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Assessment of Optimisation Techniques for Sliding Mode Control of an Inverted Pendulum

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Abstract

Stabilization of an Inverted Pendulum is one of the most major problems in control engineering. The inverted pendulum is an open-loop and highly non-linear unstable system that moves toward an uncontrolled state frequently. In broad-spectrum, an upright control system for inverted pendulum should ensure stability and should produce desired response in terms of rejection of disturbances and parameter uncertainties. In this paper, Sliding Mode Controller (SMC) is developed to confirm stability of inverted pendulum at all operating conditions. But then again, the sliding mode control parameters should be chosen optimally to provide an optimal solution for efficiency. To provide the optimization, techniques such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO) are fused with SMC technique. From the simulation results, Particle Swarm Optimization - Sliding Mode Control (PSO - SMC) generates better response compared to SMC and GA-SMC control strategies and the responses are discussed in time domain analysis.

Keywords: Inverted Pendulum, SMC, GA-SMC, PSO-SMC

INTRODUCTION

The inverted pendulum is one of the fundamental problems in the theory of systems and in control engineering. The inverted pendulum is a highly nonlinear and open loop unstable system. The inverted pendulum falls over quickly, when the system is activated by a small force. This makes the characteristics of an inverted pendulum make the identification and control more challenging. For this tricky problem, many researchers [1] have developed various control methodologies. There are variety methods for inverted pendulum control that are presented since now. They can be divided generally in three groups, namely, classic methods such as PID, PI controllers; modern methods and artificial methods such as neural networks and fuzzy logic. However, it is still an active region of research [2] due to its broad applications in rocket technology, missile guidance, seismometers, wheeled-legged robots and in avionics systems such as air planes, ships, mobile robots, and automobiles. Therefore, it is proven to be a best benchmark for testing a wide range of classical as well as modern control technologies. To develop a precise model of the inverted pendulum, linear and nonlinear approaches of identification should be used. However, one of the problem encountered during modelling is collection of investigational data from the inverted pendulum system [3]. Since the output data from the unstable system does not show adequate information of the system. The quality of a control algorithm is tested and demonstrated on the inverted pendulum problem due to its inherent instability and dynamic characteristics. The first step in this work is to determine the equations of motion for the inverted pendulum, using the Euler-Lagrange equations. The next step is to find a linearized model that approximates the original nonlinear systems behaviour around the equilibrium located on the upright vertical position.

The potential of SMC methodology addresses general approach for the design and response control of variable structure control of a class of non-linear plants [4]. The feedback coefficients in the sliding mode control of an inverted pendulum, as well as any other dynamic system, can be expressed as a non-linear programming problem with appropriately selected objective function and constraints. They use modified sub-gradient algorithm to solve this non-linear programming problem. However, SMC produces chattering in control variable. So, in order to optimise SMC method, optimization techniques like genetic algorithm (GA) and practical swarm optimization (PSO) techniques are combined with SMC.

Presently Genetic Algorithm (GA) has been receiving a lot of attention due to its efficient and cost effective results when compared to many conventional methods and other soft computing technologies [5]. In spite of this fact, more research has been done to study the behaviour of genetic algorithm in many linear and nonlinear process control applications. Genetic algorithm is a random search method used to solve nonlinear system and optimize complex problems which uses probabilistic transition rules instead of deterministic rules and handles a population of potential solutions iteratively known as individuals or chromosomes. Each iteration of the algorithm is termed a generation. The controller parameters can be found using GA.

Particle swarm optimization is as an alternative technique to design the controller for a single-input single output system. This evolutionary scheme is applied to design the proposed controller [3, 10]. The particle swarm optimization algorithm is used to the design both linear and non-linear PID controllers. Eberhart and Kennedy proposed the Particle Swarm Optimization (PSO) algorithm; a new, motivated evolutionary computation algorithm, which differs from other evolution techniques.

The objective of this paper is to develop a Robust Control of Inverted Pendulum and to compare the servo performance between SMC, GA-SMC, PSO-SMC for an inverted pendulum system and to determine which control strategy

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delivers better performance with respect to pendulum's angle and cart's position.

In this paper, section 2 discusses about inverted pendulum, section 3 discusses about design of sliding mode control, section 4 discusses about modelling of inverted pendulum using GA-SMC, PSO-SMC, section 5 discusses about results and discussions.

INVERTED PENDULUM

Due to integral instability and dynamic characteristics, the quality of a control algorithm is tested and demonstrated on the inverted pendulum problem habitually. Fig. 1 shows the free-bodied diagram of the inverted pendulum system.

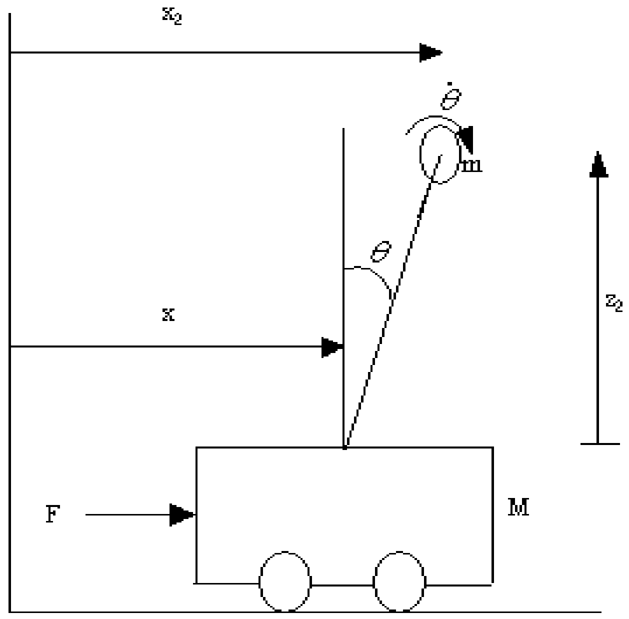

Figure 1. Inverted pendulum

Where,

- θ - Angle of the pole w.r.t the vertical axis
- $\dot{\theta}$ - Angular velocity of pole w.r.t vertical axis
- F - Force applied to the cart
- M - Mass of the cart
- m - Mass of the pole
- x - Position of the cart
- $\dot{x}$ - Velocity of the cart

It is the problem of learning, how to balance an upright pole. Its solution consists of finding the horizontal force to be applied to the cart in order to balance the pole. The cart is moving on the track with no friction. Also, the pole is tied up to the cart by a frictionless hinge. Both the cart and the pole have only one degree of freedom, i.e. each of them can move in vertical plane only. Lagrange equations can be used to derive dynamical system equations for a complicated mechanical system such as the inverted pendulum. The Lagrange equations use the kinetic and potential energy in the system to determine the dynamical equations of the inverted pendulum system.

The kinetic energy of the system is the sum of the kinetic energies of each mass. The kinetic energy Ec of the cart is

$$E_c = \frac{1}{2} M \dot{x}^2 \quad (1)$$

The pole can move in both the horizontal and vertical directions. So the pole kinetic energy is

$$E_p = \frac{1}{2} m(\dot{x}_2^{\,2} + \dot{z}_2^{\,2}) \quad (2)$$

From the free bodied diagram x2 and z2 are equal to

$$x_2 = x + L(\sin\theta) \quad (3)$$

$$z_2 = L(\cos\theta) \quad (4)$$

$$\dot{z}_2 = -L(\sin\theta)\dot{\theta} \quad (5)$$

$$\dot{x}_2 = \dot{x} + L(\cos\theta)\dot{\theta} \quad (6)$$

The total kinetic energy, E of the system is equal to

$$E = E_c + E_p \quad (7)$$

Substitute Eq. (1), (2) in (7)

$$E = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m(\dot{x}_2^{\,2} + \dot{z}_2^{\,2}) \quad (8)$$

Substitute Eq. (3), (4), (5) and (6) in (8)

$$E = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \dot{x}^2 + \frac{1}{2} mL^2\dot{\theta}^2 + mL\dot{x}(\cos\theta)\dot{\theta} \quad (9)$$

The potential energy, V of the system is stored in the pendulum,

$$V = mgz_2 \quad (10)$$

Substitute equation (4) in (9)

$$V = mgL(\cos\theta) \quad (11)$$

The Lagrangian function is $P = E - V$

$$P = \frac{1}{2}(M+m)\dot{x}^2 + mL(\cos\theta)\dot{x}\dot{\theta} + \frac{1}{2}mL^2\dot{\theta}^2 - mgL(\cos\theta) \quad (12)$$

The state-space variables of the system are x and θ, so the Lagrange equations are

$$\frac{d}{dt}\left(\frac{\partial P}{\partial \dot{x}}\right) - \frac{\partial P}{\partial x} = F \quad (13)$$

$$\frac{d}{dt}\left(\frac{\partial P}{\partial \dot{\theta}}\right) - \frac{\partial P}{\partial \theta} = 0 \quad (14)$$

Solving equation (13) and (14) the dynamic equations of the inverted pendulum is obtained given below

$$\dot{x}_1 = x_2 \tag{15}$$

$$\dot{x}_2 = \frac{\left((M+m)g\,(sinx_1) - mL\,(sinx_1)(cosx_1)x_2^2 - (cosx_1)F\right)}{L\left((M+m) - mcos^2x_1\right)} \quad 16)$$

$$\dot{x}_3 = x_4 \tag{17}$$

$$\dot{x}_4 = \frac{mL\,(sinx_1)x_2^2 - mg\,(sinx_1)(cosx_1)}{\left((M+m) - mcos^2x_1\right)} + \frac{F}{\left((M+m) - mcos^2x_1\right)} \tag{18}$$

Let the state variables

$x_1 = \theta$; Angle of pole with respect to vertical axis.

$x_2 = \dot{\theta}$; Angular velocity of pole w.r.t vertical axis

$x_3 = x$; Position of the cart

$x_4 = \dot{x}$; Velocity of the cart

To simulate the above equations, the mass of the cart, M is set to 1.2 kg, mass of the pendulum is set to 0.1 kg, length of the pendulum is 0.4 meters, gravitational force, and g is set to 9.81 m /s. The above state equations are simulated using SIMULINK software.

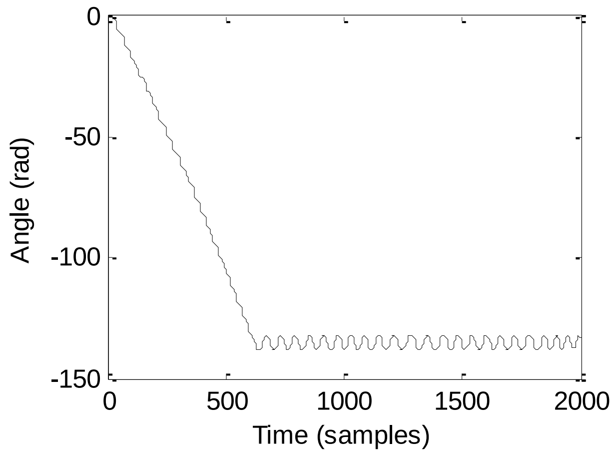

Figure 2. Open loop response of the inverted pendulum.

It is observed that the angle of the inverted pendulum shown in Fig. 2 does not give us enough information on the inverted pendulum system. The pendulum falls over quickly and it found to be unstable. One of the requirements in system identification is the collection of 'information rich' input-output data. In order to adequately model the inverted pendulum it is necessary to stabilize it using a nonlinear feedback controller.

DESIGN OF SLIDING MODE CONTROL

Sliding mode control (SMC) has been known for its capabilities in accounting for modeling imprecision, large signal stability, good dynamic response and simple implementation. It achieves robust control by adding a discontinuous control signal across the sliding surface, satisfying the sliding condition. However, the sliding mode control system has a particularly high control gain due to the non-linear compensation and can suffer from the effects of actuator chattering due to the switching and imperfect implementations. To design a sliding mode control [3,8], the time varying sliding surface is defined as

$$s = c_1\theta + \dot{\theta}\ ;\ c_1 > 0 \tag{19}$$

Let the Lyapunov function is defined as

$$v = \frac{1}{2}s^2 \tag{20}$$

If $\dot{v} = s\dot{s} < 0$ is satisfied, the state trajectory of the system will be forced to approach the sliding surface. Hence, the control law for sliding mode control is derived as follows:

From eqn. (16)

$$\dot{x}_2 = \frac{\left((M+m)g\,(sinx_1) - mL\,(sinx_1)(cosx_1)x_2^2\right)}{L\left((M+m) - mcos^2x_1\right)} - \frac{(cosx_1)}{L\left((M+m) - mcos^2x_1\right)}F \tag{21}$$

$$\dot{x}_2 - \frac{\left((M+m)g\,(sinx_1) - mL\,(sinx_1)(cosx_1)x_2^2\right)}{L\left((M+m) - mcos^2x_1\right)} = -\frac{(cosx_1)}{L\left((M+m) - mcos^2x_1\right)}F \tag{22}$$

From eqn. (18);

$$\dot{\theta} = s - c_1\theta \tag{23}$$

Substitute $x_2 = \dot{\theta}$ and sliding surface s=0 in eqn. (23)

$$x_2 = -c_1\theta \tag{24}$$

$$\dot{x}_2 = -c_1\dot{\theta} \tag{25}$$

$$-c_1\dot{\theta} - \frac{\left((M+m)g\,(sinx_1) - mL\,(sinx_1)(cosx_1)x_2^2\right)}{L\left((M+m) - mcos^2x_1\right)} = -\frac{(cosx_1)}{L\left((M+m) - mcos^2x_1\right)}F \tag{26}$$

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$$c_1\dot{\theta} + \frac{\left((M+m)g\,(\sin x_1) - mL\,(\sin x_1)(\cos x_1)x_2^2\right)}{L\left((M+m) - m\cos^2 x_1\right)}$$

$$= \frac{(\cos x_1)}{L\left((M+m) - m\cos^2 x_1\right)}F \tag{27}$$

$$F = \frac{c_1 x_2 + \dfrac{\left((M+m)g\,(\sin x_1) - mL\,(\sin x_1)(\cos x_1)x_2^2\right)}{L\left((M+m) - m\cos^2 x_1\right)}}{\dfrac{(\cos x_1)}{L\left((M+m) - m\cos^2 x_1\right)}} \tag{28}$$

$$F = \frac{c_1 x_2\left[L\left((M+m) - m\cos^2 x_1\right)\right] + \begin{pmatrix}(M+m)g\,(\sin x_1)\\ - mL\,(\sin x_1)(\cos x_1)x_2^2\end{pmatrix}}{(\cos x_1)} \tag{29}$$

The control input u is given as

$$u = F - K * \mathrm{sgn}(s) \tag{30}$$

Where, K is control gain

$$\mathrm{Sgn}(s) = \begin{cases} 1 & \text{if } s > 0 \\ 0 & \text{if } s = 0 \\ -1 & \text{if } s < 0 \end{cases} \tag{31}$$

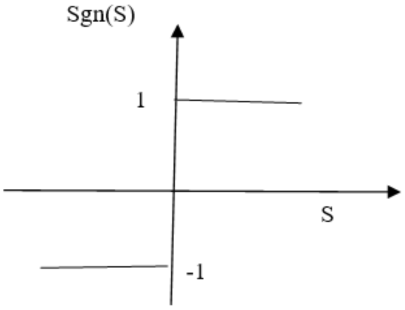

Figure 3. SGN function of inverted pendulum

The correct magnitude and force to keep the pendulum stable is calculated by the control law. If the control law 'u' as given in eqn. (30) is chosen, then negative $\dot{v}$ is guaranteed, and the system state will approach the sliding surface gradually. Although the control law as eqn. (30) can force state to the sliding surface, chattering phenomena will occur after the first time state hits the sliding surface [6]. It is a drawback in the sliding mode control behavior, and will lead to an unstable condition of the controlled system. This is owing to the discontinuous of 'sgn' function in eqn (31). In order to adequately model the inverted pendulum it is necessary to stabilize it using a nonlinear feedback controller (Guez and Selinsky, 1988). Using a nonlinear feedback controller, the output data will contain more information for describing the process. The following equations are the control law developed for the inverted pendulum.

$$F_{NL} = \frac{f_2}{h_2}\begin{bmatrix} h_1 + k_1(\theta - \theta_d) + k_2\dot{\theta} + \\ c_1(x - x_d) + c_2\dot{x} \end{bmatrix} - f_1 \tag{32}$$

Eqn. (32) calculates the required force, 'F' to keep the pendulum stable. For simulation, k1=25, K2=10, c1=1 and c2=2.6 are taken by repeated simulation studies. Here x_d = 0 meters and θ_d = 0, which are the desired position of the cart and angle of the pendulum, respectively. The inputs to this controller are the four output states of the non-liner pendulum model. The correct magnitude and force to keep the pendulum stable is calculated by the control law. The system is disturbed by a band limited white noise after the system is stabilized by a non-linear control law F_{NL}.

MODELING OF INVERTED PENDULUM USING GA-SMC, PSO-SMC

Genetic algorithm uses a direct analogy of natural evolution to do global optimization in order to solve highly complex problems [5, 7]. It presumes that the potential solution of a problem is an individual and can be represented by a set of parameters. These parameters are regarded as genes of a chromosome and can be structured by a string of concatenated values. The form of variables representation is defined by the encoding scheme and can be represented by binary, real numbers or other forms, depending on the application data. Its range, the search space, is usually defined by the problem. In the beginning, an initial chromosome is randomly generated. The chromosomes are candidate solutions to the problem. Then, the fitness values of all chromosomes are evaluated by calculating the objective function in decoded form. So, based on the fitness of each individual, a group of the best chromosomes is selected through the selection process. The Genetic operators, crossover, and mutation, are applied to this surviving population in order to improve the next generation solution. The process continues until the population converges to the global maximum or another stopping criterion is reached. During the reproduction phase, the fitness value of each chromosome is assessed and it is used in the selection process to provide a bias towards fitness individuals. Then crossover algorithm is initiated once the selection process is completed. The background operator in the genetic algorithm is the mutation. The probability of mutation is normally low since high mutation rate will destroy fit strings and degenerate GA into random search. The sequence of evolution is repeated until a termination criterion is reached. An illustrative flowchart of the GA algorithm implementation is shown in the Figure.4.

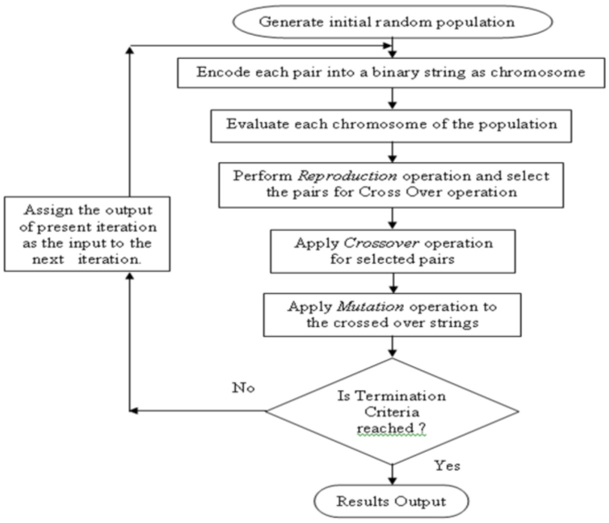

Fig.4 Flowchart of GA algorithm

In PSO algorithm, the system is initialized with a population of random solutions, which are called particles, and each potential solution is also allocated a randomized velocity. PSO depend on the exchange of data between particles of the population called swarm. Every particle adjusts the situation trajectory towards its best solution (fitness) that is achieved so far. Every particle also modifies its trajectory towards the best previous position attained by any member of its neighbourhood. Each particle moves in the search space with an adaptive velocity. The fitness function evaluates the performance of particles to determine whether the best fitting solution is achieved.

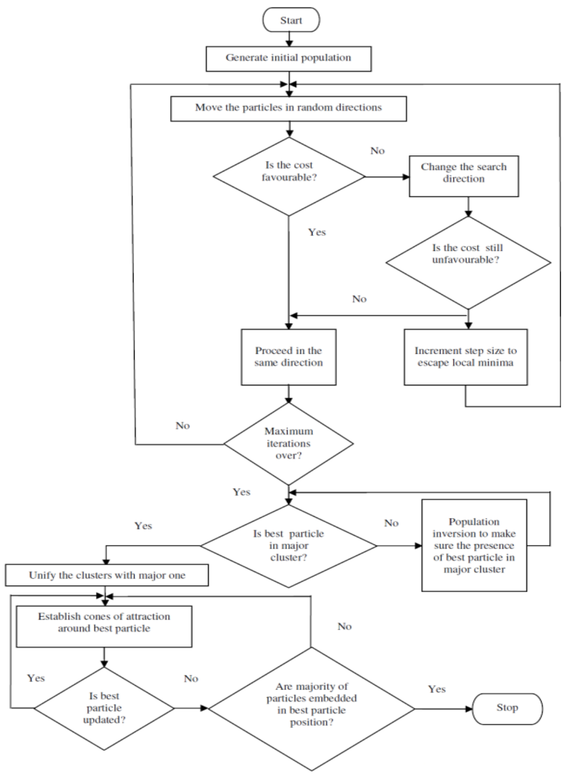

Figure 5. Flowchart of PSO algorithm

During the run, the fitness of the best individual improves over time and typically tends to stagnate towards the end of the run. Ideally, the stagnation of the process coincides with the successful discovery of the global optimum.

The PSO algorithm consists of three steps, which are repeated until some stopping condition is met:

- Evaluate the fitness of each particle
- Update individual ;global best fitness & positions
- Update velocity and position of each particle

RESULTS & DISCUSSION

The simulation results of servo response of inverted pendulum for proposed controller methods are presented below. The time domain specification comparison is done for the SMC, GA-SMC and PSO-SMC based controllers. Figure 6 shows the angle of an Inverted pendulum system with SMC, GA-SMC and PSO-SMC. Figure 7 shows the angular velocity of the inverted pendulum system with SMC, GA-SMC and PSO-SMC.

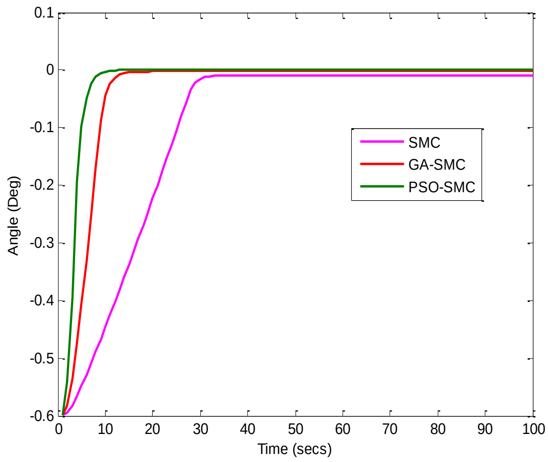

Figure 6. Angle of an Inverted pendulum system with SMC, GA-SMC and PSO-SMC

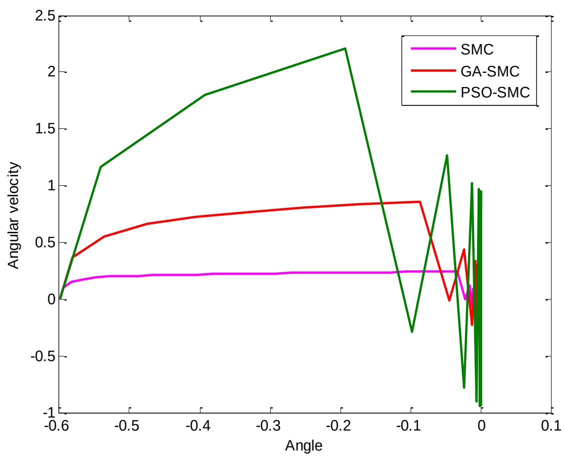

Figure 7. Angle vs Angular velocity of an Inverted pendulum system with SMC, GA-SMC and PSO-SMC

Figure 8 shows the controller output of an Inverted pendulum system with SMC, GA-SMC and PSO-SMC.

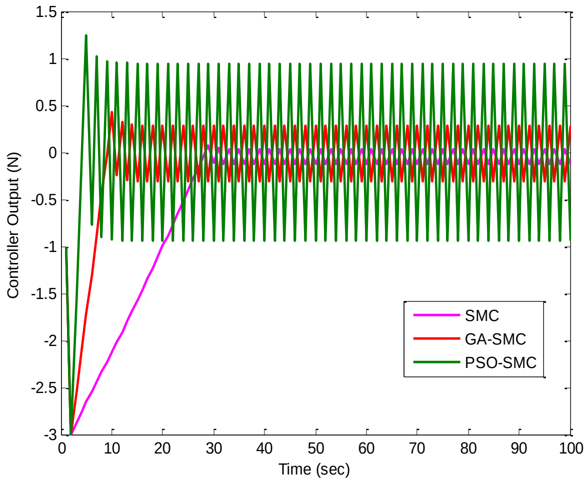

Figure 8. Controller output of an Inverted pendulum system with SMC, GA-SMC and PSO-SMC

It is clear from the responses that the PSO based controller has the advantage, which enables the controller to act faster with a minimum settling time. PSO-SMC method reduces the oscillations when compared to plain SMC and GA-SMC methods. PSO based SMC also track the set point, $\theta_d = 0°$. The performance index of each controller in terms of integral of average error and integral of square error criteria of the controller for inverted pendulum system with SMC, GA-SMC and PSO-SMC are tabulated in table 1.

Table 1. Performance measure for inverted pendulum system with SMC, GA-SMC and PSO-SMC

Controller	ISE	IAE	Settling Time
SMC	0.8581	0.4449	36
GA-SMC	0.4326	0.2527	18
PSO-SMC	0.3154	0.1965	14

The developed controller tuning for various set points can be suitably tracked by providing a program which can allow the system to choose that value based on the set point selected. The various results presented prove the healthiness of the PSO tuned SMC. The simulation responses for the models validated reflect the effectiveness of the PSO based controller in terms of time domain specifications. The performance index under the various error criterions for the PSO-SMC controller is always less than the SMC, GA-SMC tuned controllers.

CONCLUSION

In this paper, SMC controller with GA and PSO optimisation algorithms are successfully designed. Based on the results and the analysis, a conclusion has been made that all three control methods are capable of controlling the nonlinear inverted pendulum system. Simulation results in Fig. 6, Fig.7 and Fig. 8 show that PSO-SMC controller has better performance compared to GA-SMC and SMC controller in controlling the nonlinear inverted pendulum system. The performance index of each controller in terms of integral of average error and integral of square error criteria of the controller using PSO-SMC provided is less than SMC &GA-SMC and the same were summarized in Table 1. This methodology has less settling time as well as the best response, when it reaches to the unstable point. However, due to applied discontinuous control signal, fast and finite chattering are generated, which travel to the plant in a controlled manner. Further improvement need to be done so that the overshoot for the linear and angular positions do not have very high range as required by the design criteria.

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