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Valuation of exotic options through Monte Carlo simulation in the Colombian local market

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Abstract

The disadvantage of offering exotic options in the local market is the valuation method. However, the Monte Carlo simulation method allows to obtain a value for the options including the exotic options. This article will focus on determining the value of exotic options with local underlying assets to obtain an offer premium for these financial instruments. To achieve this, a Wiener process will be applied in the valuation.

Keywords: Valuation; Asian options; Barrier options; Monte Carlo.

INTRODUCTION

After the creation of the Colombian Stock Exchange in 2001, various projects have been built with the aim of consolidating or strengthening the country's stock market [1]. One of the key elements of financial markets, which currently takes on special importance, are financial instruments and with them, the derivatives market. This is due to the ability of the latter to offer coverage to market participants, that is, it enables investors to manage the risk associated with their investments, an important function in the context of globalization [1], [2]. In addition to offering other benefits such as the possibility of greater efficiency in the flow of financial resources and increase the interest of the country for investment.

For all of the above, and in order to further develop the stock markets, modernize them and innovate in them, in 2008, Colombia became the third country in Latin America to create a platform for the market of standardized derivatives through the Stock Exchange of Colombia [3], [4]. However, this market only trades futures contracts, that is, standardized options contracts are not negotiated, so negotiations on these must be done in the OTC market [4]. The standardized market for options has not been created due to the problems that have to do with valuation methods, so the need to propose these methods is important for the country context.

When performing an analysis in the literature, there are several studies that demonstrate the importance in the development of valuation models in order to reduce uncertainty; Hong [5]; for example, it indicates that setting the price of exotic derivatives, specifically options, becomes a challenge for investment professionals, because the standard approach focuses on the use of parametric or traditional models, where the price is always fixed as the discount of instrument payments, in neutral risk scenarios. However, there is also a broad spectrum of alternative models, which also generates uncertainty in the choice of the model.

Ribeiro and Webber [6] propose to use the Lévy process in the Montecarlo valuation method, in order to reduce the variance of the estimate, which generates a positive result in its investigation; and considers it a good solution for options that do not have an apparent analytical solution. A similar approach is made by Shevchenko [7] who also uses the Monte Carlo simulation but in this case sequentially, and observes a small improvement in the price estimate, however it indicates that it is not very significant, and that both the estimation by the traditional method and the sequential continue to introduce biases.

On the other hand, computational sciences are also presented as an alternative for the valuation of exotic options; Dempster and Richards [8], for example, perform an analysis of the options assessment literature that focuses on the adoption of techniques such as linear programming based on the simplex method and Partial Differential Equations -EDP; both for traditional options and exotic options, and as a result, it expresses that this type of tools generate greater precision in the estimation of volatility with constant or variable trends; which thus generates better long-term coverage.

Based on the literature and the needs expressed above; this work proposes to study the financial options in a general way, to then delve into the exotic options, which take special importance due to their ability to adapt to the needs of different investors. According to this, the article presents the following structure: First, a theoretical contextualization is made about financial derivatives, financial options, exotic options and methods for valuation; In the following section, the methodological proposal for a valuation model of exotic purchasing options is made, through the use of the Weiner process or Geometric Brownian Motion; applied to the underlying asset of the TRM (exchange rate USD/COP) and finally, the results and conclusions of the study are presented.

FINANCIAL OPTIONS

Derivative products are used for various economic purposes, in general, these are within three major uses: coverage, speculation and arbitration. Coverage occurs when the agents participating in the contracts intervene in them in order to avoid market risks or credit risks due to future changes in a market variable. The advantage of these contracts is that they provide insurance, since it allows an investor to transfer the risk of loss, but at the same time allows him to take advantage of the beneficial changes; this makes the option contracts flexible tools [4].

Speculation occurs when the agents participating in the contracts deliberately decide to take a position in the market assuming greater risks in order to obtain profitability, that is, speculators bet on a future direction of a market variable. To make this bet, speculators use option contracts as a form of leverage, which magnifies the financial consequences; the advantage of using options in this situation is that, if the consequences are highly negative, the speculator may decide not to exercise the option and must only assume the amount paid in the option premium. It should be noted that speculators enter the market thanks to the risk that the hedgers want to convey [4].

Finally, arbitration seeks to achieve profits through simultaneous transactions on the same underlying asset in two or more markets. In effect, what arbitrageurs seek is to ensure a risk-free return by taking compensation positions in the different markets [4].

Financial options are instruments that allow to reach any of the purposes previously expressed and are defined as: a contract that gives the holder or buyer the right, but not the obligation, to buy or sell any stock at a predetermined date already a pre-established price; and, it gives the seller or subscriber the obligation, in case the buyer exercises the right, to sell or buy, correspondingly [4], [9] [10].

Based on the foregoing, it can be said that the rights and obligations of the buyer and the seller in the options contracts are asymmetric. In other words, the buyer has a right that he can exercise or not, according to what he considers pertinent or convenient on the expiration date of the contract; while the seller only has obligations, since he must sell or buy if the buyer decides to exercise the option [4].

The options are issued on a wide variety of securities, the most common of which are shares, indexes of stock markets, currencies, futures, treasury certificates and swaps [4], [9]. Some important elements of option contracts are the date, the price and the premium of the option. The pre-established date in the contract is the date on which the option contract expires, which is why it is known as the expiration date; the price of the contract is the price agreed upon in the option contract and is known as strike price or strike price, commonly it is called K; and, finally, the premium of the option is the advance payment that must be made by the agent participating in the contract.

Additionally, there are two important aspects that should be considered about the options. The first is the classification of the options and the second is the position that the investor takes in the option contract. In general, the options can be classified according to the right granted by the option to the buyer of the same and according to the time in which the right that they grant can be exercised. According to this, the options can be purchase (Call) or sale (Put). Purchase options give your holder the right to buy an asset at a certain price on the due date; while the put options give the holder the right to sell an asset at a certain price on the due date. In the classification according to time, the options can be American or European. The American options can be exercised at any time of his life until his expiration date; while European options can be exercised only on their due date. Commonly, the options that are traded in the derivatives markets are the American options, although these are more difficult to analyze [11].

Within each option contract there are two parts. On the one hand, the long position is found and on the other hand there is the short position.

$$\max(0, S\_prom - K) \quad (1)$$

$$\max(0, K - S\_prom) \quad (2)$$

In the Asian options with an average exercise price, unlike those discussed above, the exercise price does vary, and is calculated through an average of the values reached by the underlying asset [12]. This variety of Asian options guarantees that the price paid for an asset in frequent negotiations during a certain period is not greater, nor less, than the final price [11]. As in the Asian options with average value of the underlying, the payment depends on the type of option. For purchase options, the payment is calculated according to equation 3; while, for the sale options, the payment is given by equation 4.

$$\max(0, S\_T - S\_prom) \quad (3)$$

$$\max(0, S\_prom - S\_T) \quad (4)$$

The barrier options or conditional options are those whose payment depends on the price of the underlying asset reaching a certain level during a certain period, that is, its possibility of exercise will depend on the underlying to reach a certain level, known as a barrier [12]. In general, one could say that they are options that are canceled or activated depending on the values taken by the underlying. The main advantage of this type of options is its lower price compared to traditional options; this saving in cost is due to three factors: the proximity of the barrier, the life of the option and volatility [12]–[14]. There are two types of barrier options: knock-in options and knock-out options. The knock in occurs when the price of the underlying asset reaches the barrier, and, therefore, the conditional option immediately becomes a simple purchase or sale option. Within these options there are two additional classifications, which are: up-and-in options and down-and-in options [12]. In the options up-and-in, the right to exercise the option is obtained when the value of the underlying asset is located above the barrier at some point during the life of the option; while in the options down-and-in the exercise right is activated if the value of the underlying falls below the barrier.

Knock out options have the right to be exercised only if the value of the underlying does not reach the barrier, that is, the option ceases to exist if the value of the underlying touches the barrier at some point in its life. As in the knock in options, the knock out options are divided into two types: up-and-out options and down-and-out options. In the options up-and-out, the right to exercise the option is lost when the value of the underlying asset is located above the barrier at some point during the life of the option; while down-and-out options cease to exist if the value of the underlying falls below the barrier [12].

As for the barrier options in their entirety, that is, knock-in options and knock-out options, it should be noted that their prices are closely related to the prices of the regular options. For example, the price of a down-and-out purchase option plus the price of a down-and-in purchase option must be equal to the price of a regular European option (when exercise prices, expiration times and barrier levels are same). In the same way, the price of a sale option down-and-out plus the price of a sale option down-and-in, must be equal to the price of a regular European option [11].

MATERIALS AND METHODS

After presenting the most common methods for the valuation of financial options, and with the aim of solving the problem in the local market, a model of valuation of exotic purchase options is proposed, which focuses on the valuation of Asian options with average value of the underlying and options barrier up-and-in that have as an underlying asset currency COP/USD.

As already mentioned, the problem that arises in Colombia with the options contracts, and what does not allow them to be traded in the local market, is the valuation method of them. In the case of options, their theoretical value is the expected value of the benefits that would be received [4]. In this way, there is a need to study the existing valuation models and analyze which are the best for this type of derivative instruments.

In this article we will study the Black-Scholes method, the binomial tree method, the Geometric Brownian Motion and the Monte Carlo simulation. The Black-Scholes model was proposed in the early 1970s by Fischer Black, Myron Scholes and Robert Merton for the valuation of stock options. This model has strongly influenced the way in which traders value and hedge options, and has been fundamental to the growth and success of financial engineering over the past 30 years [11]. The assumptions for this model are:

1. The price behavior of the underlying corresponds to a normal logarithmic model with μ and σ constants, in addition to a continuous stochastic Gauss-Wiener evolution process defined by:

   $$\frac{dS}{S} = \mu dt + \sigma dZ \quad (5)$$

   Where dS is the variation of the price of S at time dt, μ is the mathematical expectation of the instantaneous yield of the underlying, σ is the standard deviation, and dZ is a standard Gauss-Wiener process [4], [15].

2. There are no transaction costs or taxes. All assets are perfectly divisible.
3. There are no dividends on the stock during the life of the option.
4. There are no risk-free arbitrage opportunities.
5. The negotiation of values is continuous.
6. Investors can acquire or grant loans at the same risk-free interest rate.
7. The short-term risk-free interest rate, r, is constant.

The Black-Scholes formulas for calculating the prices of European purchase and sale options on shares that do not pay dividends are:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (6)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \quad (7)$$

Where c and p are the prices of the European call and put options, S_0 is the share price, K is the exercise price, r is the constant risk free interest rate (expressed as a continuous composition), N(x) is the cumulative probability function for a standardized normal variable and T is the time to expiration. In addition, d_1 and d_2 are calculated as follows:

$$d_1 = \frac{Ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (8)$$

$$d_2 = \frac{Ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (9)$$

Where σ is the volatility of the share price. This volatility is calculated as follows:

$$\sigma = \frac{s}{\sqrt{\tau}} \quad (10)$$

Where τ is the duration of the interval in years and s is calculated in the following way:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\mu_i - \bar{\mu})^2} \quad (11)$$

Where n is the number of observations and μ_i is calculated as follows:

$$\mu_i = Ln\left(\frac{S_i}{S_{i-1}}\right) \quad (12)$$

Where S_i action price at the end of interval i, where i = 0, 1, ..., n [15], [16].

As mentioned from the beginning, and in the definition of the assumptions of the model, all the equations presented are applicable for European stock options that do not pay dividends. But, in order to make this model applicable to other types of options, other authors have made different modifications to the initial model, changing in a way the equations already proposed. For example, if the share pays dividends, it is considered that on the date on which the dividend is paid the share price is reduced by the amount of the dividend and its effect will be to reduce the value of the call option and increase the value of the dividend the put option [4].

A very used technique to value an option is the construction of a binomial tree, method developed by Cox, Ross, and Rubinstein in 1979, which consists of the creation of a diagram that represents the different trajectories that the price of an action could follow during the life of the option ([15], [16]. According to its authors, this method allows the agents participating in the options contracts to determine, at any time, whether it is more beneficial to exercise the option at that moment or wait until the expiration date.

This model assumes that the due date can be divided into discrete periods, represented by Δt. Additionally, it proposes that the underlying asset is shares and its price depends on a behavior associated with two coefficients: μ, if the behavior is upward, or d, if the behavior is downward, reflecting favorable or unfavorable market conditions; these coefficients depend on the volatility (σ) and the length of the periods (Δt), and are by:

$$\mu = e^{\sigma\sqrt{\Delta t}} \tag{13}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{\mu} \tag{14}$$

Figure 1 shows a binomial tree where the price evolution for an underlying asset is observed. There is represented the distribution that the possible future values of the underlying asset can take.

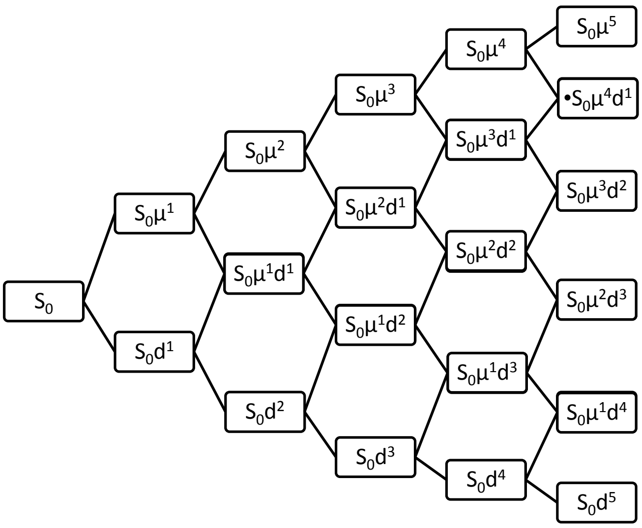

Figure 1. Binomial tree for the evolution of the price of an underlying asset.

To be able to construct the binomial tree, it is necessary to know the probability that the price of the asset has to grow or decrease. The probability of growing is given by equation 15 and the probability of decreasing is shown in equation 16 [15].

$$p = \frac{e^{rT} - d}{\mu - d} \tag{15}$$

$$q = 1 - p \tag{16}$$

Where r is the risk-free rate and T is time.

After knowing this probability, it is possible to value the option applying equation 17.

$$f = e^{-rT}(pf_u + qf_d) \tag{17}$$

Where f_u is the benefit that would be obtained if the underlying price rises to S_0u and f_d is the benefit that would be obtained if the underlying price drops to S_0d.

It should be noted that, from the value of the options given in the nodes on the right of the tree, it is possible to calculate the other values by applying the probability function in each pair of vertically adjacent values [15]. In addition, this method can be used to value other types of options, through changes in equation 15, that is, modifying the way to calculate p.

On the other hand, the Geometric Brownian Motion (GBM) is a continuous time stochastic process that is used mainly in the simulation of future prices of a financial asset based on the historical data of this one [17]. The GBM part of the following properties for prices:

1. They are continuous in time and value.
2. They follow a Markov process, that is, only the current price is relevant to predict future prices.
3. The proportional profitability of a financial asset during a very short period of time is normally distributed.
4. The price follows a normal logarithmic distribution.
5. The return (with continuous composition) follows a normal distribution.

The mathematical formulation of the Geometric Brownian Motion is shown below:

$$S_T = S_0 * e^{\left(\mu - \sigma^2/_2\right)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon} \tag{18}$$

Where S_T is the future price of the financial asset; S_0 is the last known price of the asset, μ and σ represent the average of returns and volatility, respectively (both are constant); Δt represents the period of time in which the prices will be presented (be it weekly, monthly, quarterly, etc.) and ε is a random number that follows a normal distribution with zero mean and standard deviation of 1.

The returns and volatility of the asset are calculated as follows:

$$R_i = \mathrm{Ln}\left(\frac{S_i}{S_{i-1}}\right) \tag{19}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(S_i - \mu)^2} \tag{20}$$

Monte Carlo simulation, on the other hand, is a tool widely used in practice, which consists especially of artificial or simulated sampling; that is, to generate random numbers and turn them into observations of the random variable of the model [18]. More specifically, it can be said that the Monte Carlo simulation is responsible for estimating the expectation of functions of random variables through the average of a considerable number of samples obtained from said simulations [14]. In this work, the random variable whose distribution is unknown is the average of the values of the options. This calculation can be seen in equation 21.

$$E(S_T) = \frac{1}{n}\sum_{i=1}^{n} S(T_i) \tag{21}$$

Where $E(S_T)$ is the expectation of the value of the option, $S(T_i)$ is the value of the quotations of the stock in time i and n is the number of samples obtained from the simulations [18], [19].

In general, what is sought with this method is to simulate the different values of S (T_i) through the generation of random numbers, known with the symbol ε, and thus obtain $E(S_T)$. By repeating this procedure a large number of times, an optimal sample will be obtained and an appropriate estimate can be obtained for the value of the option.

It should be noted that this method has an important drawback, but this can be addressed. This drawback is that the result obtained is not exact but an approximation, which produces an error when using the estimate, but as already mentioned, this can be counteracted; to do so, the number of simulations generated must be increased to estimate the value of financial assets [18], [19]

To achieve this, the Geometric Brownian Motion, presented in equation 18, will be used, which will model the prices of the underlying asset, which in this work will be the exchange rate of the Colombian peso against the dollar, measured by the indicator TRM. For the modeling of currencies, the equation of the Brownian Geometric Motion must be made the following modification, which will be the change of μ by the risk-free rate (r) and the inclusion of the foreign risk-free rate (rf) [20]. The modified equation is presented below.

$$S_T = S_0 * e^{\left(r-rf-\sigma^2/_2\right)\Delta t+\sigma\sqrt{\Delta t}\,\epsilon} \tag{22}$$

After having modeled the future behavior of the prices for the days of validity of the option, the Monte Carlo simulation with 50,000 iterations will be applied to obtain the expected prices, that is, the premium of each option.

For the application of the model, the data of the TRM prices will be taken from the databases of the Colombian Stock Exchange [21]. The historical information will contain the daily variation of the prices of 3 years, beginning on November 18, 2014 and ending on November 17, 2017. To avoid errors in the estimation of the premiums of the options, the days where it was not The TRM is quoted, that is, Saturdays, Sundays and holidays. From this information, the last known price of the TRM is extracted, which is S_0 in the model and is equivalent to $ 3,003.19.

In addition, based on this information, we also obtain the value of the volatility that would be present in the model, this is done through the calculation of the standard deviation of the continuous returns that the TRM had in the period described; based on this, the volatility that have is of 14.7573% continuous annual.

On the other hand, the domestic risk-free rate that will be worked on will be the current IBR for one month, because the effective term of the options will be established as 30 days; this rate is 4.8038% continuous annual and was obtained from the Colombian Stock Exchange [22]. As for the foreign risk-free rate, this was defined as the one in effect in the United States Treasury Bonds; its value is 1.0742% continuous annual and was taken from the US Department of the Treasury [23].

Finally, as already mentioned, Δt represents the period of time in which prices will be presented, therefore, as we will work with continuous daily rates, the value of Δt will be 30 days and T0 will be defined as the date in which the simulations were carried out, that is to say, November 17, 2017.

RESULTS

The Asian options, as mentioned above, are options in which their value is calculated based on some kind of average on the values reached by the underlying during all or part of the life of the option [14]. In this case, the average of the whole life of the option will be taken and it will be done for options in which the price of the underlying is obtained with the average and the exercise price remains fixed.

For the application, S_0 was taken as $ 3,003.19, based on this the modeling of the prices was made and the possible values of the TRM were found during the life of the option. Based on these values, S_T was obtained, that is, the average of the TRM in the 30 days. When having S_T, the expected compensation and the present value of it were calculated. Finally, as already mentioned, when performing the 50,000 iterations of the Monte Carlo simulation, the option premium was obtained, which is the average of the present values of the compensations [19].

In order to carry out an exhaustive analysis, various values were taken for K, starting from $ 2,800 to $ 3,050 with increases of $ 50. This allows visualizing the variations suffered by the premium due to the difference between S_0 and K. Table 1 shows the results obtained.

Table 1. Variation of the value of the premium in Asian options due to changes in the strike price.

***Strike price* (K)**	**Premium**
$ 2,800	$ 209.1910
$ 2,850	$ 159.4143
$ 2,900	$ 109.6376
$ 2,950	$ 59.8677
$ 3,000	$ 14.0539
$ 3,050	$ 0.1904

As can be seen in the results obtained, Asian options are a type of option that is usually low cost, because their profit margin is lower than that of other options. The above is due to the fact that the value of S_T is less volatile, since, as seen during this work, it is obtained by means of an average and not only a final value [19]. This represents an advantage for conservative investors, since it would have little sensitivity to significant and unexpected changes at the end of the option life [14].

In a complementary way, it can be observed that as the value of K increases, the option price decreases. This occurs because the option is initially in the money, that is, $S_0 > K$, which is a very advantageous and therefore expensive position; but as K increases, this position becomes less favorable until it becomes an out of money option, that is, $S_0 > K$, which makes the option less likely to be exercised and this makes it an option less attractive and more economical.

The barrier option is a conditional option whose payment depends on the price of the underlying asset reaching a certain level to be activated. The option selected is the up-and-in option, which is activated when the price of the underlying asset is above the barrier [12].

The price modeling was done the same as that proposed for the Asian options, that is, S_0 was taken as $ 3,003.19 and based on this the possible values of the TRM were found during the life of the option. Then, the fulfillment of a condition that guarantees that the option surpassed the barrier was verified, which was established as $ 3,000, otherwise, the option was simply deactivated or ceased to exist. The options that exceeded the barrier were calculated the expected compensation and the present value of this. Finally, the option premium was obtained by Monte Carlo simulation.

As in the analysis performed for the Asian options, in this case several values were also taken for K, starting from $ 2,800 to $ 3,050 with increases of $ 50, to study the changes presented by the premium due to the difference between S_0 and K The results are shown in Table 2.

Table 2. Variation of the premium value in barrier options due to changes in the strike price.

Strike price **(K)**	**Premium**
$ 2,800	$ 220.1962
$ 2,850	$ 176.2571
$ 2,900	$ 133.9662
$ 2,950	$ 99.32642
$ 3,000	$ 70.05812
$ 3,050	$ 46.40391

Based on the results presented in Table 2, it is highlighted that the barrier options are more expensive than the Asian options, due to the differences in the S_T calculation that these types of options have. However, a similarity with the situation presented in the Asian options is evident since the premium has the same behavior, that is, the value of the premium is lower as the exercise price agreed between the parties increases. As mentioned, this is because of the position in which the option is against K, which establishes that there are so many possibilities that the option is exercised, since it will only be exercised in the position in the money.

CONCLUSIONS

In the present work, we initially defined generalities about financial options and exotic options, to then define more thoroughly the Asian and barrier options, which were the main object of study. After this contextualization, the most commonly used methodologies to evaluate options were presented and a valuation methodology based on the Geometric Brownian Motion and the Monte Carlo simulation was implemented. This methodology was selected because it is simple to understand and offers greater flexibility to be used in all types of options and not limited to just one type, as is the case of the Black-Scholes method that only allows to evaluate European options.

Regarding the application of the methodology, satisfactory results were obtained with the modeling, since the premiums behaved as expected; they decreased as the difference between S_0 and K became smaller because having a smaller difference between them the possibilities of exercising the option were reduced and a location in an in the money scenario became more difficult to reach.

The results of this work can be considered as significant contributions to the process of including options in the Colombian derivatives market, as it contributes to the development, implementation and offering of new and varied alternatives for investors. It should be noted that these alternatives attract new investors because they represent a possibility to overcome the limits of standard options and offer greater flexibility.

For the purposes of future research, it would be interesting to implement this methodology or a variation of it to evaluate sale options of the same types of exotic options and make comparisons about the behavior and variation of their premiums. It would also be interesting to evaluate plain vanilla options to establish the differences in prices between traditional options and exotic options.

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