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ON FUZZY ∂- SPACES

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Abstract

In this paper, a new class of fuzzy topological spaces, namely fuzzy ∂- spaces, are introduced and studied .Several characterizations of fuzzy ∂- spaces are established. The conditions under which fuzzy P-spaces become fuzzy ∂ -spaces and fuzzy ∂ -spaces become fuzzy Baire spaces, are obtained.

Keywords: Fuzzy G_δ - set, fuzzy nowhere dense set, fuzzy residual set fuzzy simply open set, fuzzy P-space, fuzzy submaximal space, fuzzy globally disconnected space, fuzzy hyper connected space, fuzzy Baire space.

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INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A. Zadeh** [15] in 1965. Any application of mathematical concepts depends firmly and closely how one introduces basic ideas that may yield various theories in various directions. If the basic idea is suitably introduced, then not only the existing theories stand but also the possibility of emerging new theories increases and on these lines, in 1967 **C.L.Chang** [4], introduced the notion of fuzzy topological spaces by means of fuzzy sets and his work paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study various types of fuzzy topological spaces. In classical topology **F.Azarpanah** and **M. Karavan** [2] introduced the notion of ∂- spaces (a space in which the boundary of any zeroset is contained in a zero set with empty interior). The concept of fuzzy simply open sets was introduced and studied by **G.Thangaraj** and **K.Dinakaran** [11]. The purpose of this paper is to introduce the concept of fuzzy ∂- spaces by means of fuzzy G_δ -sets and fuzzy simpy open sets. Several characterizations of fuzzy ∂- spaces are established. The conditions under which fuzzy P-spaces become fuzzy ∂ –spaces and fuzzy ∂ -spaces become fuzzy Baire spaces, are obtained. The conditions under which fuzzy residual sets become fuzzy simply open sets in fuzzy ∂ –spaces, are also obtained in this paper.

PRELIMINARIES

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given . In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [4] : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ, are defined respectively as follows :

(i). Int (λ) = $\vee$ { μ / μ $\leq$ λ, μ $\in$ T} and (ii). Cl (λ) = $\wedge$ { μ / λ $\leq$ μ, 1−μ $\in$ T }.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{Int}(\lambda) = \text{Cl}(1 - \lambda)$ and (ii). $1 - \text{Cl}(\lambda) = \text{Int}(1 - \lambda)$.

Definition 2.2 [3] : A fuzzy set λ in a fuzzy topological space (X,T) is called

(i). a fuzzy G_δ - set in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in$ T for i $\in$ I.

(ii). a fuzzy F_σ - set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in$ T for i $\in$ I.

Definition 2.3 [6] : A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X,T).

Definition 2.4 [6] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, int cl (λ) = 0, in (X,T).

Definition 2.5 [6]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.6 [7]: Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then, $1 - \lambda$ is called a fuzzy residual set in (X,T).

Definition 2.7 [11]: Let λ be a fuzzy set in a fuzzy topological space (X,T). The fuzzy boundary of λ is defined as $\text{Bd}(\lambda) = \text{cl}(\lambda) \wedge \text{cl}(1 - \lambda)$.

Definition 2.8 [17] : A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply open set if Bd (λ) is a fuzzy nowhere dense set in (X,T).

Definition 2.9 [9]: A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy σ-nowhere dense set if λ is a fuzzy F_σ-set in (X,T) with int (λ) = 0.

Definition 2.10 [7] : A fuzzy topological space (X,T) is called a fuzzy Baire space if $\text{int}\,(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T).

Definition 2.11 [3] : A fuzzy topological space (X, T) is called a fuzzy submaximal space if for each fuzzy set λ in (X, T) such that cl (λ) = 1, λ ∈ T.

Definition 2.12 [5] : A fuzzy topological space (X,T) is said to be fuzzy hyper‑connected if every non‑ null fuzzy open subset of (X,T) is fuzzy dense.

Definition 2.13 [14] : A fuzzy topological space (X,T) is called a fuzzy globally disconnected space if each fuzzy semi-open set in (X,T) is fuzzy open.

Definition 2.14 [8] : A fuzzy topological space (X, T) is called a fuzzy P-space if every non- zero fuzzy G_δ-set in (X,T), is fuzzy open.

Theorem 2.1 [12] : If λ is a fuzzy dense and fuzzy G_δ set in a fuzzy topological space (X,T), then λ is a fuzzy residual set in (X,T).

Theorem 2.2 [10] : If λ is a fuzzy residual set in a fuzzy submaximal space (X,T), then λ is a fuzzy G_δ -set in (X,T).

Theorem 2.3 [11] : If $\lambda = \mu \vee \delta$, where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Theorem 2.4 [11] : If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then $1 - \lambda$ is also a fuzzy simply open set in (X,T).

FUZZY ∂- SPACES

Defintion 3.1: A fuzzy topological space (X,T) is called a fuzzy ∂-space if each fuzzy G_δ - set in (X, T) is a fuzzy simply open set in (X,T). That is, (X,T) is a fuzzy ∂ -space if int cl (bd (λ)) = 0, for each fuzzy G_δ - set λ in (X,T) .

Example 3.1 : Let X = { a, b, c}. Consider the fuzzy sets λ, μ and γ defined on X as follows :

λ : X → [0, 1] is defined as λ(a) = 0.5 ; λ (b) = 0.6 ; λ (c) = 0.4,

μ : X → [0, 1] is defined as μ(a) = 0.4 ; μ (b) = 0.7 ; μ (c) = 0.5,

γ : X → [0, 1] is defined as γ(a) = 0.5 ; γ (b) = 0.8 ; γ (c) = 0.6.

Then, T = { 0, λ, μ, γ, λ ∨ μ, λ ∧ μ, 1 } is a fuzzy topology on X. Now $\lambda \wedge \mu = \lambda \wedge \gamma \wedge [\lambda \vee \mu]$ and $\mu = [\lambda \vee \mu] \wedge \gamma \wedge \mu$. Then, λ ∧ μ and μ are fuzzy G_δ - sets in (X,T). On computation cl (λ ∧ μ) = 1 and cl (μ) = 1. Now int cl [bd (μ)] = int cl [cl(μ) ∧ cl (1 − μ)] = int cl [cl(μ) ∧ (1 − int(μ)] = int cl [1 ∧ (1− int (μ)] = int cl [(1− int (μ)] = int cl [1− μ] = 1 − cl int [μ] = 1 − cl [μ] = 1 − 1 = 0, in (X,T) and hence μ is a fuzzy simply open set in (X,T) . Also int cl [bd (λ ∧ μ)] = int cl [cl (λ ∧ μ) ∧ cl (1 − [λ ∧ μ])] = int cl [cl (λ ∧ μ) ∧ (1 − int (λ ∧ μ)] = int cl [1 ∧ (1− int (λ ∧ μ)] = int cl [(1 − int (λ ∧ μ)] = int cl [1 − (λ ∧ μ)] = 1 − cl int [λ ∧ μ] = 1 − cl [λ ∧ μ] = 1 − 1 = 0, in (X,T) and hence λ ∧ μ is a fuzzy simply open set in (X,T) . Thus the fuzzy G_δ - sets λ ∧ μ and μ in (X,T), are fuzzy simply open sets in (X,T). Hence (X,T) is a fuzzy ∂–space.

Proposition 3.1: If λ is a fuzzy G_δ - set in a fuzzy ∂ -space (X,T), then

(i). int [bd(λ)] = 0 in (X,T)

(ii). 1 – bd (λ) is a fuzzy dense set in (X,T).

Proof : **(i)** Let λ be a fuzzy G_δ - set in (X,T). Since (X,T) is a fuzzy ∂-space, the fuzzy G_δ - set λ is a fuzzy simply open set in (X,T) and then int cl (bd (λ)) = 0 in (X,T). Now int [bd(λ)] ≤ int cl (bd (λ)) in (X,T) implies that int [bd (λ)] ≤ 0. That is, int [bd (λ)] = 0, in (X,T)

(ii). By (i) int (bd (λ) = 0, for a fuzzy G_δ - set λ in (X,T) . Then 1 − int (bd (λ)) = 1 and hence cl (1 − bd (λ)) = 1, in (X, T). Thus, 1 – bd (λ) is a fuzzy dense set in (X,T).

Proposition 3.2 : If a fuzzy topological space (X,T) is a fuzzy ∂ -space, then int cl (λ) ≤ cl int (λ) for a fuzzy G_δ-set λ in (X,T).

Proof : Let λ be a fuzzy G_δ-set in (X,T) . Since (X,T) is a fuzzy ∂ -space, by proposition 3.1, int [bd (λ)] = 0, in (X,T). Then, int [cl(λ) ∧ cl (1 − λ)] = 0 in (X,T). This implies that int cl (λ) ∧ int cl (1 − λ) = 0 in (X, T) and then int cl (λ) ∧ [1 − cl int(λ)] = 0 in (X, T) .Thus, int cl (λ) ≤ 1 − (1− cl int (λ)) in (X,T). Therefore int cl (λ) ≤ cl int (λ) for a fuzzy G_δ-set λ in (X,T).

Proposition 3.3 : If a fuzzy topological space (X,T) is a fuzzy ∂ -space, then λ ∧ (1 − λ) is a fuzzy nowhere dense set in (X,T), for a fuzzy G_δ-set λ in (X,T).

Proof : Let λ be a fuzzy G_δ-set in the fuzzy ∂ -space (X,T) . Now cl [λ ∧ (1 − λ)] ≤ cl (λ) ∧ cl (1 – λ) in (X,T) and then, int cl [λ ∧ (1 − λ)] ≤ int [cl (λ) ∧ cl (1 – λ)] in (X,T). Thus, int cl [λ ∧ (1 − λ)] ≤ int [bd (λ)] in (X,T). By proposition 3.2, int [bd (λ)] = 0 in (X,T). This implies that int cl [λ ∧ (1 − λ)] = 0 in (X,T). Hence λ ∧ (1 − λ) is a fuzzy nowhere dense set in (X,T), for a fuzzy G_δ-set λ in (X,T).

Proposition 3.4 : If a fuzzy topological space (X,T) is a fuzzy ∂ -space, then int [λ ∧ (1 − λ)] = 0, for a fuzzy G_δ-set λ in (X,T).

Proof : Let λ be a fuzzy G_δ-set in the fuzzy ∂ -space (X,T). Then, by proposition 3.3, λ ∧ (1 − λ) is a fuzzy nowhere dense set in (X,T), for the fuzzy G_δ-set λ in (X,T) and hence int cl [λ ∧ (1 − λ)] = 0 in (X,T). But int [λ ∧ (1 − λ)] ≤ int cl [λ ∧ (1 − λ)] implies that int [λ ∧ (1 − λ)] = 0, in (X,T).

Proposition 3. 5 : If λ is a fuzzy G_δ-set with int (λ) = 0, in a fuzzy ∂ -space (X,T), then λ is a fuzzy nowhere dense set in (X,T).

Proof : Let λ be a fuzzy G_δ-set with int (λ) = 0 in the fuzzy ∂ -space (X,T). Since (X,T) is a fuzzy ∂ -space, by proposition 3.1, for the fuzzy G_δ-set λ in (X,T), int [bd(λ)] = 0 . Then, int [cl (λ) ∧ cl (1 – λ)] = 0, in (X,T). This implies that int [cl (λ) ∧ (1 – int(λ)] = 0, in (X,T). Then, int [cl (λ) ∧ (1 – 0)] = 0, in (X,T) and hence int [cl (λ) ∧ (1)] = 0 and thus int cl(λ) = 0 in (X,T) . Therefore λ is a fuzzy nowhere dense set in (X,T).

Proposition 3.6 : If μ is a fuzzy σ-nowhere dense set in a fuzzy ∂ -space (X,T), then μ is a fuzzy simply open set with int (μ) = 0 in (X,T).

Proof : Let μ be a fuzzy σ-nowhere dense set in (X,T). Then, μ is a fuzzy F_σ-set with int (μ) = 0 in (X,T). Now 1 − μ is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy ∂ -space, the fuzzy G_δ-set 1 − μ is a fuzzy simply open set in (X,T) and then by theorem 2.4, 1 − [1 − μ], is also a fuzzy simply open set in (X,T). That is, μ is a fuzzy simply open set with int (μ) = 0 in (X,T).

The following proposition gives condition for a fuzzy topological space to become a fuzzy ∂ –space .

Proposition 3.7 : If each fuzzy G_δ-set is fuzzy open and dense in a fuzzy topological space (X,T), then (X,T) is a fuzzy ∂ -space.

Proof : Let μ be a fuzzy G_δ-set in (X,T) with cl (μ) = 1 and int (μ) = μ. Now int cl [bd (μ)] = int cl [cl(μ) ∧ cl (1 − μ)] ≤ int [cl cl(μ) ∧ cl cl (1 − μ)] = int [cl (μ) ∧ cl (1− μ)] = int [cl(μ) ∧ (1− int (μ))] = int [1 ∧ (1− μ)] Thus, int cl [bd (μ)] ≤ int [1− μ] = 1− cl (μ) = 1 − 1= 0, in (X,T). This implies that int cl [bd (μ)] = 0 in (X,T) and thus μ is a fuzzy simply open set in (X,T). Hence the fuzzy G_δ-set μ is a fuzzy simply open set in (X,T) implies that (X,T) is a fuzzy ∂ -space.

Proposition 3.8 : If λ is a fuzzy G_δ set such that cl (λ) = 1 in a fuzzy

∂-space (X,T), then λ is a fuzzy residual and fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy G_δ set such that cl (λ) = 1, in (X,T). Then, by theorem 2.1, λ is a fuzzy residual set in (X,T). Also since (X,T) is a fuzzy ∂ -space, the fuzzy G_δ-set λ is a fuzzy simply open set in (X,T). Thus λ is a fuzzy residual and fuzzy simply open set in (X,T).

Theorem 3.1 [13] : If λ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ –set η in (X,T) such that η ≤ λ.

Theorem 3.2 [9] : If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space (X, T), then 1 − λ is a fuzzy residual set in (X, T).

Proposition 3.9 : If λ is a fuzzy residual set in a fuzzy ∂-space (X, T), then there exists a fuzzy simply open set η in (X,T) such that η ≤ λ.

Proof : Let λ be a fuzzy residual set in (X, T). Then, by theorem 3.1, there exists a fuzzy G_δ –set η in (X,T) such that η ≤ λ. Since (X,T) is a fuzzy ∂ -space, the fuzzy G_δ-set η is a fuzzy simply open set in (X,T). Thus, for a fuzzy residual set λ in (X,T), there exists a fuzzy simply open set η in (X,T) such that η ≤ λ.

Proposition 3.10 : If λ is a fuzzy σ-nowhere dense set in a fuzzy ∂-space (X, T), then there exists a fuzzy simply open set η in (X,T) such that η ≤ 1 − λ.

Proof : Let λ be a fuzzy σ-nowhere dense set in (X,T). Then, by theorem 3.2, 1 − λ is a fuzzy residual set in (X, T). Since (X,T) is a fuzzy ∂ -space, by proposition 3.9, there exists a fuzzy simply open set η in (X,T) such that η ≤ 1 − λ.

Proposition 3.11 : If cl int (μ) = 1 for each fuzzy G_δ -set μ in a fuzzy topological space (X,T), then (X,T) is a fuzzy ∂ -space.

Proof : Let μ be a fuzzy G_δ set such that cl int (μ) = 1, in (X,T). Now int cl [bd (μ)] = int cl [cl(μ) ∧ cl (1 − μ)] ≤ int [cl cl(μ) ∧ cl cl (1 − μ)] = int [cl (μ) ∧ cl(1− μ)] = [int cl(μ)] ∧ [int cl (1− μ)] = int cl(μ) ∧ (1−cl int(μ)] Thus, int cl [bd (μ)] ≤ [int cl(μ)] ∧ [1 − 1] = [int cl(μ)] ∧ 0 = 0, in (X,T). This implies that int cl [bd (μ)] = 0 in (X,T) and thus μ is a fuzzy simply open set in (X,T) . Therefore the fuzzy G_δ set μ is a fuzzy simply open set in (X,T) implies that (X,T) is a fuzzy ∂ -space.

Proposition 3.12 : If for each fuzzy G_δ -set λ in a fuzzy topological space (X,T), λ = μ ∨ δ, where μ is a fuzzy open and fuzzy dense set and δ is a fuzzy nowhere dense set in (X,T), then (X,T) is a fuzzy ∂ -space.

Proof : Let λ be a fuzzy G_δ set in (X,T) such that λ = μ ∨ δ, where μ ∈ T, cl (μ) = 1 and int cl (δ) = 0 in (X,T) . Then, by theorem 2.3, λ is a fuzzy simply open set in (X,T). Therefore the fuzzy G_δ set λ is a fuzzy simply open set in (X,T), implies that (X,T) is a fuzzy ∂ -space.

FUZZY ∂- SPACES and OTHER FUZZY TOPOLOGICAL SPACES

Since a fuzzy simply open set need be a fuzzy open set in a fuzzy topological space, a fuzzy ∂ -space need not be a fuzzy P -space. The following two propositions give conditions for fuzzy P-spaces to become fuzzy ∂ –spaces.

Proposition 4.1: If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy P -space (X,T), then (X,T) is a fuzzy ∂-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T) such that cl (λ) = 1. Since the fuzzy topological space (X,T) is a fuzzy P-space, the fuzzy G_δ-set λ is a fuzzy open set in (X,T). Now int cl [bd (λ)] = int cl [cl(λ)∧ cl (1− λ)] ≤ int [cl cl(λ) ∧ cl cl(1 − λ)] = int [cl(λ) ∧ cl(1− λ)] = int [cl(λ) ∧ (1− int (λ))] = int [1 ∧ (1− λ)] (since cl (λ) = 1 and int(λ) = λ in (X,T)). Thus, int cl [bd (λ)] ≤ int [1− λ] = 1− cl (λ) = 1−1= 0, in (X,T).

This implies that int cl [bd (λ)] = 0 in (X,T) and thus λ is a fuzzy simply open set in (X,T). Hence the fuzzy G_δ-set λ is a fuzzy simply open set in (X,T) implies that (X,T) is a fuzzy ∂ -space .

Proposition 4.2 : If each fuzzy F_σ-set is a fuzzy nowhere dense set in a fuzzy P- space (X,T), then (X,T) is a fuzzy ∂ -space.

Proof : Let λ be a fuzzy F_σ-set in (X,T) such that int cl (λ) = 0. Then, $1 - \lambda$ is a fuzzy G_δ-set in (X,T) . Now int (λ) ≤ int cl (λ) in (X,T), implies that int (λ) = 0 and then cl (1 – λ) = 1− int (λ) = 1 – 0 = 1. Thus, the fuzzy G_δ-set 1 – λ is a fuzzy dense set in the fuzzy P-space (X,T). Then, by proposition 4.1, (X,T) is a fuzzy ∂ -space .

The following two propositions give conditions for fuzzy residual sets to become fuzzy simply open sets in fuzzy ∂ –spaces.

Proposition 4.3 : If λ is a fuzzy residual set in a fuzzy submaximal and fuzzy ∂ -space (X,T), then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy residual set in (X,T). Since (X,T) is a fuzzy submaximal space, by theorem 2.2, λ is a fuzzy G_δ -set in (X,T). Also since (X,T) is a fuzzy ∂ -space, the fuzzy G_δ-set λ is a fuzzy simply open set in (X,T). Thus the fuzzy residual set λ is a fuzzy simply open set in (X,T).

Theorem 4.1 [14] : If λ is a fuzzy residual set in a fuzzy globally disconnected space (X,T), then λ is a fuzzy G_δ- set in (X,T).

Proposition 4.4 : If λ is a fuzzy residual set in a fuzzy globally disconnected and fuzzy ∂ -space (X,T), then λ is a fuzzy simply open set in (X,T).

Proof : Let λ be a fuzzy residual set in (X,T). Since (X,T) is a fuzzy globally disconnected space by theorem 4.1, λ is a fuzzy G_δ -set in (X,T). Also since (X,T) is a fuzzy ∂ -space, the fuzzy G_δ-set λ is a fuzzy simply open set in (X,T). Thus the fuzzy residual set λ is a fuzzy simply open set in (X,T).

The following two propositions give conditions for fuzzy ∂ -spaces to become fuzzy Baire spaces.

Proposition 4.5 : If int ($\bigvee_{i=1}^{\infty}[\lambda_i \wedge (1 - \lambda_i)]$) = 0, where (λ_i)'s are fuzzy G_δ-sets in a fuzzy ∂ -space (X,T), then (X,T) is a fuzzy Baire space.

Proof : Let (λ_i)'s (i = 1 to ∞) be fuzzy G_δ-sets in (X,T) such that int ($\bigvee_{i=1}^{\infty}[\lambda_i \wedge (1 - \lambda_i)]$) = 0. Since (X,T) is a fuzzy ∂ -space, by proposition 3.3, $[\lambda_i \wedge (1 - \lambda_i)]'$s are fuzzy nowhere dense sets, for the fuzzy G_δ-sets (λ_i)'s in (X,T). Thus, int ($\bigvee_{i=1}^{\infty}[\lambda_i \wedge (1 - \lambda_i)]$) = 0, where $[\lambda_i \wedge (1 - \lambda_i)]'$s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space.

Proposition 4.6 : If int ($\bigvee_{i=1}^{\infty}[\lambda_i]$) = 0, where (λ_i)'s are fuzzy G_δ-sets with int (λ_i) = 0, in a fuzzy ∂ -space (X,T), then (X,T) is a fuzzy Baire space.

Proof : Let (λ_i)'s (i = 1 to ∞) be fuzzy G_δ-sets with int (λ_i) = 0 in (X,T) such that int ($\bigvee_{i=1}^{\infty}[\lambda_i]$) = 0. Since (X,T) is a fuzzy ∂ -space, by proposition 3.5, $(\lambda_i)'$ s are fuzzy nowhere dense sets, for the fuzzy G_δ-sets (λ_i)'s in (X,T). Thus, int ($\bigvee_{i=1}^{\infty}[\lambda_i]$) = 0, where $(\lambda_i)'$s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space.

Proposition 4.7 : If a fuzzy topological space (X,T) is a fuzzy hyperconnected and fuzzy P –space, then (X,T) is a fuzzy ∂ -space.

Proof : Let λ be a fuzzy G_δ set in (X,T). Since the fuzzy topological space (X,T) is a fuzzy P-space, the fuzzy G_δ-set λ is a fuzzy open set in (X,T). Also since (X,T) is a fuzzy hyperconnected space, the fuzzy open set λ is a fuzzy dense set in (X,T) and thus the fuzzy G_δ set λ is fuzzy open and fuzzy dense in (X,T). Then, by proposition 3.7, (X,T) is a fuzzy ∂ -space.

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