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Generalized (α, β)-derivation on Lie Ideals in σ-prime rings

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Abstract

Let R will be a 2-torsion free σ-prime ring and $\alpha, \beta \in Aut\ R$. F be a non-zero generalized (α, β)-derivation of R with associated nonzero (α, β)-derivation d of R which commutes with σ and L a non-zero σ-Lie ideal and a subring of R, then R is commutative if any one of the following conditions holds. (i) $[F(u), u]_{\alpha,\beta} = 0$, (ii) $F(u)\alpha(u) = \beta(u)d(u)$, (iii) $F(u^2) = \pm\alpha(u^2)$, (iv) $F(u^2) = 2d(u)u$, (v) $d(u^2) = 2F(u)\alpha(u)$, for all $u \in L$.

Keywords: σ-prime ring, σ-Lie ideal, $(\alpha, \beta)-$ *derivation*, generalized $(\alpha, \beta) -$ *derivation*.

INTRODUCTION

Let R will be an associative ring with center Z. For any $x, y \in R$ the symbol $[x, y]$ represents commutator $xy - yx$ and the Jordan product $xoy = xy + yx$. R is a prime ring if $xRy = 0$ implies either $x = 0$ or $y = 0$. An additive mapping $\sigma: R \to R$ is called the involution if $\sigma(xy) = \sigma(y)\sigma(x)$ and $\sigma(\sigma(x)) = x$ for all $x, y \in R$. A ring equipped with an involution σ is called a ring with involution or σ-ring. A ring with an involution is said to σ-prime if $xRy = \sigma(x)Ry = 0$ implies either $x = 0$ or $y = 0$. Every prime ring with involution is σ-prime but converse need not be hold in general. An example due to Oukhtite [9], justifies the above statement that is, R be a prime ring, and let $S = R \times R°$, $R°$ is the opposite ring of R. Define involution σ on S as $\sigma(x, y) = (y, x)$. Then S is σ-prime, but not prime. This example shows that σ-prime ring constitute a more general class of prime rings. In all that follows the symbol $S_a(R)$, first introduced by Oukhtite, will denote the set of symmetric and skew-symmetric elements of R, that is $S_{a_\sigma}(R) = \{x \in R / \sigma(x) = \pm\ x\}$.

A Lie ideal L of R is an additive subgroup of R such that $[u, r] \in L$, for all $u \in L$, $r \in R$. A Lie ideal L is said to be a σ-Lie ideal if $\sigma(L) = L$. Let I be the ring of integers. Set

$R = \left\{\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} / x, y, z \in I\right\}$ and $L = \left\{\begin{bmatrix} 0 & z \\ 0 & 0 \end{bmatrix} / z \in I\right\}$

We define a map σ on R such that

$$\sigma\begin{bmatrix} x & y \\ 0 & w \end{bmatrix} = \begin{bmatrix} -w & y \\ 0 & x \end{bmatrix}.$$

Then it is easy to see that L is a non-zero σ-Lie ideal of R.

An additive mapping $d: R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. For a fixed $a \in R$, the mapping $I_a: R \to R$ given by $I_a(x) = [a, x]$ is a derivation and is called an inner derivation of R. An additive mapping $F: R \to R$ is called a generalized derivation if there exists a derivation $d: R \to R$ such that $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$. This definition was given by Bresar in [6]. Let α and β be any two automorphisms of R. An additive mapping $d: R \to R$ is called a (α, β)-derivation on R if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$. Inspired by this definition, the notion of generalized (α, β)-derivation was extended as follows: Let α and β be any two automorphisms of R. An additive mapping $F: R \to R$ is called a generalized (α, β)-derivation on R if there exists a (α, β)-derivation $d: R \to R$ such that $F(xy) = F(x)\alpha(y) + \beta(x)d(y)$, for all $x, y \in R$. Several authors have proved many commutativity theorems for prime rings, semi prime rings, σ- prime rings admitting derivations, automorphisms which are centralizing and commuting on appropriate subsets of R (see, e.g.,[3, 4, 5, 8] and references therein). In this direction, Bell and Mason [2] initialized the study using the notion of derivation in a prime ring. Argac [1] continued on a same line and he introduced the notion of two sided α-derivation. Recently Oukhtite et al.[10, theorem-1] proved Posner's second theorem to rings with involution in case of characteristic not 2. They generalized this theorem to generalized derivations centralizing on Jordan ideals in ring with involution. Nadeem ur Rehman et al. [14] proved some theorems concerning to σ-Lie ideals in a 2-torsion free σ-prime ring admitting a generalized (α, β)-derivation on R. Our attempt in this present paper is to prove commutativity conditions for a 2-torsion free σ-prime ring with a generalized (α, β)- derivation of R on σ-Lie ideals.

PRELIMINARIES

Throughout this paper, R will be a 2-torsion free σ-prime ring and $\alpha, \beta \in Aut\ R$ and F is a non-zero generalized (α, β)-derivation of R with a non-zero (α, β)-derivation d which commutes with σ and L is a non-zero σ- Lie ideal of R. Also we will make some extensive use of the basic commutator identities.

$[x, yz] = y[x, z] + [x, y]z$

$[xy, z] = x[y, z] + [x, z]y$

$[xy, z]_{\alpha,\beta} = x[y, z]_{\alpha,\beta} + [x, \beta(z)]y$

$\quad = x[y, \alpha(z)] + [x, z]_{\alpha,\beta}\, y$

$[x, yz]_{\alpha,\beta} = \beta(y)[x, z]_{\alpha,\beta} + [x, y]_{\alpha,\beta}\, \alpha(z)$

$x\ o\ yz = (x\ o\ y)z - y[x, z] = y(x\ o\ z) + [x, y]z$

$xy\ o\ z = x(y\ o\ z) - [x, z]y = (x\ o\ z)y + x[y, z]$

$(x\ o\ yz)_{\alpha,\beta} = (x\ o\ y)_{\alpha,\beta}\, \alpha(z) - \beta(y)[x, z]_{\alpha,\beta}$

$\quad = \beta(y)(x\ o\ z)_{\alpha,\beta} + [x, y]_{\alpha,\beta}\, \alpha(z)$

$(xy\ o\ z)_{\alpha,\beta} = x(y\ o\ z)_{\alpha,\beta} - [x, \beta(z)]y$
$= (x\ o\ z)_{\alpha,\beta}\, y + x[y,\ \alpha(z)]$

Lemma-1: *If R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal of R, then $2[R, R] \subseteq L$, and $2L[R, R] \subseteq L$.*

Proof: Let $p, q \in R$ and $u \in L$. Then $[p, q] - [[u, p], q] + [[u, q], p] = 2pqu - 2qpu \in L$ and we get, $2\,[p, q]u \in L$, for all $p, q \in R$ and $u \in L$, that is, $2[R, R] \subseteq L$. Similarly it is easy to see that, $[p, q] - [[u, q], p] + [[u, p], q] = 2u[p, q] \in L$, for all $p, q \in R$ and $u \in L$ and hence $2L[R, R] \subseteq L$. □

Lemma-2: *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal of R. If $aLb = \sigma(a)Lb = 0$, for $a, b \in R$, then either $a = 0$ or $b = 0$.*

Proof: Assume that $a \neq 0$. Since $2[R, R] \subseteq L$, by lemma-1, then $2a[p, q]ub = 0$, for all $p, q \in R$ and $u \in L$. This implies that, $[p, q]ub = 0$, for all $p, q \in R$ and $u \in L$ (2.1). Replacing q by qa in (2.1) because of $aub = 0$, we find that $aqapub = o$, and thus $aRapub = 0$, for all $p \in R$ and $u \in L$ (2.2). On the other hand, from $\sigma(a)Lb = 0$, it follows that, $\sigma(a)[p, qa]ub = 0$. Which leads to $\sigma(a)qapub = 0$ for all $p, q \in R$ and therefore, $\sigma(a)Rapub = 0$, for all $p \in R$ and $u \in L$ (2.3). From equations (2.2) and (2.3), because of $a \neq 0$, the σ-primeness of R yields $apub = 0$, for all $p \in R$. Accordingly, $aRub = 0$, for all $u \in L$ (2.4). Writing $q\sigma(a)$ instead of q in (2.1), we obtain $a[p, q\sigma(a)]ub = 0$. Because of $\sigma(a)ub = 0$, we get, $aq\sigma(a)pub = 0$. So that, $aR\sigma(a)pub = 0$ for all $u \in L$ (2.5). In view of $\sigma(a)ub = 0$, we find that $\sigma(a)[p, q\sigma(a)]ub = 0$, and thus $\sigma(a)q\sigma(a)pub = 0$ for all $p, q \in R$ and $u \in L$. Hence $\sigma(a)R\sigma(a)pub = 0$, for all $p \in R$ and $u \in L$ (2.6). Using (2.5) and (2.6), because of $a \neq 0$, the σ-primeness of R yields $\sigma(a)pub = 0$, for all $p \in R$. Accordingly, $\sigma(a)Rub = 0$, for all $u \in L$ (2.7).
Again, because of (2.4) and (2.7), σ-primeness of R assures that $ub = 0$, for all $u \in L$. Hence it follows, $Lb = 0$ (2.8). From $[u, p] = 0$, by view of (2.8), we get $upb = 0$ for all $p \in R$, $u \in L$. And thus, $uRb = 0$, for all $u \in L$ (2.9).
Since L is invariant under σ, from (2.9) it follows that, $\sigma(u)Rb = 0$, for all $u \in L$ (2.10). Using the σ-primeness of R, because of $u \neq 0$, equations (2.9) and (2.10) assures that $b = 0$. □

Lemma-3: *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal of R. If $[L, L] = 0$, then $L \subseteq Z(R)$.*

Proof: From $[2[p, q], v] = 0$ it follows that $[u[p, q], v] = 0$ and thus $[u[p, q], v] = [u, v][p, q] + u[[p, q], v] = 0$, for all $p, q \in R$ and $u, v \in L$. Hence $[[p, q], v] = 0$ for all $p, q \in R$ and $v \in L$ (2.11). Since equation (2.11) is analogous to equation (2.8), arguing as in the proof of Lemma-2, we arrive at $[[p, q], v] = 0$ for all $v \in R$ (2.12). Replacing q by qp in (2.12) we obtain $[[p, qp], v] = 0$ and hence $[p, q][p, v] = 0$ for all $p, q \in R$ and $v \in L$ (2.13). Writing qu instead of q in equation (2.13) where $u \in L$ we obtain $[p, qu][p, v] = 0$ and thus $[p, u]q[p, v] = 0$ for all $q \in R$. Hence we get $[p, u][p, v] = 0$ for all $u, v \in L$, $p \in R$ (2.14). Since $\sigma(L) = L$, replacing v by $\sigma(v)$ in equation (2.14), we get $[p, u][p, \sigma(v)] = 0$, for all $u, v \in L$, $p \in R$ (2.15).
Let $p \in S_a(R)$. From equation (2.15) it follows that $[p, u]R[p, v] = 0$, for all $u, v \in L, p \in R$ (2.16). Using equations (2.14) together with (2.16), the σ -primeness of R forces $[p, u] = 0$ for all $u \in L$. Accordingly, $[p, u] = 0$ for all $p \in S_a(R)$ and $u \in L$ (2.17). Let $p \in R$, since $[p - \sigma(p), p] \in S_{a_\sigma}(R)$. Equation (2.17) yields that $[p - \sigma(p), p] = 0$, for all $u \in L$. And therefore, $[p, u] = [\sigma(p), u]$, for all $p \in R$ and $u \in L$ (2.18). Substituting $\sigma(p)$ for p in equation (2.15) and using equation (2.18), we get $[\sigma(p), u][\sigma(p), \sigma(u)] = 0$, for all $u, v \in L, p \in R$. Which leads to $[p, u]R[u, v] = 0$ for all $u, v \in L, p \in R$ (2.19). Using the σ-primeness of R, equations (2.14) and (2.19) assure that $[p, u] = 0$ for all $u \in L, p \in R$.This proves that $L \subseteq Z(R)$. □

Lemma-4: *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal of R. If $L \subseteq Z(R)$, then R is commutative.*

Proof: From $L \subseteq Z(R)$ it follows that $pu = up$, implies $[p, u] \in L$ for all $p \in R$ and $u \in L$. And thus $pqu = p(qu) = q(pu)$ (2.20). Hence, equation (2.20) yields $[p, q]u = 0$ for all $p, q \in R$ and $u \in L$ (2.21). Replacing q by qt in equation (2.21) where $t \in R$ we get $[p, qt] = 0$ and hence we get $[p, q]tu = 0$, and thereby $[p. q]u = 0$ for all $p, q \in R$ and $u \in L$ (2.22). Since L is a non-zero σ-Lie ideal of R then eq.(2.22) implies that $[p, q]R\ \sigma(u) = 0$ for all $p, q \in R$ and $u \in L$ (2.23). In view of lemma-2, because of $0 \neq L$, equation (2.22) together with equation (2.23) force $[p, q] = 0$ for all $p, q \in R$. Accordingly R is commutative. □

Lemma-5: *Let R be a 2-torsion free σ-prime ring, L a non-zero σ-Lie ideal of R and d a non-zero (α, β)-derivation of R. If d commutes with σ and $d(L) = 0$, then R is commutative.*

Proof: By hypothesis we have $[u, p] \in L$, for all $u \in L, p \in R$ $d[u, p] = 0$. This implies $d(up - pu) = 0$. Expanding this term and using the hypothesis we get $[d(p), u]_{\alpha,\beta} = 0$ for all $u \in L$, $p \in R$ (2.24). Replacing p by $2pv$, $v \in L$ in eq.(2.24), using eq.(2.24) and $d(L) = 0$ we have $d(p)\alpha[v, u] = 0$, for all $u, v \in L, p \in R$. Substituting $pq, q \in R$ for p in this eq. and using this, we find that $d(p)\alpha(q)\alpha[v, u] = 0$ and so, $d(p)R\alpha[v, u] = 0$, for all $u, v \in L$, $p \in R$ (2.25). Writing $\sigma(p)$ by p in the last equation, we get $d(\sigma(p))R\alpha[v, u] = 0$, for all $u, v \in L, p \in R$. Since d commutes with σ, the last equation follows $\sigma(d(p))R\alpha[v, u] = 0$, for all $u, v \in L, p \in R$ (2.26). Applying the σ-primeness of R, because of eq.(2.25) and eq.(2.26), we conclude that $d(p) = 0$ or $\alpha[v, u] = 0$, for all $u, v \in L, p \in R$. Since d a non-zero (α, β)-derivation of R, we arrive at $[L, L] = 0$, and so R is commutative by lemma-3 and lemma-4. □

RESULTS

Theorem-1: *Let R be a 2-torsion free σ-prime ring, L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero generalized (α, β)-derivation F associated with a non-zero (α, β)-derivation d which commutes with σ such that $[F(u), u]_{\alpha,\beta} = 0$ for all $u \in L$, then R is commutative.*

Proof: Suppose that $[F(u), u]_{\alpha,\beta} = 0$, for all $u \in L$ (3.1). Linearizing (3.1) and using (3.1) for $u, v \in L$, we get $[F(u), v]_{\alpha,\beta} + [F(v), u]_{\alpha,\beta} = 0$ for all $u, v \in L$ (3.2). Replacing v by vu in (3.2) we get $[F(u), vu]_{\alpha,\beta} + [F(v)\alpha(u) + \beta(v)d(u), u]_{\alpha,\beta} = 0$ for all $u, v \in L$. That is $\{[F(u), v]_{\alpha,\beta} + [F(v), u]_{\alpha,\beta}\}\alpha(u) + \beta(v)[F(u), u]_{\alpha,\beta} + F(v).[\alpha(u), \alpha(u)] + \beta(v)[d(u), u]_{\alpha,\beta} + [\beta(v), \beta(u)]d(u) = 0$ for all $u, v \in L$. Substituting (3.1) and (3.2) in this equation we get $\beta(v)[d(u), u]_{\alpha,\beta} + [\beta(v), \beta(u)]d(u) = 0$ for all $u, v \in L$ (3.3). Again replacing v by vw in (3.3) and using (3.3) for $u, w \in L$, we get $[\beta(v), \beta(u)]\,\beta(w)d(u) = 0$ for all $u, v, w \in L$. Since β is an automorphism of R we have $[v,u]w\beta^{-1}(d(u)) = 0$. This implies $[v, u]L\beta^{-1}(d(u)) = 0$ for all $u, v \in L$. Since L is a non-zero σ-Lie ideal of R yields that $[v, u]L\beta^{-1}(d(u)) = 0$ for all $v \in L$, $u \in L \cap S_{a_\sigma}(R)$ (3.4). By lemma-1 we have either $[v, u] = 0$ for all $v \in L$ or $d(u) = 0$ for all $u \in L \cap S_{a_\sigma}(R)$. Let $u \in L$, as $u + \sigma(u), u - \sigma(u) \in L \cap S_{a_\sigma}(R)$ and $[v, u \pm \sigma(u)] = 0$ for all $v \in L$ or $d(u \pm \sigma(u)) = 0$. Hence we have $[v, u] = 0$ or $d(u) = 0$ for all $u, v \in L$. That is L is the union of two additive subgroups J and K of U such that $J = \{u \in L \,/\, d(u) = 0\}$ and $K = \{u \in L \,/\, [v, u] = 0$ for all $v \in L\}$. But a group L cannot be expressed as the union of two proper subgroups. Hence either $J = L$ or $K = L$. If $J = L$ then $d(L) = 0$, we get R is commutative by lemma-5. If $K = L$ then $[L, L] = 0$. That is $L \subseteq Z(R)$ by lemma-4. By lemma-5 R is commutative. □

Corollary 1: *Let R be a 2-torsion free σ-prime ring, L a non-zero σ-Lie ideal and a subring of R. If R admits a non- zero (α, β)-derivation d which commutes with σ such that* $[d(u), u]_{\alpha,\beta} = 0$ *for all* $u \in L$ *then R is commutative.*

Theorem 2: *Let R be a 2-torsion free σ-prime ring, L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero generalized (α, β) - derivation F associated with non-zero (α, β)-derivation d which commutes with σ such that* $F(u)\alpha(u) = \beta(u)d(u)$ *for all* $u \in L$*, then R is commutative.*

Proof: We have $F(u)\alpha(u) = \beta(u)d(u)$ for all $u \in L$ (3.5). Replacing u by $u + v$, $v \in L$ in (3.5) using (3.5), we get $F(u)\alpha(v) + F(v)\alpha(u) = \beta(u)d(v) + \beta(v)d(u)$, for all $u, v \in L$ (3.6). Replacing v by vu in (3.6) and using (3.6) for $u, v \in L$, we get $2\beta(v)d(u)\alpha(u) = \beta(uov)d(u)$ for all $u, v \in L$. Taking wv instead of v in above equation and using this equation, we obtain $[w, u]\beta(v)d(u) = 0$ for all $u, v, w \in L$. Since β is an automorphism of R, we get $[w, u]v\beta^{-1}(d(u)) = 0$ for all $u, v, w \in L$. Hence we arrive to $[w, u]\beta^{-1}(d(u)) = 0$ for all $u, w \in L$. Since L a non-zero σ-Lie ideal of R we get also $[w, u]L\beta^{-1}(d(u)) = 0$ for all $w \in L$, $u \in L \cap S_{a_\sigma}(R)$. That is $[w, u]L\beta^{-1}(d(u)) = 0$ and $\sigma[w, u]L\beta^{-1}(d(u)) = 0$ for all $w \in L, u \in L \cap S_{a_\sigma}(R)$. If we apply the similar arguments as used after equation (3.4) of theorem-1, we get the result. □

Theorem-3*:* *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero generalized (α, β) −derivation F associated with non-zero (α, β)-derivation d which commutes with σ such that* $F(u^2) = \pm\alpha(u^2)$ *for all* $u \in L$*, then R is commutative.*

Proof: We have, $F(u^2) = \pm\alpha(u^2)$ for all $u \in L$. By linearizing this equation and using the above equation for $u, v \in L$, we get $F(u)\alpha(v) + \beta(u)d(v) + F(v)\alpha(u) + \beta(v)d(u) = \alpha(uv) + \alpha(vu)$ for all $u, v \in L$. (3.7) Replacing v by vu in equation (3.7) and using (3.7), we get $(uov)d(u) = 0$, for all $u, v \in L$ (3.8). Replacing v by vw in equation (3.8) and using (3.8) for $u, w \in L$, we get $[u, v]\beta(w)d(u) = 0$, for all $u, v, w \in L$ and so, $[u, v]w\beta^{-1}(d(u)) = 0$, for all $u, v, w \in L$. Hence, $[u, v]\beta^{-1}(d(u)) = 0$, for all $u, v \in L$. Since L is a non-zero σ-Lie ideal of R yields that $[u, v]L\beta^{-1}(d(u)) = 0$ for all $v \in L$, $u \in L \cap S_a(R)$. By similar argument made in theorem-1 after (3.4), we get the required result. □

Theorem-4: *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero generalized (α, β) −derivation F associated with non-zero (α, β)-derivation d which commutes with σ such that* $F(u^2) = 2d(u)\alpha(u)$*, for all* $u \in L$*, then R is commutative.*

Proof: We have, $F(u^2) = 2d(u)\alpha(u)$, for all $u \in L$. $F(u)\alpha(u) + \beta(u)d(u) = 2d(u)\alpha(u)$, for all $u \in L$ (3.9). Linearizing equation (3.9) and using equation (3.9), we get $F(v)\alpha(u) + F(u)\alpha(v) + \beta(v)d(u) + \beta(u)d(v) = 2d(u)\alpha(v) + 2d(v)\alpha(u)$, for all $u, v \in L$ (3.10). Replacing v by vu in equation (3.10) and using equation (3.10), we get $\beta(v)\beta(u)d(u) + \beta(u)\beta(v)d(u) = 2\beta(v)d(u)\alpha(u)$ (3.11). Replacing v by wv in equation (3.11) and using (3.11), we get $[u, w]\beta(v)d(u) = 0$, for all $u, v, w \in L$ (3.12). Since β is an automorphism of R, we get $[u, w]v\beta^{-1}(u) = 0$, for all $u, v, w \in L$. That is $[u, w]L\beta^{-1}(d(u)) = 0$ for all $u, w \in L$. Since L is a non-zero σ-Lie ideal of R yields that $[u, v]L\beta^{-1}(d(u)) = 0$ for all $v \in L$, $u \in L \cap S_a(R)$. By similar argument made in theorem-1 after equation (3.4), we get the required result. □

Theorem-5*:* *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero generalized (α, β)-derivation F associated with non-zero (α, β)-derivation d which commutes with σ such that* $d(u^2) = 2F(u)\alpha(u)$*, for all* $u \in L$*, then R is commutative.*

Proof: We have $d(u^2) = 2F(u)\alpha(u)$, for all $u \in L$. That is $d(u)\alpha(u) + \beta(u)d(u) = 2F(u)\alpha(u)$, for all $u \in L$ (3.13). Linearizing equation (3.13) and using equation (3.12), we get $d(v)\alpha(u) + d(u)\alpha(v) + \beta(v)d(u) + \beta(u)d(v) = F(u)\alpha(v) + 2F(v)\alpha(u)$, for all $u, v \in L$ (3.14). Replacing v by vu in equation (3.14) and using equation (3.14), we get $\beta(v)\beta(u)d(u) + \beta(u)\beta(v)d(u) = 2\alpha(v)d(u)\alpha(u)$. This is similar to equation (3.12) in theorem-4. Hence we get the required result. □

Corollary-2: *Let R be a 2-torsion free σ-prime ring and L a non-zero σ-Lie ideal and a subring of R. If R admits a non-zero (α, β)-derivation d which commutes with σ such that $d(u^2) = 2d(u)\alpha(u)$, for all $u \in L$, then R is commutative.*

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