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A Mathematical Exponeniated Weibull Model for Altered Brain and Gut Responses to CRH in Patients with Irritable Bowel Syndrome

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Abstract

The Weibull distribution is one of the best-known life-time distributions. It adequately describes observed failures of many different types of components and phenomena. Over the last three decades, numerous articles have been written on this distribution. The purpose of the paper is to give a brief introduction for exponentiated Weibull model and its applications.

Keywords: ACTH, IBS, CRH, stress, Exponentiated Weibull distribution.

2010 AMS Classification: 62XX, 60EXX.

MATHEMATICAL MODEL:

Hallinan gives an insightful review by presenting a number of historical facts, the many forms of this distribution as used by practitioners, and possible confusions and errors that arise due to this non-uniqueness. Johnson et.al devote a comprehensive chapter to a systematic study of this distribution. More recently, Murthy et.al presented a monograph that contains every facet relating to the Weibull distribution and its extensions. In sec 1.1, we first define the historical development and relations to other distributions. Sec 1.2 studies the properties of the Weibull distribution, in particular those relevant to reliability.

Historical Development:

The **Weibull distribution** is named after its originator, the Swedish physicist **Waloddi Weibull**, who in 1939 used it to model the distribution of the breaking strength of materials [7,8] **(Murthy,Weibull 1939)** and in 1951 for a wide range other applications [7,9] **(Murthy, Weibull 1951)** . The distribution has been known that Weibull may not be the first to propose this distribution. The name Frechet distribution is also sometimes used due to the fact that it was Frechet distribution who first identified this distribution to be an external distribution. According to **Hallian** [4], It was Weibull who suggested a scale parameter and a location parameter that made the distribution meaningful and useful [7,8] **(Murthy, Weibull 1939).**

Basic properties:

Density Function:

The probability density function (PDF) of (3) and (4) are

$$f(t) = \beta\alpha^{-\beta}(t-\tau)^{\beta-1}exp\left[-\left(\frac{t-\tau}{\alpha}\right)^{\beta}\right], t \geq \tau \qquad (5)$$

and

$$f(t) = \beta\lambda(t-\tau)^{\beta-1}exp\left[-\lambda(t-\tau)^{\beta}\right], \quad t \geq \tau \qquad (6)$$

Weibull-Derived Models:

There are many extensions, generalizations and modifications to the Weibull distribution. They arise out of the need to model features of empirical data sets that cannot be adequately described by a three-parameter Weibull model.

Exponentiated Weibull Distribution:

The starting point is the two-parameter Weibull model with distribution function $F(t)$. Let $G(t)$ denote the de-goodness of fit test for two-parameter Weibull and its power is compared with other traditional goodness-of-fit tests [5].

$$G(t) = [F(t)]^{\nu} = \left\{1 - exp\left[-(t/\alpha)^{\beta}\right]\right\}^{\nu}, \qquad t \geq 0.$$

The density function is

$$g(t) = \frac{\beta\nu}{\alpha^{\beta}}t^{\beta-1}e^{-(t/\alpha)^{\beta}}\left\{1 - e^{-(t/\alpha)^{\beta}}\right\}^{\nu-1}$$

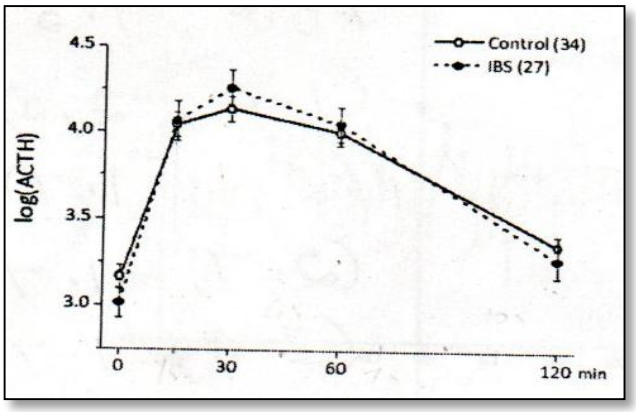
The failure rate function is given by

$$h(t) = \frac{\beta\nu}{\alpha^{\beta}}t^{\beta-1}e^{-(t/\alpha)^{\beta}}\frac{\left(1-e^{-(t/\alpha)^{\beta}}\right)^{\nu-1}}{\left[1-\left(1-e^{-(t/\alpha)^{\beta}}\right)^{\nu}\right]}$$

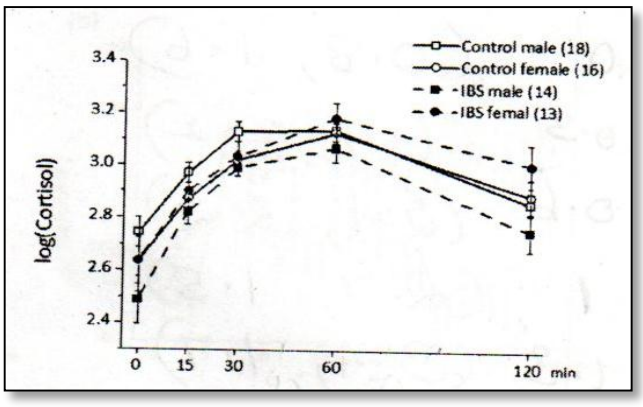
APPLICATION

Stress is known as a trigger of irritable bowel syndrome (IBS) and exacerbates its gastrointestinal symptoms [1]. However, underlying the physiological mechanism remains unknown. Here, we investigated hypothalamic-pituitary-adrenal (HPA) axis, colonic motility, and autonomic responses to corticotropin-releasing hormone (CRH) administration as well as brain activity alterations in IBS [2]. The study is the first to

comprehensively assess HPA axis, autonomic, and colonic responses to CRH in IBS and healthy controls and link these responses to brain responses to rectal distension in regions known to be involved in top-down control of the above mentioned peripheral stress responses [6]. The study included 28 IBS patients and 34 age sex-matched healthy controls subjects. healthy control subjects. Blood samples for ACTH and cortisol assays were drawn immediately before and 15,30,60,120 min after CRH injection. In the first experiment, plasma adrenocorticotropic hormone (ACTH) and cortisol, and electrocardiography (ECG) were measured from 20 min before until 12 min after intravenous CRH administration(2µg/kg). After eliminating non-significant interaction effects, the final model for cortisol included a significant main effect of time and significant sex-by-time and group-by-sex interaction effects. The main effects of group and sex were not significant. The interaction effect showed that the increases in cortisol relative to the pre-infusion time point were not significantly different between sexes at any post-infusion time point [3]. Separate analysis according to group revealed a significant difference between groups for male subjects(i.e lower levels of cortisol in male patients with IBS relative to male control subjects, but not for female subjects).



(A)



(B)

Figure1.1. (A) ACTH responses to intravenous CRH administration in patients with IBS and healthy control subjects. **(B)** Cortisol responses to intravenous CRH administration in patients with IBS and healthy control subjects.

MATHEMATICAL RESULTS:

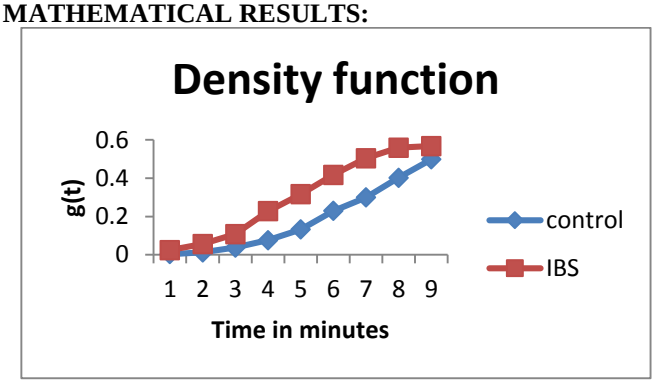


Figure 2.1

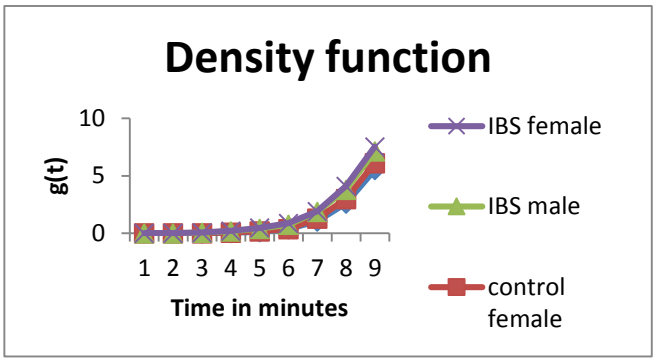


Figure 2.2

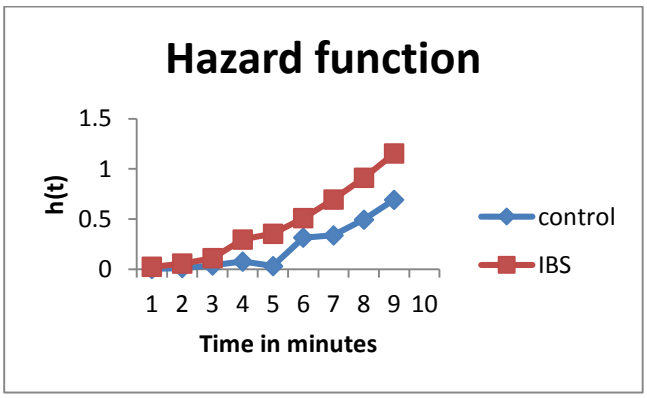


Figure 2.3

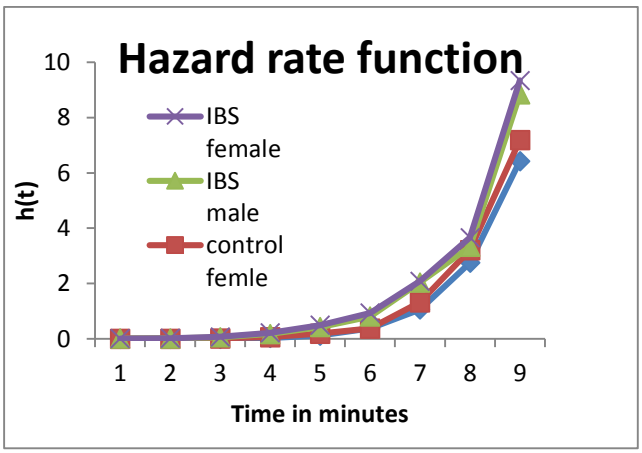


Figure 2.4

CONCLUSION

Using exponentiated Weibull distribution, we have found the values for both cases IBS control subjects. At the early morning, the values for control is 0.0039 and for IBS group is 0.0242. After 15 minutes, the values are 0.0124 and for IBS is 0.0564. After 30 minutes, the corresponding values for control and IBS are 0.0379 and 0.1080 respectively. After 45 minutes the values are 0.0765 and 0.2273. At one hour the range is 0.1320 and 0.3161. After 75 minutes the values for control and IBS group is 0.2293 and 0.4170. After one and half an hour the values are 0.2998 and 0.5041. After one hour and 45 minutes the values lie as 0.4025 and .5590. At 2 hours, the values shown are 0.4984 and 0.5675. Among these, we come to know that IBS group shows greater response than control groups. Similarly when we compare the groups such as control male, control female, IBS male, IBS female, the IBS male group show greater reactivity than other subjects. The same as Hazard rate function also reflects the same result for both the categories i.e, the IBS group reacts more than the control group. As well as, the IBS male group reacts pronounced manner than the control male, control female, IBS female. Our mathematical results show that IBS patients demonstrate greater ACTH responses to CRH than control subjects.

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