi$

$C_7 = -l\,C_3$

$C_8 = -l\,C_3$

$C_9 = \frac{i}{(m_1+1)\,(m_1+2)}$

$C_{10} = \frac{i\,D}{(m_2+1)\,(m_2+2)}$

$C_{11} = \frac{Bi}{3!}$

$C_{12} = \frac{i\,BD}{3!}$

$C_{13} = \frac{Bi}{(m_1+1)}$

$C_{14} = \frac{i\,BD}{(m_2+1)}$

$C_{15} = \frac{-i\,C_7}{m_1\,(m_1+1)}$

$C_{16} = \frac{-i\,C_8}{m_2\,(m_2+1)}$

$C_{17} = \frac{-i\,BC_7}{m_1}$

$C_{18} = \frac{-iB\,C_8}{m_2}$

$C_{19} = \frac{-i\,BC_7}{3!}$

$C_{20} = \frac{-i\,BC_8}{3!}$

$C_{21} = \frac{B}{(m_1+1)\,(m_1+2)}$

$C_{22} = \frac{B\,D}{(m_2+1)\,(m_2+2)}$

$C_{23} = m_1 + i\,l\,C_7$

$C_{24} = D\,m_2 + i\,l\,C_8$

$C_{25} = \frac{2\,C_{23}}{m_1}$

$C_{26} = \frac{2\,C_{24}}{m_2}$

$C_{27} = \frac{C_{23}}{m_1(m_1+1)}$

$C_{28} = \frac{C_{24}}{m_2(m_2+1)}$

$C_{29} = m_1(m_1-1)$

$$C_{30} = i\,(1+l^2)\,C_1$$

$$C_{31} = \frac{\left(l^2 - i\,(1+l^2)\right)C_{23}(m_1-1)A^2}{2}$$

$$C_{32} = \frac{2\,AB\left(l^2 - i\,(1+l^2)\right)C_{23}(m_1-1)}{3!}$$

$$C_{33} = \frac{B^2\left(l^2 - i\,(1+l^2)\right)C_{23}(m_1-1)}{3!}$$

$$C_{34} = m_2(m_2-1)$$

$$C_{35} = i\,(1+l^2)\,C_2$$

$$C_{36} = \frac{A^2\left(l^2 - i\,(1+l^2)\right)C_{24}(m_2-1)}{2}$$

$$C_{37} = \frac{2\,AB\left(l^2 - i\,(1+l^2)\right)C_{24}(m_2-1)}{3!}$$

$$C_{38} = \frac{B^2\left(l^2 - i\,(1+l^2)\right)C_{24}(m_2-1)}{3!}$$

$$C_{39} = -BC_{11}\,m_1 + \left(\frac{2\,C_{23}}{3} + \frac{C_{25}\,m_1}{2}\right)B^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{40} = \left(\frac{3\,C_{25}\,m_1}{2} + C_{23}\right)AB\left(l^2 - i\,(1+l^2)\right)$$

$$C_{41} = (C_{25}\,m_1 + C_{23})A^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{42} = -BC_{12}\,m_2 + \left(\frac{2\,C_{24}}{3} + \frac{C_{26}\,m_2}{2}\right)B^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{43} = \left(\frac{3\,C_{26}\,m_2}{2} + C_{24}\right)AB\left(l^2 - i\,(1+l^2)\right)$$

$$C_{44} = (C_{26}\,m_2 + C_{24})A^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{45} = -3\,BC_{11} - C_{13}B(\,m_1+1) + \left(\frac{3\,C_{25}}{2} + C_{27}\,(m_1+1)\right)B^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{46} = \left(3\,C_{25} + 2\,C_{27}(m_1+1)\right)AB\left(l^2 - i\,(1+l^2)\right)$$

$$C_{47} = \left(C_{25} + C_{27}(\,m_1+1)\right)A^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{48} = -3\,BC_{12} - C_{14}B(\,m_2+1) + \left(\frac{3\,C_{26}}{2} + C_{28}\,(m_2+1)\right)B^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{49} = \left(3\,C_{26} + 2\,C_{28}(m_2+1)\right)AB\left(l^2 - i\,(1+l^2)\right)$$

$$C_{50} = \left(C_{26} + C_{28}(\,m_2+1)\right)A^2\left(l^2 - i\,(1+l^2)\right)$$

$$C_{51} = (-C_9)B^2(m_1+2) - 2\,BC_{\,13} + \left(2B^2\,C_{27} + C_{21}B(m_1+2)\right)\left(l^2 - i\,(1+l^2)\right)$$

$$C_{52} = (-C_9)AB(m_1+2) + \left(2AB\,C_{27} + C_{21}A(m_1+2)\right)\left(l^2 - i\,(1+l^2)\right)$$

$$C_{53} = (-C_{10})B^2(m_2+2) - 2\,BC_{\,14} + \left(2\,B^2C_{28} + C_{22}\,B(m_2+2)\right)\left(l^2 - i\,(1+l^2)\right)$$

$$C_{54} = (-C_{10})AB(m_2+2) + \left(2\,ABC_{28} + AC_{22}\,(m_2+2)\right)\left(l^2 - i\,(1+l^2)\right)$$

$$C_{55} = -B^2C_9 + \left(l^2 - i\,(1+l^2)\right)B\,C_{21}$$

$$C_{56} = -B^2C_{10} + \left(l^2 - i\,(1+l^2)\right)BC_{22}$$

$$C_{57} = \frac{C_{29}}{(m_1-1)^2 - (m_1-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{58} = \frac{C_{30}}{(m_1-1)^2 - (m_1-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{59} = \frac{C_{31}{\sigma_0}^2 + C_{32}{\sigma_0}^3 + C_{33}{\sigma_0}^4}{(m_1-1)^2 - (m_1-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{60} = \frac{C_{34}}{(m_2-1)^2 - (m_2-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{61} = \frac{C_{35}}{(m_2-1)^2 - (m_2-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{62} = \frac{C_{36}{\sigma_0}^2 + C_{37}{\sigma_0}^3 + C_{38}{\sigma_0}^4}{(m_2-1)^2 - (m_2-1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{63} = \frac{\sigma_0(2\,C_{31} + C_{41}) + {\sigma_0}^2(3\,C_{32} + C_{40}) + {\sigma_0}^3(4\,C_{33} + C_{39})}{{m_1}^2 - m_1 + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{64} = \frac{\sigma_0(2\,C_{36} + C_{44}) + {\sigma_0}^2(3\,C_{37} + C_{43}) + {\sigma_0}^3(4\,C_{38} + C_{42})}{{m_2}^2 - m_2 + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{65} = \frac{(C_{31} + C_{41} + C_{47}) + \sigma_0\,(3\,C_{32} + 2\,C_{40} + C_{46}) + {\sigma_0}^2\,(6\,C_{33} + 3\,C_{39} + C_{45})}{(m_1+1)^2 - (m_1+1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{66} = \frac{(C_{36} + C_{44} + C_{50}) + \sigma_0\,(3\,C_{37} + 2\,C_{43} + C_{49}) + {\sigma_0}^2\,(6\,C_{38} + 3\,C_{42} + C_{48})}{(m_2+1)^2 - (m_2+1) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{67} = \frac{(C_{32} + C_{40} + C_{46} + C_{52}) + \sigma_0\,(4\,C_{33} + 3\,C_{39} + 2\,C_{45} + C_{51})}{(m_1+2)^2 - (m_1+2) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{68} = \frac{(C_{37} + C_{43} + C_{49} + C_{54}) + \sigma_0\,(4\,C_{38} + 3\,C_{42} + 2\,C_{48} + C_{53})}{(m_2+2)^2 - (m_2+2) + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)}$$

$$C_{69} = C_{33} + C_{39} + C_{45} + C_{51}$$

$$C_{70} = C_{38} + C_{42} + C_{48} + C_{53}$$

$$C_{71} = -1 - \sigma_0$$

$$C_{72} = 1 - \sigma_0$$

$$C_{73} = \frac{C_{57}\left({C_{71}}^{m_1-1}{C_{72}}^{m_2} - {C_{72}}^{m_1-1}{C_{71}}^{m_2}\right) + C_{60}\left({C_{71}}^{m_2-1}{C_{72}}^{m_2} - {C_{72}}^{m_2-1}{C_{71}}^{m_2}\right)}{{C_{71}}^{m_2}{C_{72}}^{m_1} - {C_{71}}^{m_1}{C_{72}}^{m_2}}$$

$$C_{74} = \frac{C_{58}\left({C_{71}}^{m_1-1}{C_{72}}^{m_2} - {C_{72}}^{m_1-1}{C_{71}}^{m_2}\right) + C_{61}\left({C_{71}}^{m_2-1}{C_{72}}^{m_2} - {C_{72}}^{m_2-1}{C_{71}}^{m_2}\right)}{{C_{71}}^{m_2}{C_{72}}^{m_1} - {C_{71}}^{m_1}{C_{72}}^{m_2}}$$

$$\begin{aligned}C_{75} = \Big(&C_{59}\left({C_{71}}^{m_1-1}{C_{72}}^{m_2} - {C_{72}}^{m_1-1}{C_{71}}^{m_2}\right)\\
&+C_{62}\left({C_{71}}^{m_2-1}{C_{72}}^{m_2} - {C_{72}}^{m_2-1}{C_{71}}^{m_2}\right)\\
&+C_{63}\left({C_{71}}^{m_1}{C_{72}}^{m_2} - {C_{72}}^{m_1}{C_{71}}^{m_2}\right)\\
&+C_{65}\left({C_{71}}^{m_1+1}{C_{72}}^{m_2} - {C_{72}}^{m_1+1}{C_{71}}^{m_2}\right)\\
&+C_{66}\left({C_{71}}^{m_2+1}{C_{72}}^{m_2} - {C_{72}}^{m_2+1}{C_{71}}^{m_2}\right)\\
&+C_{67}\left({C_{71}}^{m_1+2}{C_{72}}^{m_2} - {C_{72}}^{m_1+2}{C_{71}}^{m_2}\right)\\
&+C_{68}\left({C_{71}}^{m_2+2}{C_{72}}^{m_2} - {C_{72}}^{m_2+2}{C_{71}}^{m_2}\right)\\
&+C_{69}\left({C_{71}}^{m_1+3}{C_{72}}^{m_2} - {C_{72}}^{m_1+3}{C_{71}}^{m_2}\right)\\
&+C_{70}\left({C_{71}}^{m_2+3}{C_{72}}^{m_2} - {C_{72}}^{m_2+3}{C_{71}}^{m_2}\right)\Big)/\end{aligned}$$

$$(C_{72}{}^{m_1}C_{71}{}^{m_2} - C_{71}{}^{m_1}C_{72}{}^{m_2})$$

$$C_{76} = \frac{C_{57}\left(C_{71}{}^{m_1-1}C_{72}{}^{m_1} - C_{72}{}^{m_1-1}C_{71}{}^{m_1}\right) + C_{60}\left(C_{71}{}^{m_2-1}C_{72}{}^{m_1} - C_{72}{}^{m_2-1}C_{71}{}^{m_1}\right)}{C_{71}{}^{m_2}C_{72}{}^{m_1} - C_{71}{}^{m_1}C_{72}{}^{m_2}}$$

$$C_{77} = \frac{C_{58}\left(C_{71}{}^{m_1-1}C_{72}{}^{m_1} - C_{72}{}^{m_1-1}C_{71}{}^{m_1}\right) + C_{61}\left(C_{71}{}^{m_2-1}C_{72}{}^{m_1} - C_{72}{}^{m_2-1}C_{71}{}^{m_1}\right)}{C_{71}{}^{m_1}C_{72}{}^{m_2} - C_{71}{}^{m_2}C_{72}{}^{m_1}}$$

$$C_{78} = \Big(C_{59}\left(C_{71}{}^{m_1-1}C_{72}{}^{m_1} - C_{72}{}^{m_1-1}C_{71}{}^{m_1}\right)$$
$$+C_{62}\left(C_{71}{}^{m_2-1}C_{72}{}^{m_1} - C_{72}{}^{m_2-1}C_{71}{}^{m_1}\right)$$
$$+C_{64}(C_{71}{}^{m_2}C_{72}{}^{m_1} - C_{72}{}^{m_2}C_{71}{}^{m_1})$$

$$+C_{65}\left(C_{71}{}^{m_1+1}C_{72}{}^{m_1} - C_{72}{}^{m_1+1}C_{71}{}^{m_1}\right)$$
$$+C_{66}\left(C_{71}{}^{m_2+1}C_{72}{}^{m_1} - C_{72}{}^{m_2+1}C_{71}{}^{m_1}\right)$$
$$+C_{67}\left(C_{71}{}^{m_1+2}C_{72}{}^{m_1} - C_{72}{}^{m_2+2}C_{71}{}^{m_1}\right)$$
$$+C_{68}\left(C_{71}{}^{m_2+2}C_{72}{}^{m_1} - C_{72}{}^{m_2+2}C_{71}{}^{m_1}\right)$$
$$+C_{69}\left(C_{71}{}^{m_1+3}C_{72}{}^{m_1} - C_{72}{}^{m_1+3}C_{71}{}^{m_1}\right)$$
$$+C_{70}\left(C_{71}{}^{m_2+3}C_{72}{}^{m_1} - C_{72}{}^{m_2+3}C_{71}{}^{m_1}\right)\Big)$$
$$/(C_{72}{}^{m_2}C_{71}{}^{m_1} - C_{71}{}^{m_2}C_{72}{}^{m_1})$$

$$C_{79} = (C_{75} + C_{63})C_{71}{}^{m_1} + (C_{78} + C_{64})\,C_{71}{}^{m_2}$$
$$+C_{59}\,C_{71}{}^{m_1-1} + C_{62}\,C_{71}{}^{m_2-1}$$
$$+C_{65}\,C_{71}{}^{m_1+1} + C_{66}\,C_{71}{}^{m_2+1}$$
$$+C_{67}\,C_{71}{}^{m_1+2} + C_{68}\,C_{71}{}^{m_2+2}$$
$$+C_{69}\,C_{71}{}^{m_1+3} + C_{70}\,C_{71}{}^{m_2+3}$$

$$C_{80} = C_{73}\,C_{71}{}^{m_1} + C_{76}\,C_{71}{}^{m_2}$$
$$+C_{57}\,C_{71}{}^{m_1-1} + C_{60}\,C_{71}{}^{m_2-1}$$

$$C_{81} = C_{74}\,C_{71}{}^{m_1} + C_{77}\,C_{71}{}^{m_2}$$
$$+C_{58}\,C_{71}{}^{m_1-1} + C_{61}\,C_{71}{}^{m_2-1}$$

$$C_{82} = \frac{C_{31}\sigma_0{}^4}{(m_1-1)^2 - (m_1-1) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{83} = \frac{C_{34}\sigma_0{}^4}{(m_2-1)^2 - (m_2-1) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{84} = \frac{\sigma_0{}^3(4\,C_{31} + C_{82})}{m_1{}^2 - m_1 + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{85} = \frac{\sigma_0{}^3(4\,C_{34} + C_{83})}{m_2{}^2 - m_2 + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{86} = \frac{\sigma_0{}^2\,(6\,C_{31} + 3\,C_{82} + C_{84})}{(m_1+1)^2 - (m_1+1) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{87} = \frac{\sigma_0{}^2\,(6\,C_{34} + 3\,C_{83} + C_{85})}{(m_2+1)^2 - (m_2+1) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{88} = \frac{(C_{27}) + \sigma_0\,(4\,C_{31} + 3\,C_{39} + 2\,C_{45} + C_{51})}{(m_1+2)^2 - (m_1+2) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{89} = \frac{(C_{28}) + \sigma_0\,(4\,C_{34} + 3\,C_{42} + 2\,C_{48} + C_{53})}{(m_2+2)^2 - (m_2+2) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{90} = \frac{C_{31} + C_{39} + C_{45} + C_{51} + C_{55}}{(m_1+3)^2 - (m_1+3) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{91} = \frac{C_{34} + C_{42} + C_{48} + C_{53} + C_{56}}{(m_2+3)^2 - (m_2+3) + \frac{Ri\,N^2}{N_0{}^2}(1+l^2)}$$

$$C_{92} = \Big(C_{82}\left(C_{71}{}^{m_1-1}C_{72}{}^{m_2} - C_{72}{}^{m_1-1}C_{71}{}^{m_2}\right)$$
$$+C_{83}\left(C_{71}{}^{m_2-1}C_{72}{}^{m_2} - C_{72}{}^{m_2-1}C_{71}{}^{m_2}\right)$$
$$+C_{84}(C_{71}{}^{m_1}C_{72}{}^{m_2} - C_{72}{}^{m_1}C_{71}{}^{m_2})$$
$$+C_{86}\left(C_{71}{}^{m_1+1}C_{72}{}^{m_2} - C_{72}{}^{m_1+1}C_{71}{}^{m_2}\right)$$
$$+C_{87}\left(C_{71}{}^{m_2+1}C_{72}{}^{m_2} - C_{72}{}^{m_2+1}C_{71}{}^{m_2}\right)$$
$$+C_{88}\left(C_{71}{}^{m_1+2}C_{72}{}^{m_2} - C_{72}{}^{m_1+2}C_{71}{}^{m_2}\right)$$
$$+C_{89}\left(C_{71}{}^{m_2+2}C_{72}{}^{m_2} - C_{72}{}^{m_2+2}C_{71}{}^{m_2}\right)$$
$$+C_{90}\left(C_{71}{}^{m_1+3}C_{72}{}^{m_2} - C_{72}{}^{m_1+3}C_{71}{}^{m_2}\right)$$
$$+C_{91}\left(C_{71}{}^{m_2+3}C_{72}{}^{m_2} - C_{72}{}^{m_2+3}C_{71}{}^{m_2}\right)\Big)$$
$$/(C_{72}{}^{m_1}C_{71}{}^{m_2} - C_{71}{}^{m_1}C_{72}{}^{m_2})$$

$$C_{93} = \Big(C_{82}\left(C_{71}{}^{m_1-1}C_{72}{}^{m_1} - C_{72}{}^{m_1-1}C_{71}{}^{m_1}\right)$$
$$+C_{83}\left(C_{71}{}^{m_2-1}C_{72}{}^{m_1} - C_{72}{}^{m_2-1}C_{71}{}^{m_1}\right)$$
$$+C_{85}(C_{71}{}^{m_2}C_{72}{}^{m_1} - C_{72}{}^{m_2}C_{71}{}^{m_1})$$
$$+C_{86}\left(C_{71}{}^{m_1+1}C_{72}{}^{m_1} - C_{72}{}^{m_1+1}C_{71}{}^{m_1}\right)$$
$$+C_{87}\left(C_{71}{}^{m_2+1}C_{72}{}^{m_1} - C_{72}{}^{m_2+1}C_{71}{}^{m_1}\right)$$
$$+C_{88}\left(C_{71}{}^{m_1+2}C_{72}{}^{m_1} - C_{72}{}^{m_1+2}C_{71}{}^{m_1}\right)$$
$$+C_{89}\left(C_{71}{}^{m_2+2}C_{72}{}^{m_1} - C_{72}{}^{m_2+2}C_{71}{}^{m_1}\right)$$
$$+C_{90}\left(C_{71}{}^{m_1+3}C_{72}{}^{m_1} - C_{72}{}^{m_1+3}C_{71}{}^{m_1}\right)$$
$$+C_{91}\left(C_{71}{}^{m_2+3}C_{72}{}^{m_1} - C_{72}{}^{m_2+3}C_{71}{}^{m_1}\right)\Big)/$$
$$(C_{72}{}^{m_2}C_{71}{}^{m_1} - C_{71}{}^{m_2}C_{72}{}^{m_1})$$

$$C_{94} = (C_{92} + C_{84})C_{71}{}^{m_1} + (C_{93} + C_{85})\,C_{71}{}^{m_2}$$
$$+C_{82}\,C_{71}{}^{m_1-1} + C_{83}\,C_{71}{}^{m_2-1} + C_{86}\,C_{71}{}^{m_1+1}$$
$$+C_{87}\,C_{71}{}^{m_2+1} + C_{88}\,C_{71}{}^{m_1+2} + C_{89}\,C_{71}{}^{m_2+2}$$
$$+C_{90}\,C_{71}{}^{m_1+3} + C_{91}\,C_{71}{}^{m_2+3}$$