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Variance of Time to Recruitment for a Single Grade Manpower System using Order Statistics for Inter-decision Times and Wastages

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Abstract

In this paper, a single grade organization is considered in which successive exit of personnel leads to wastage in the form of man hours due to its policy decisions. A mathematical model is constructed using order statistics for the loss of man hours and inter-decision times based on shock model approach with univariate CUM policy of recruitment. The mean and variance of time to recruitment is obtained by assuming specific distribution for breakdown threshold. The influence of nodal parameters on the performance measures are studied numerically with relevant findings and conclusions are presented.

Keywords: Single grade manpower system, Univariate CUM policy, Order statistics, Shock model, Variance of time to recruitment.

AMS Subject Classification (2010): Primary: 90B70, Secondary: 60H30, 60K05.

INTRODUCTION

In real time, once the exit of personnel happens, the recruitment cannot be introduced as it is time overwhelming and costly. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. Many models have been discussed considering different kinds of wastages and different types of distributions for the loss of man hours, the threshold and the inter-decision times. Such models could be seen in [1],[2],[4],[5],[6],[7],[9] and [10]. In [8], a single grade manpower system with mandatory exponential threshold for the loss of manpower, the author studied system characteristics when inter-decision times form an order statistics and the loss of manpower forms a sequence of independent and identically distributed exponential random variables. In [11] and [3] considering three different distribution for thresholds, the author has obtained system characteristics when the inter-decision times form an order statistics and loss of man hours are correlated exponential random variables and vice versa. The present paper extends the result when order statistics applied to loss of man hours and inter-decision times.

MODEL DESCRIPTION AND ANALYSIS FOR MODEL-I

Consider an organization taking decision at random epochs $[0,\infty)$ and at every decision epoch a random number of persons exit from the organization. There is an associated loss of man hours if a person exits and it is linear and cumulative. Let X_i be the loss of man hours (wastage) due to the i^{th} decision epoch $i=1,2,3...k$. Assume that this population is a sequence of independent and identically distributed random variables

with distribution $G(x)=1-e^{-\lambda x}$ and density function $g(.)$ with mean $\frac{1}{\lambda}(\lambda>0)$. Assume that $\{X_i\}_{i=1}^{n}$ be a sample of size $'n'$ selected from the population. Let $X_{(1)},X_{(2)},X_{(3)},....X_{(n)}$ be the order statistics corresponding to this sample with respective density functions $g_{X(1)}(.),g_{X(2)}(.),g_{X(3)}(.),....g_{X(n)}(.)$. Here $X_{(1)}$ is the first order statistics (smallest) with probability density function $g_{X(1)}(.)$ and $X_{(n)}$ is the n^{th} order statistics (largest) with probability density function $g_{X(n)}(.)$. Note that the random variables $X_{(1)},X_{(2)},X_{(31)},....X_{(n)}$ are not independent. Let $U_i, i=1,2,3,....$ be the time between $(i-1)^{th}$ and i^{th} decisions with distribution function $F(.)$ and probability density function $f(.)$. Let $F_k(t)$ $(f_k(t))$ be the distribution (probability density) function of $\sum_{i=1}^{k}U_i$. Let Y be a random variable denoting the threshold for the loss of man hours with cumulative distribution function $H(.)$ and probability density function $h(.)$. Let T be a continuous random variable denoting the time to recruitment in the organization with probability density function and distribution function $l(.)$ and $L(.)$ respectively. Let $l^*(.), f^*(.)$ and $f_k^*(.)$ be the Laplace transforms of $l(.), f(.)$ and $f_k(.)$ respectively. Let $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let $E(T)$ and $V(T)$ be the mean and variance of time to recruitment. The univariate CUM policy of recruitment employed in this paper is as follows ***"Recruitment is done as and when the cumulative loss of man hours in the organization exceeds threshold"***.

MAIN RESULTS

The survival function of T is given by

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] P\left[\sum_{i=1}^{\infty} X_i \le Y\right] \quad (1)$$

By using the law of total probability and on simplification we get

$$P(T > t) = \int_0^{\infty} G_k(y) h(y) dy \quad (2)$$

Here Y follows the exponential distribution with parameter θ.

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_0^{\infty} G_k(y) h(y) dy \quad (3)$$

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] [g^*(\theta)]^k \quad (4)$$

$$P(T > t) = 1 - \sum_{k=1}^{\infty} F_k(t) [g^*(\theta)]^{k-1} [1 - g^*(\theta)] \quad (5)$$

The probability distribution function of T is given by

$$L(t) = 1 - P(T > t) \text{ and } l(t) = \frac{d}{dt} L(t) \quad (6)$$

From (5) and (6) we have

$$l(t) = [1 - g^*(\theta)] \sum_{k=1}^{\infty} f_k(t) [g^*(\theta)]^{k-1} \quad (7)$$

The Laplace transform of density function of T is

$$l^*(s) = [1 - g^*(\theta)] \sum_{k=1}^{\infty} [f^*(s)]^k [g^*(\theta)]^{k-1} \quad (8)$$

(on simplification)

$$l^*(s) = \frac{[1 - g^*(\theta)] f^*(s)}{1 - f^*(s) g^*(\theta)} \quad (9)$$

It is known that,

$$E(T) = -\frac{d}{ds}\left(l^*(s)\right)\Big|_{s=0}, \; E(T^2) = \frac{d^2}{ds^2}\left(l^*(s)\right)\Big|_{s=0} \quad (10)$$

and $V(T) = E(T^2) - [E(T)]^2$ (11)

The probability density function of X_i is given by

$$G_{X(r)}(x) = r\binom{n}{r} [G(x)]^{r-1} g(x) [1 - G(x)]^{n-r}$$

where $r = 1,2,3....n$ (12)

If $g(x) = G_{X(1)}(x)$ then $g_{X(1)}(x) = ng(x)[1 - G(x)]^{n-1}$ (13)

If $g(x) = G_{X(n)}(x)$ then $g_{X(n)}(x) = ng(x)[G(x)]^{n-1}$ (14)

By hypothesis $g(x) = \alpha e^{-\alpha x}$, we get

$$g^*(\gamma) = \begin{cases} \dfrac{n\alpha}{n\alpha + \gamma} & \text{if} \quad g(t) = g_{X(1)}(t) \\ \dfrac{n!\alpha^n}{(\gamma+\alpha)(\gamma+2\alpha)(\gamma+3\alpha)...(\gamma+n\alpha)} & \text{if} \quad g(t) = g_{X(n)}(t) \end{cases}$$

where $\gamma = \theta, \theta_1, \theta_2$ and θ_3. (15)

Consider the population $\{U_i\}_{i=1}^{\infty}$ of independent and identically distributed inter-decision times with exponential cumulative distribution $F(t) = 1 - e^{-\lambda t}$ and the corresponding density function $f(t)$. Assume that $\{U_i\}_{i=1}^{m}$ be a sample of size 'm' selected from the population. Let $U_{(1)}, U_{(2)}, U_{(3)}, ...U_{(m)}$ be the order statistics corresponding to this sample with respective density functions $f_{U(1)}(.), f_{U(2)}(.), f_{U(3)}(.), ...f_{U(m)}(.)$. Here $U_{(1)}$ is the first order statistics (smallest) with probability density function $f_{U(1)}(.)$ and $U_{(m)}$ is the m^{th} order statistics (largest) with probability density function $f_{U(m)}(.)$. Note that the random variables $U_{(1)} \le U_{(2)} \le U_{(3)} \le ... \le U_{(m)}$ and hence $U_{(1)}, U_{(2)}, U_{(3)}, ..., U_{(m)}$ are not independent. The probability density function of the general j^{th} order statistics is given by

$$f_{U(j)}(t) = j\binom{m}{j} [F(t)]^{j-1} f(t) [1 - F(t)]^{m-j}$$

where $j = 1,2,3,...m$ (16)

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Case (i):

The probability density functions of $U_{(1)}$ and $U_{(m)}$ are given by

$$f_{U(1)}(t) = mf(t)[1-F(t)]^{m-1} \text{ and}$$
$$f_{U(m)}(t) = mf(t)[F(t)]^{m-1} \quad (17)$$

If $f(t) = f_{U(1)}(t)$ then $f^*(s) = f^*_{U(1)}(s)$ (18)

By hypothesis $f(t) = \lambda e^{-\lambda t}$, the probability density function of the first order statistics is given by

$$f^*_{U(1)}(s) = \frac{m\lambda}{s+m\lambda} \quad (19)$$

From (9), (10), (15) and (19) we get

$$E(T) = \frac{1}{m\lambda[1-g^*(\theta)]} \quad (20)$$

and $E(T^2) = \dfrac{2}{(m\lambda[1-g^*(\theta)])^2}$ (21)

Using (20) and (21) in (11) the variance of time to recruitment for Case (i) of model I is obtained.

Case (ii):

Suppose that $f(t) = f_{U(m)}(t)$.

In this case $f^*(s) = f^*_{U(m)}(s)$ (22)

The probability density function of the m^{th} order statistics is given by

$$f^*_{U(m)}(s) = \frac{m!}{\left(s/_{\lambda}+1\right)\left(s/_{\lambda}+2\right)\ldots\left(s/_{\lambda}+m\right)} \quad (23)$$

Substituting (22) in (9) we get

$$l^*(s) = \frac{[1-g^*(\theta)]f^*_{U(m)}(s)}{1-f^*_{U(m)}(s)g^*(\theta)} \quad (24)$$

Now, $\dfrac{d}{ds}\left[f^*_{U(m)}(s)\right]_{s=0} = -\sum_{r_1=1}^{m}\dfrac{1}{r_1}$ (25)

and $\dfrac{d^2}{ds^2}\left[f^*_{U(m)}(s)\right]_{s=0} = \dfrac{1}{\lambda^2}\left[\left(\sum_{r_1=1}^{m}\dfrac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\dfrac{1}{r_1^2}\right]$ (26)

From (9), (10), (15), (23), (24), (25) and (26) we get

$$E(T) = \frac{\sum_{r_1=1}^{m}\frac{1}{r_1}}{\lambda[1-g^*(\theta)]} \quad (27)$$

and $E(T^2) = \dfrac{\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2}{\lambda^2[1-g^*(\theta)]^2} - \dfrac{\left[\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\frac{1}{r_1^2}\right]}{\lambda^2[1-g^*(\theta)]}$ (28)

Substituting (27) and (28) in (11) the variance of time to recruitment for Case (ii) of model I is obtained.

MODEL II

Here, Y follows the extended exponential distribution with parameter θ with shape parameter 2. In this case,

$$P(T > t) = 2\left[1-\sum_{k=1}^{\infty}F_k(t)[g^*(\theta)]^{k-1}[1-g^*(\theta)]\right] \quad (30)$$
$$-\left[1-\sum_{k=1}^{\infty}F_k(t)[g^*(2\theta)]^{k-1}[1-g^*(2\theta)]\right]$$

The probability distribution function of T is given by

$$l(t) = 2[1-g^*(\theta)]\sum_{k=1}^{\infty}f_k(t)[g^*(\theta)]^{k-1} - [1-g^*(2\theta)]\sum_{k=1}^{\infty}f_k(t)[g^*(2\theta)]^{k-1}$$
(31)

The Laplace transform of density function of T is

$$l^*(s) = \frac{2[1-g^*(\theta)]f^*(s)}{1-g^*(\theta)f^*(s)} - \frac{[1-g^*(2\theta)]f^*(s)}{1-g^*(2\theta)f^*(s)} \quad (32)$$

Case (i):

Proceeding as Case (i) of model I, using equations (10), (13), (15), (19) and (32) we get

$$E(T) = \frac{2}{m\lambda[1-g^*(\theta)]} - \frac{1}{m\lambda[1-g^*(2\theta)]} \quad (33)$$

and $E(T^2) = \dfrac{4}{(m\lambda[1-g^*(\theta)])^2} - \dfrac{2}{(m\lambda[1-g^*(2\theta)])^2}$ (34)

Using (33) and (34) in (11), the variance of time to recruitment for Case (i) of model II is obtained.

Case (ii):

Proceeding as Case (ii) of model I, using equations (10), (15), (22), (23), (25), (26) and (32) we get

$E(T) = \dfrac{2\sum_{r_1=1}^{m}\frac{1}{r_1}}{\lambda[1-g^*(\theta)]} - \dfrac{\sum_{r_1=1}^{m}\frac{1}{r_1}}{\lambda[1-g^*(2\theta)]}$ and (35)

$$E(T^2) = \frac{4\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2}{\lambda^2\left[1-g^*(\theta)\right]^2} - \frac{2\left[\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\frac{1}{r_1^2}\right]}{\lambda^2\left[1-g^*(\theta)\right]} + \frac{4\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2}{\lambda^2\left[1-g^*(2\theta)\right]^2} - \frac{2\left[\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\frac{1}{r_1^2}\right]}{\lambda^2\left[1-g^*(2\theta)\right]} \quad (36)$$

Substituting (35) and (36) in (11), the variance of time to recruitment for Case (ii) of model II is obtained.

Model III

Here, Y follows the SCBZ property with parameter θ_1 , θ_2 and θ_3. In this case,

$$P[T>t] = p(\theta_1+\theta_3)\left\{1-\left[1-g^*(\theta_1+\theta_3)\right]\sum_{k=1}^{\infty}\left[F_k(t)\right]\left[g^*(\theta_1+\theta_3)\right]^{k-1}\right\} + q\theta_2\left\{1-\left[1-g^*(\theta_2)\right]\sum_{k=1}^{\infty}\left[f_k(t)\right]\left[1-g^*(\theta_2)\right]^{k-1}\right\} \quad (37)$$

where $p = \frac{\theta_1-\theta_2}{\theta_3+\theta_1-\theta_2}$ and $q = \frac{\theta_3}{\theta_3+\theta_1-\theta_2}$ with $p+q=1$.

The probability distribution function of T is given by

$$L(t) = 1-\begin{bmatrix} 1-p(\theta_1+\theta_3)\left[1-g^*(\theta_1+\theta_3)\right]\sum_{k=1}^{\infty}\left[F_k(t)\right]\left[g^*(\theta_1+\theta_3)\right]^{k-1} \\ +1-q\theta_2\left[1-g^*(\theta_2)\right]\sum_{k=1}^{\infty}\left[f_k(t)\right]\left[1-g^*(\theta_2)\right]^{k-1} \end{bmatrix} \quad (38)$$

$$l(t) = p(\theta_1+\theta_3)\left[1-g^*(\theta_1+\theta_3)\right]\sum_{k=1}^{\infty}\left[f_k(t)\right]\left[g^*(\theta_1+\theta_3)\right]^{k-1} + q\theta_2\left[1-g^*(\theta_2)\right]\sum_{k=1}^{\infty}\left[f_k(t)\right]\left[1-g^*(\theta_2)\right]^{k-1} \quad (39)$$

The Laplace transform of density function of T is

$$l^*(s) = p(\theta_1+\theta_3)\left[1-g^*(\theta_1+\theta_3)\right]\sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[g^*(\theta_1+\theta_3)\right]^{k-1} + q\theta_2\left[1-g^*(\theta_2)\right]\sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[1-g^*(\theta_2)\right]^{k-1} \quad (40)$$

Case (i):

Proceeding as Case (i) of model I, using equations (10), (15), (19) and (40) we get

$$E(T) = \frac{p(\theta_1+\theta_3)}{m\lambda\left[1-g^*(\theta_1+\theta_3)\right]} + \frac{q\theta_2}{m\lambda\left[1-g^*(\theta_2)\right]} \quad (41)$$

and $$E(T^2) = \frac{2p(\theta_1+\theta_3)}{\left[m\lambda\left[1-g^*(\theta_1+\theta_3)\right]\right]^2} + \frac{2q\theta_2}{\left[m\lambda\left[1-g^*(\theta_2)\right]\right]^2} \quad (42)$$

Equations (11), (41) and (42) give the mean time to recruitment for Case (i) of model III.

Case (ii):

Proceeding as Case (ii) of model I, using equations (10), (15), (22), (23), (25), (26) and (40) we get

$$E(T) = \frac{p(\theta_1+\theta_3)\sum_{r_1=1}^{m}\frac{1}{r_1}}{\lambda\left[1-g^*(\theta_1+\theta_3)\right]} + \frac{q\theta_2\sum_{r_1=1}^{m}\frac{1}{r_1}}{\lambda\left[1-g^*(\theta_2)\right]} \text{ and} \quad (43)$$

$$E(T^2) = \frac{2p(\theta_1+\theta_3)\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2}{\lambda^2\left[1-g^*(\theta_1+\theta_3)\right]^2} - \frac{p(\theta_1+\theta_3)\left[\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\frac{1}{r_1^2}\right]}{\lambda^2\left[1-g^*(\theta_1+\theta_3)\right]} + \frac{2q\theta_2\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2}{\lambda^2\left[1-g^*(\theta_2)\right]^2} - \frac{q\theta_2\left[\left(\sum_{r_1=1}^{m}\frac{1}{r_1}\right)^2 - \sum_{r_1=1}^{m}\frac{1}{r_1^2}\right]}{\lambda^2\left[1-g^*(\theta_2)\right]} \quad (44)$$

Substituting (43) and (45) in (11), the mean time to recruitment for Case (ii) of model III is obtained.

NUMERICAL ILLUSTRATION

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically. The parameters $\theta_1 = 0.4, \theta_2 = 0.2, \theta_3 = 0.1, \theta = 0.2$ and the sample sizes $m = 3$ and $n = 5$ are fixed.

Table 1. (Effect of α on the performance measures E(T) and V(T))

α	λ	Model I				Model II				Model III			
		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$	
		E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
$f(t)=f_{U(1)}(t)$													
0.1	0.5	1.50	2.25	0.67	0.45	0.83	3.61	0.01	0.03	0.60	1.55	0.27	0.29
0.2	0.5	2.33	5.44	0.70	0.49	1.66	10.00	0.06	0.15	0.93	3.93	0.28	0.30
0.3	0.5	3.17	10.03	0.75	0.56	2.50	19.17	0.12	0.34	1.27	7.42	0.29	0.33
0.4	0.5	4.00	16.00	0.80	0.64	3.33	31.11	0.20	0.56	1.60	12.02	0.30	0.37
0.5	0.5	4.83	23.36	0.86	0.74	4.17	45.83	0.28	0.80	1.93	17.73	0.32	0.41
$f(t)=f_{U(m)}(t)$													
0.1	0.5	8.25	50.06	3.70	5.60	4.58	331.25	0.05	76.36	3.30	39.72	1.48	7.77
0.2	0.5	12.83	136.69	3.85	6.42	9.17	755.00	0.30	80.91	5.13	107.65	1.55	8.69
0.3	0.5	17.42	265.34	4.10	7.86	13.75	1346.81	0.68	88.93	6.97	209.19	1.65	10.19
0.4	0.5	22.00	436.00	4.40	9.76	18.33	2106.66	1.10	99.52	8.80	344.34	1.77	12.15
0.5	0.5	26.58	648.67	4.73	12.06	22.92	3034.58	1.53	112.16	10.63	513.10	1.90	14.51

Table 2. (Effect of λ on the performance measures E(T) and V(T))

α	λ	Model I				Model II				Model III			
		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$		$g(x)=G_{X(1)}(x)$	
		E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
$f(t)=f_{U(1)}(t)$													
0.5	0.1	24.17	584.03	4.30	18.50	20.83	1145.83	1.39	20.08	9.67	443.22	1.60	10.24
0.5	0.2	12.08	146.01	2.15	4.63	10.42	286.46	0.70	5.02	4.83	110.81	0.80	2.56
0.5	0.3	8.06	64.89	1.43	2.06	6.94	127.31	0.46	2.23	3.22	49.25	0.53	1.14
0.5	0.4	6.04	36.50	1.08	1.16	5.21	71.61	0.35	1.25	2.42	27.70	0.40	0.64
0.5	0.5	4.83	23.36	0.86	0.74	4.17	45.83	0.28	0.80	1.93	17.73	0.32	0.41
$f(t)=f_{U(m)}(t)$													
0.5	0.1	132.92	16216.84	23.66	301.60	114.58	75864.58	7.66	2804.00	53.17	12827.47	9.52	362.75
0.5	0.2	66.46	4054.21	11.83	75.40	57.29	18966.15	3.83	701.00	26.58	3206.87	4.76	90.69
0.5	0.3	44.31	1801.87	7.89	33.51	38.19	8429.40	2.55	311.56	17.72	1425.27	3.17	40.31
0.5	0.4	33.23	1013.55	5.91	18.85	28.65	4741.54	1.91	175.25	13.29	801.72	2.38	22.67
0.5	0.5	26.58	648.67	4.73	12.06	22.92	3034.58	1.53	112.16	10.63	513.10	1.90	14.51

FINDINGS

(a) From table 1, if α increases, the average loss of man hour decreases and hence the mean and variance of time to recruitment increase when the loss of man hour process forms first order as well as n^{th} order statistics.

(b) From table 2, if λ increases, the average inter decision times decrease and hence the mean and variance of time to recruitment decrease when the inter-decision time process forms first order as well as m^{th} order statistics.

CONCLUSION

From the numerical illustration we conclude that model I is preferable than models II and III. In model I, time to recruitment is postponed when loss of man hour process and the process of inter-decision times forms first order statistics.

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