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Effect of Estimation Error on Adaptive L-MRC Receiver over Nakagami-m Fading Channels

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Abstract

One of the key challenges in wireless communication is to deliver high speed data, using a limited bandwidth, among a large group of user. Adaptive diversity receiver is a very popular potential solution in wireless system design, to achieve this goal. However, the ideal estimation technique which is considered to optimize the performance of a diversity receiver is difficult to implement. Hence, in a practical scenario, the system performance of a diversity receiver is considerably degraded. In this paper the expression for average bit error rate (ABER) and outage probability has been derived and studied for an arbitrary branch maximum ratio combiner (L-MRC) system for different modulation techniques over Nakagami-m fading channels considering estimation error.

Keywords: Adaptive MRC receiver, Estimation Error, Nakagami-m fading, ABER, Outage Probability, SNR, QAM, PS

INTRODUCTION

Diversity technique is one of the very effective techniques to reduce the adverse effect of fading at the receiver. Though there are numbers of diversity techniques are there, among these, the maximum ratio combining (MRC) is considered to be best for its optimal potential. Adaptive techniques further increase the performance of a receiver. Here the transmitter power and the rate of data transmission can be adapted depending on the channel information from the receiver. Therefore, theoretically for optimal outcome, the receiver should have the perfect estimation of the phase and the envelope information of the received signal [1]. However, in practice, the perfect estimation of phase or envelop is quite arduous and so the design of an optimal receiver is also challenging [4]. In this study, the effect of phase and envelop estimation error has been analyzed in terms of ABER and outage probability for an arbitrary branch MRC receiver (L-MRC), considering different modulation schemes over Nakagami-m fading channels. From the analysis, it can be distinguished that, for different modulation schemes, there is a very remarkable effect of phase and envelop estimation error, on the receiver performance.

In [2] a study on capacity analysis has been carried out for an L-MRC receiver over Nakagami-m fading channels for different power and rate adaptation schemes. In [3] the capacity analysis of an adaptive single antenna receiver has been shown over Nakagami-m fading distribution. Then closed-form expressions has been evaluated for the outage probability, spectral efficiency and average bit-error-rate (ABER) assuming perfect channel estimation and negligible time delay between channel estimation and signal set adaptation. The impact of time delay on the BER of an adaptive M-QAM also has been presented in this paper. The effective receiver output SNR statistics and outage probability have been presented for an MRC receiver, over Rician fading channels with channel estimation error in [4]. In [5], the performance analysis of a generalized selection combining (GSC) and an equal gain combining (EGC) receiver has been given, considering Gaussian weighting errors over Rayleigh, Rician, Nakagami-m and Nakagami-q fading channels. In [6] different fading channels including Nakagami-m and different system have been described and analyzed. Capacity analysis for an adaptive SC and MRC receiver has been done considering correlated Hoyt fading distribution in [8]. Performance analysis of a dual MRC receiver has been presented over correlated Hoyt fading channels is presented in [9]. Mathematical expression of the SNR PDF of the receiver, outage probability and ABER has been evaluated in the paper with a perfect channel estimation. In [10] the performance of an L-independent branch and dual correlated branch, SC receivers are analyzed over Hoyt fading channels. A new bit error probability expression for m-ary phase-shift keying (M-PSK) has been presented for both additive white Gaussian noise (AWGN) and fading channels in [11]. In [12] the outage probability and ABER of arbitrary branches of an arbitrary branch MRC receiver over TWDP fading channel has been discussed. Capacity analysis of a dual-MRC system for the adaptive schemes, optimum rate adaptation (ORA) and channel information with fixed rate (CIFR) schemes has been studied in [13], over correlated Nakagami-m fading channels with non-identical fading parameters. In [14] an optimal power allocation scheme has been studied for spectrum sharing between transmitter and receiver with estimation error over Rayleigh fading channel. Two asymptotic closed form bit error rate (BER) formulas have been presented in [15] considering a TWDP fading channel. Though a few works on imperfect channel estimation are available in literature still more investigation is required for

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different scenarios. This generates a motive to derive the unknown ABER and outage probability performance measures of a arbitrary branch MRC receiver over Nakagami-m fading model.

The rest of the paper is organized as follows. In section 2 channel and system model is discussed. ABER for the considered system has been evaluated in section 3. In section 4 outage probability of the system is evaluated. The result and analysis are presented in section 5. Finally the paper is concluded in section 6.

CHANNELS AND SYSTEM MODEL

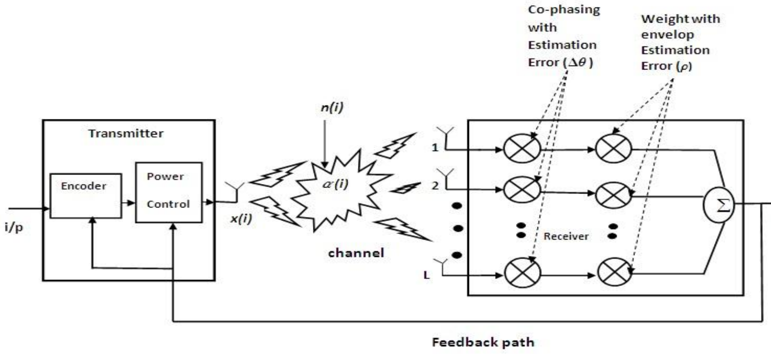

Figure 1: System model for imperfect channel estimation

The mentioned wireless system in Figure 1 is an L-MRC system with arbitrary ‘L’ number of antennas with a feedback path from the receiver to the transmitter. In this considered model, receiver estimates the channel condition depends on the received multipath signals and sends the estimated information to the transmitter. From the estimated channel information transmitter adjust its transmitted signal power and rate of data transmission [10]**.** In our analysis the channel is treated as slow and frequency non selective with Nakagami-m distribution which is the best fit for the multi cluster channels. For the channel, the received signal over ‘ith’ bit duration can be represented as [10],

$$y(i)=\alpha e^{j\varphi}x(i)+n(i) \tag{1}$$

where $x(i)$ is the transmitted symbol in ‘ith’ interval, with energy $E_x = E[|x(i)|^2]$ and noise vector $n(i) = [n_1(i), n_2(i), \ldots, n_L(i)]^T$, is the complex Gaussian noise having zero mean and two sided power spectral density $2N_0$. In the considered model, φ is the phase and α represents the Nakagami-m distributed fading amplitude. The envelop pdf ($\rho_\alpha(\alpha)$) of a Nakagami-m distributed received signal is given by [6],

$$\rho_\alpha(\alpha)=\frac{2m^m\alpha^{2m-1}}{\Omega^m\Gamma(m)}\exp\left(-\frac{m\alpha^2}{\Omega}\right);\alpha\geq 0 \tag{2}$$

where, $\Omega = E\left|\alpha^2\right|$ is the mean square value of a random variable (RV) is ‘ α ’ and ‘m’ is the Nakagami fading parameter ($m\geq\frac{1}{2}$). For imperfect channel estimation (ICE), if the channel vector at the receiver side is denoted as $\hat{\alpha}(i)=\left[\hat{\alpha}_1(i),\ldots\ldots..,\hat{\alpha}_L(i)\right]^T$, the estimation error at each branch can be represented as, $e_l(i)=\hat{\alpha}_1(i)-\alpha_l(i)$. A flexile model for such estimation error, of an arbitrary linear channel estimation is presented in [4] as,

$$\alpha_{f,l}(i)=\rho_l\hat{\alpha}_{f,l}(i)+z_{f,l}(i) \tag{3}$$

where, l = 1, 2, 3,….., L , ‘f’ is the diffused component and the errors $\left\{z_{f,l}(i)\right\}_{l=1}^{L}$ are the iid (independent identically distributed) equivalent estimation error terms with zero mean (μ=0) and variance σ_z^2 . In the equation, ρ_l is symbolized as the standardized estimation correlation coefficient between the envelope $\alpha_l(i)$ and $\hat{\alpha}_l(i)$, and can be represented as $\rho_l=|\rho_l|e^{j\Delta\theta_l}$, where $\Delta\theta_l$ is the phase offset of ρ_l . In case of imperfect channel estimation, $|\rho_l|<1$ or $\Delta\theta_l\neq 0$ or both, which degrades the receiver performance [4]**.**

Effective Output SNR with ICE

In presence of estimation error, to detect the transmitted symbol $x(i)$ at the MRC receiver, the help of the complex decision variable (DV) have to take, which is given as, $\tilde{D}=\sum_{l=1}^{L}\hat{\alpha}_l(i)r_l(i)$ [5]. Now applying the half plane decision method as given in [11]**,** the complex DV ‘ $\tilde{D}$ ’ will be rotated with a plane angle β to obtain a new DV as,

$$D(\beta)=\Re\left(\tilde{D}e^{-j\beta}\right) \tag{4}$$

where, $\beta = \pm\left(\frac{\pi}{2} - \frac{\pi}{N}\right)$, N is the constellation size. So considering the half plane decision method for a DV D , the effective output SNR of a MRC receiver is given as [4],

$$\gamma_{ICE}^{MRC} = B(\beta)\sum_{l=1}^{L}\hat{\gamma}_l \tag{5}$$

where, B is a function of β and given by, $B(\beta) = \frac{|\rho_l|^2 \cos^2(\Delta\theta_l - \beta)\log_2 N}{\left[\left(1-|\rho_l|^2\right)\bar{\gamma}_l \log_2 N + 1\right]}, \hat{\gamma}_l = |\hat{\alpha}_l|^2 / N_0$.

PDF of L-MRC receiver output SNR, over Nakagami-m fading channel, with error estimation

To diminish the uncomplimentary effect of fading in an MRC scheme, non-identical, attenuated and time delayed multipath components are weighted adequately and co-phased before combining. The expression of the output SNR of an 'L' branch MRC system can be represented as [12],

$$\gamma_{ICE}^{MRC} = \sum_{l=1}^{L}\gamma_l \tag{6}$$

The SNR pdf for *l* th branch of the MRC system can be expressed as [2],

$$\rho_\gamma(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \tag{7}$$

where, instantaneous SNR per symbol is given by $\gamma = \alpha^2 E_s / N_0$, the average SNR per symbol is $\bar{\gamma} = \Omega E_s / N_0$, $\Gamma(.)$ is a Gamma function and E_s is the energy per symbol. From (7), doing some mathematical calculation, followed by a random variable transformation a closed-form expression of SNR PDF for Nakagami-m distribution of MRC receiver is given by [2],

$$\rho_\gamma(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^{Lm} \frac{\gamma^{Lm-1}}{\Gamma(Lm)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right) \tag{8}$$

In this analysis, it has been considered that the variation in the combiner output SNR is not tracked perfectly by the receiver and this variation has been sent back to the transmitter via a time varying feedback path. So eq.(8) can be expressed in terms of imperfect channel estimation considering the random variable multiplication with error parameter 'B' as per the equation given in eq.(5). Then the modified SNR pdf for Nakagami-m fading with estimation error can be represented as,

$$\rho_{\hat{\gamma}}(\hat{\gamma}) = \frac{1}{B}\left(\frac{m}{\bar{\gamma}}\right)^{Lm} \frac{B^{-(Lm-1)}\hat{\gamma}^{Lm-1}}{\Gamma(Lm)}\exp\left(-m\frac{B^{-1}\hat{\gamma}}{\bar{\gamma}}\right) \tag{9}$$

ABER OF L-MRC RECEIVER OVER NAKAGAMI-M FADING WITH ESTIMATION ERROR

ABER expression for an L-MRC receiver can be derived by substituting eq.(9) in the following eq. as given in [6],

$$P_e = \int_0^\infty P(e/\hat{\gamma})\,\rho_{\hat{\gamma}}(\hat{\gamma})d\hat{\gamma} \tag{10}$$

where, $P(e/\hat{\gamma}) = aQ\left(\sqrt{b\hat{\gamma}}\right) = \frac{a}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2}, \frac{b\hat{\gamma}}{2}\right)$ for different coherent modulation. Values of 'a' and 'b' are given in Table:1.

Substituting eq. (9) and the values of $P(e/\hat{\gamma})$ in eq. (10) it can be represented as,

$$P_e = \frac{a}{2B\Gamma(Lm)\sqrt{\pi}} B^{-(Lm-1)}\left(\frac{m}{\bar{\gamma}}\right)^{Lm}\int_0^\infty \hat{\gamma}^{Lm-1}\Gamma\left(\frac{1}{2}, \frac{b\hat{\gamma}}{2}\right)\exp^{\left(\frac{-mB^{-1}}{\bar{\gamma}}\right)\hat{\gamma}} d\hat{\gamma} \tag{11}$$

Now solving the integral as given in [7; 6.455(1)], the ABER expression for Nakagami-m fading with m-array coherent modulation can be expressed as,

$$P_e = \frac{aB^{-(Lm-1)}}{2B\Gamma(Lm)\sqrt{\pi}}\left(\frac{m}{\bar{\gamma}}\right)^{Lm} \frac{\left(\frac{b}{2}\right)^{\frac{1}{2}}\Gamma\left(Lm+\frac{1}{2}\right)}{Lm\left(\frac{mB^{-1}}{\bar{\gamma}}+\frac{b}{2}\right)^{Lm+\frac{1}{2}}}\,{}_2F_1\left(1, Lm+\frac{1}{2}, Lm+1, \frac{2mB^{-1}}{2mB^{-1}+\bar{\gamma}b}\right) \tag{12}$$

Table 1: Values of ‘a’ and ‘b’ for considered coherent *m*-ary modulations

b	a		
-	0.5	1	4(√M-1)/√M
1	BFSK	-	-
2	-	BPSK	-
2sin^2(π/M)	-	M-PSK	-
3/(M-1)	-	-	M-QAM

OUTAGE PROBABILITY OF L-MRC RECEIVER OVER NAKAGAMI-M FADING WITH ESTIMATION ERROR

Outage probability can be defined as the instantaneous error probability which exceeds a specified value or equivalently, the output SNR, $\hat{\gamma}$,falls below a particular threshold value γ_{th}. The outage probability can be expressed from the equation,as given in [9],

$$P_{out} = \int_0^{\gamma_{th}} \rho_{\hat{\gamma}}\left(\hat{\gamma}\right) d\hat{\gamma} \tag{13}$$

Now substituting eq.(9) in eq.(13) and solving the integral using [7, 3.381(1)], the final outage probability expression for Nakagami-m fading with estimation error can be derived as,

$$P_{out} = \frac{1}{B}\left(\frac{m}{\bar{\gamma}}\right)^{Lm} \frac{B^{-(Lm-1)}}{\Gamma\left(Lm\right)} \left(\frac{mB^{-1}}{\bar{\gamma}}\right)^{-Lm} g\left(Lm, \frac{mB^{-1}\gamma_{th}}{\bar{\gamma}}\right) \tag{14}$$

Eq. (14) further can be simplified as,

$$P_{out} = \frac{1}{\Gamma\left(Lm\right)} g\left(Lm, \frac{mB^{-1}\gamma_{th}}{\bar{\gamma}}\right) \tag{15}$$

RESULTS AND ANALYSIS

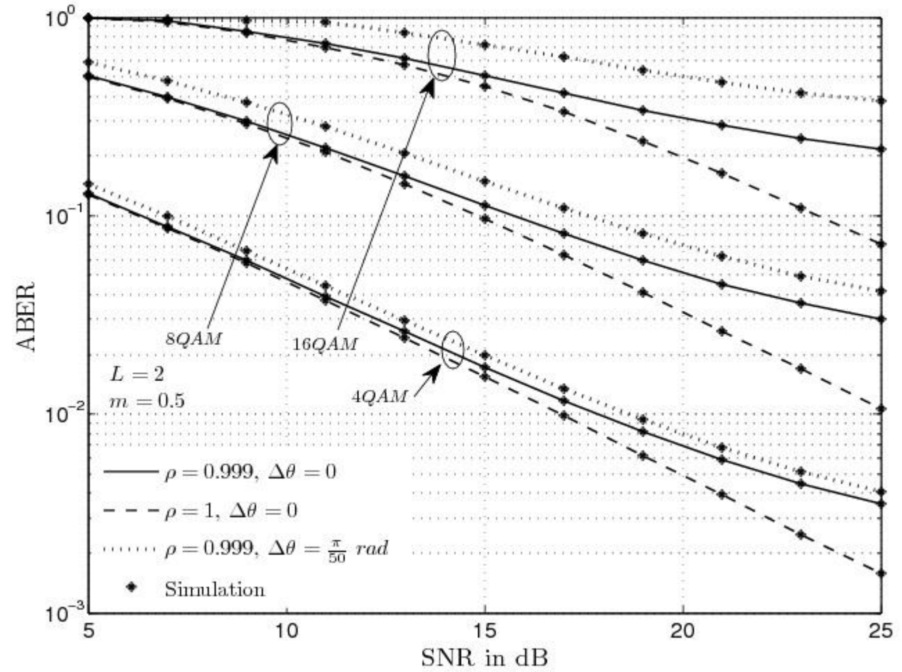

Figure 2: ABER analysis with ICE for different QAM schemes

The obtained results in the above sections has been numerically evaluated and plotted to analyze the behavior of the L-MRC adaptive receiver with estimation error over Nakagami-m fading channels. From fig:2 to fig:4 the ABER analysis has been presented. In fig:2 for a constant 'm' (m=0.5), fixed number of receiver branch (L=2) and different M-QAM, ABER analysis has been performed for various envelop (ρ) and phase estimation error ($\Delta\theta$). In this fig it can be clearly observed that, with estimation error (ρ = 0.999 and $\Delta\theta$ = 0) the ABER is remarkably increased than the perfect channel estimation (ρ = 1 and $\Delta\theta$ = 0). The ABER further increases, if both envelop and phase estimation errors have been considered (ρ = 0.999 and $\Delta\theta = \pi/50\, rad$). On the other hand, with the increase of constellation size the ABER is also increased. In fig:3, for m=1 and m=0.5 the effect of ICE has been analyzed in terms of ABER for arbitrary number of antennas. From the fig: 3 it can be observed that for a constant 'm' parameter (m=1 or m=0.5) and estimation error, the ABER is decreased with the increase of the number of antennas at the receiver (In this case when L-=3). But for a particular receiver (L=2 or L=3) , the increase of 'm' parameter indicates the decrease of ABER values accordingly. The effect of M-QAM and M-PSK modulation on ABER with ICE has been presented in fig:4, for a constant envelop error (ρ = 0.980) and 'm' parameter (m=3) considering a dual MRC receiver. Here it can be observed that, in both modulation schemes the ABER is considerably increased with increase of constellation size. However in case of M-PSK the ABER is comparatively low than the M-QAM scheme. Therefore ABER becomes quite low when we consider BPSK (2-PSK) modulation. In fig:5 outage probability analysis for a L-MRC receiver has been presented considering estimation error at the receiver. From the figure it can be observed that, like ABER with the increase of constellation size the outage probability of the receiver is also increased. On the other hand considering envelop estimation error or phase estimation error or considering both, the outage probability increased remarkably than the perfect channel estimation (when ρ = 1and $\Delta\theta$ = 0). However from the figure it is also clearly observable that, irrespective of modulation scheme, the effect of envelop error is very high than the effect of phase error on the system. In fig:5, for 8-QAM modulation, for all considered SNR, the difference between the outage probability for ρ = 0.999 and ρ = 0.980 (with no phase error) is higher than the outage probability where $\Delta\theta = \pi/30$ and $\pi/70$ (with no envelop error).

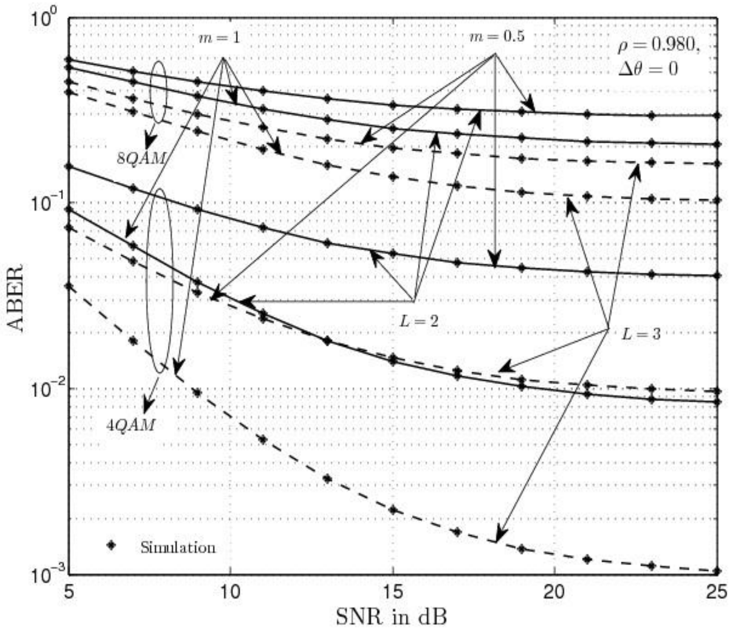

Figure 3: ABER analysis with ICE for different M-QAM schemes & 'm' parameters considering L=2,3

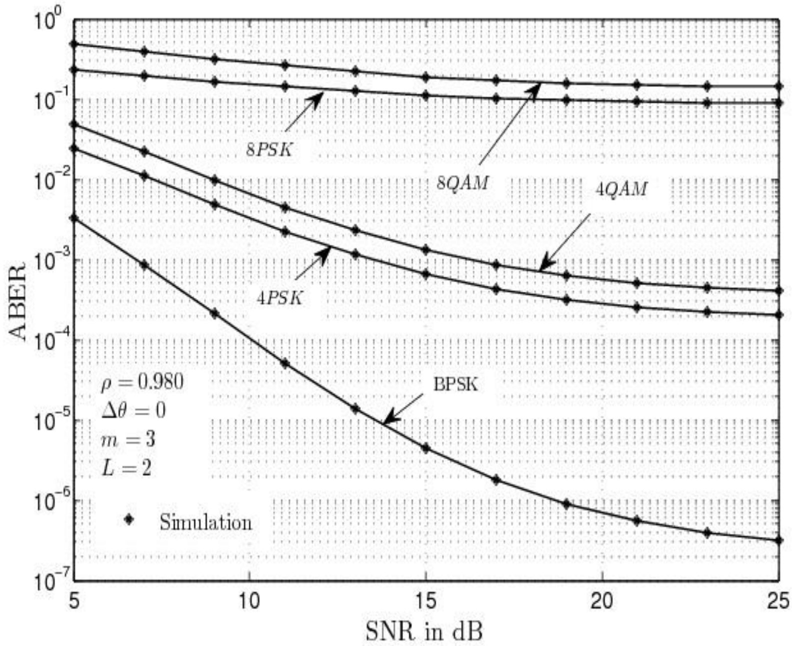

Figure 4: Effect of M-QAM & M-PSK modulation schemes on ABER with ICE

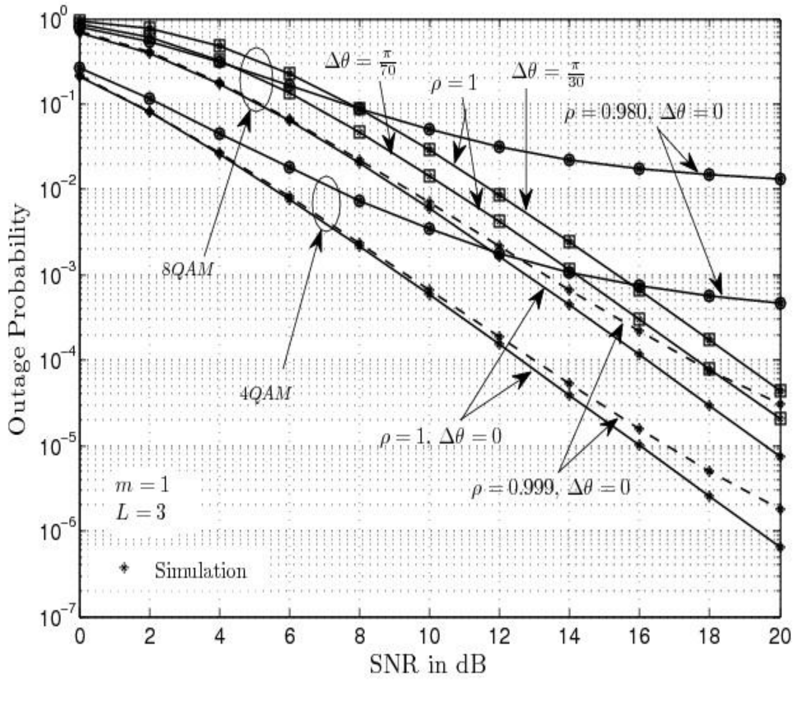

Figure 5: Outage probability analysis with ICE for a L-MRC receiver (L=3)

CONCLUSION

In this paper with ICE the expressions of ABER and outage probability of an L-MRC receiver has been derived over Nakagami-m fading channels for different coherent m-ary modulation schemes. The ABER and outage probability expression has been obtained, using SNR PDF and conditional error probability of coherent modulation schemes. From the analysis, it can be concluded that, the estimation error has a very strong impact on the wireless system performance. On the other hand the used modulation scheme has a remarkable impact on it. The obtained results are verified with Monte Carlo simulation.

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