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Medical Image Fusion using Interval Type 2 Fuzzy Logic

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Abstract

In order to improve clinical accuracy and to take better decisions, medical image fusion is used.It involves integration of the essential features present in different medical images into a single image.Different imaging modalities like, Positron Emission Tomography (PET), Computed Tomography (CT), Magnetic Resonance Imaging (MRI) and others capture different details. Dense tissue structures are visible in CT scan whereas soft tissues are visible in MRI scan.If these two scan images are fused into a single image, doctors will be able to diagnose and plan treatment for patients. In this paper, an interval type 2 fuzzy logic based image fusion technique for fusing CT and MRI images is presented.Otsu's segmentation method is used to segment dense tissue from CT image and soft tissue from MRI images. Later fusion is performed using interval type 2 fuzzy logic. Discrete wavelet transform is used to perform multiresolution fusion. Results are compared with type 1 fuzzy logic system and are found to outperform for most of the performance metrics. Sugeno type 2 fuzzy inference system produces better results compared to Mamdani Inference system.

Keywords: Computed Tomography, Image Fusion, Mamdani FLS, Magnetic Resonance Imaging, Sugeno FLS, Type 2 Fuzzy Logic

INTRODUCTION

Various techniques in image processing have been developed due to advances in medical imaging in order to provide better diagnosis, monitoring and analysis. One such technique gaining importance currently is the medical image fusion. This helps in providing more accurate and relevant information [1]. In order to take better decisions, multimodal medical image fusion is used to produce a single image by integrating the essential features present in different medical images to improve the clinical accuracy. This helps doctors to diagnose and plan treatment for patients.

MRI scan provides details on soft tissues whereas CT scan provides detailed information on bony/dense structures. PET scanning shows the functioning of the body. It provides the details of the blood flow using nuclear imaging technique [2]. The challenging task for the image fusion algorithms is to extract the relevant features from each individual image. The extracted important details are then used to form a new fused image.The fused image thus obtained using the images from various modalities reduces the memory requirement and is also of high quality compared to any of the individual images.

Several techniques and approaches have been used in developing sophisticated medical image fusion algorithms. Most of these algorithms fuse images obtained from any two modalities.

According to the survey performed in [1], medical image fusion is categorized based on (i) Image Fusion Method (ii) Modality used for imaging (iii) Imaging of organs under study.

The brute force fusion methods include maximum value, averaging, weighted averaging, additions etc. Commonly used soft computing techniques include neural networks, fuzzy logic, principal component analysis and morphology methods. According to [3] fuzzy logic based methods provide better results from subjective analysis. Fusion of PET and MRI images using weighted least squares filter method is performed in [4].

Wavelet transform based approaches are proposed by few researchers in [5], [6], [7] and [8].Regional information entropy contrast degree is used to fuse high frequency components and maximum value is used to low frequency components in [5]. In [6], approximation coefficients are fused based on higher visibility and detailed components are fused based on variance. Maximum selection rule is used at each level in [7] to fuse MRI and CT images. Entropy of 3x3 blocks is used as selection criteria to fuse detailed coefficients in [8] and average value is used to fuse the approximation coefficients.

Pulse coded neural network based medical image fusion is proposed in [9].Fuzzy logic based fusion of MRI and PET images in performed in [2].

A. CT Images

Computed tomography or CT gives information about size, shape and location of bony structures in human anatomy. It is a medical imaging technology that creates cross-sectional images of the inside of the body using X-rays. CT scanning is done using an X-ray source and detector that are situated 180 degrees across from each other. They rotate 360 degrees around the patient, continuously detecting and sending information about the attenuation of X-rays as they pass through the body. To minimize the degree of scatter or blurring, very thin X-ray beams are used. X-ray attenuation is detected by detector & data acquisition system. A computer manipulates and integrates the acquired data by assigning numerical values based on the subtle differences in X-ray attenuation. Based on these values, a gray-scale axial image is generated that can distinguish between objects with even small differences in density [10].

B. MRI Images

MRI takes detailed images of the soft tissues of the body using the power of magnets. It is a non-invasive diagnostic test that creates images using magnetic field, radio waves and a computer.The magnet creates a strong magnetic field that aligns the protons of hydrogen atoms in the body. They are then exposed to short burst of radio waves. This spins the various protons of the body and they produce a signal that is detected by the receiver portion of the MRI scanner. The signal emitted from different body tissues varies. The signal is processed by a computer and an image is produced. Images are produced as slices. MRI scans are used to diagnose a variety of conditions from torn ligaments to tumours. MRIs are very useful for examining the brain and spinal cord. Air and bone appear black in the MRI scan as they do not respond to MRI signal. Bone marrow, spinal fluid, blood and soft tissues vary from the intensity of black to white. Bones and air appear black and soft tissues appear white in MRI scan [10].

Type 1 Fuzzy Logic System

Fuzzy logic based image fusion is gaining momentum recently.Fuzzy sets were proposed in 1965 [11]. Various researchers are applying fuzzy logic technique for image fusion. To perform image fusion, the main task is to select the most suitable regions from the input images and copy them to the output image. To decide the best regions human reasoning is the most suitable technique. Fuzzy logic is the best tool to convert human reasoning into a set of rules. The reason is that the logic used for performing image fusion is fuzzy rather than crisp. When no mathematical relations are easily available and there is uncertainty fuzzy logic approaches are used [12].It is considered as a logic of approximation and an extension of Boolean logic to handle vagueness. The source for writing fuzzy rules is human reasoning.

Type 1 Fuzzy Logic System (T1FLS) consists of four components (i) Fuzzifier (ii) Rules (iii) Inference Engine (iv) Defuzzifier [13]. Fuzzifier is the component which converts crisp inputs into fuzzy values. Rules are a collection of if-then statements. Rules and membership functions are domain dependent and form the essence of fuzzy logic. They can be extracted by numerical data or provided by domain experts. The inference engine maps input fuzzy sets into output fuzzy sets by evaluating the rules based on the fuzzy operators defined on the fuzzy sets. It combines the rules using inferential procedures. The two types of fuzzy inference methods are (i) Mamdani’s Fuzzy Inference method introduced in 1975 and (ii) Takagi-Sugeno Fuzzy Inference method introduced in 1985. Mamdani’s Fuzzy Inference method produces a fuzzy value which has to be defuzzified further. Sugeno’s method produces a crisp value and hence does not require defuzzification. Sugeno's method is used to model inference systems in which the output is a linear function or constant.

Fuzzy inference process maps the fuzzy inputs to a fuzzy or crisp output using fuzzy logic. This process consists of three steps.

1. The first step is to apply fuzzy operators such as complementation, intersection and union. If the antecedent part of a rule has more than one part, these operators are applied. Complementation operation on a set A is given by 1-A. Intersection operation or the AND operations is implemented using minimum(A,B) or product(A,B). Union operation or the OR operation of two fuzzy values A and B is implemented using maximum(A,B) or probor(A,B)=A+B-AB.
2. The second step is to apply implication method to obtain the output fuzzy set. Using the antecedent of the rule, a single number is obtained as the output of the first step. To convert this number into a fuzzy set, implication method is applied for each rule using the consequent fuzzy set. Two commonly used implication methods are minimum and product. Before applying the implication method, the weight of the rule must be taken into consideration.
3. The output fuzzy values obtained for each rule must be aggregated into a single fuzzy value for each output variable. The common aggregation methods applied are (i) taking maximum value across the range of input fuzzy set (ii) summing the output fuzzy set of each rule (iii) applying probabilistic OR operation. The output of this step is a single fuzzy value.

Conversion of the fuzzy output produced by the aggregation process of a fuzzy inference system into crisp output is known as defuzzification. The defuzzifier maps the output fuzzy sets of the inference engine to crisp values. Well known defuzzification methods are weighted average method, centre of sums method, centre of gravity/centroid of area method, centre of area/bisector of area method, maxima methods: first of maxima method, last of maxima method, mean of maxima method.

Fusion of CT and MRI images is performed in [14] by selecting maximum value using Mamdani type 1 fuzzy inference system.

Type 2 Fuzzy Logic System

Type 2 Fuzzy sets were introduced by L.A.Zadeh in 1975 [15]. The fuzzy logic system that has atleast one type-2 fuzzy set (T2FS) is called a type-2 fuzzy logic system (T2FLS) [16]. Type-2 fuzzy sets are characterized by three-dimensional membership function. The third dimension gives additional degrees of freedom which helps in modeling uncertainties. Figure 2 depicts type-2 triangular membership function [17].

General type-2 fuzzy sets are characterized as shown in equation (1)

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x \subseteq [0,1]} \mu_{\tilde{A}}(x,u)/(x,u) \quad (1)$$

where$x$ is the primary variable and has domain X; $u \in$ U is the secondary variable and as domain J_x at each x∈X; Jx is the primary membership of x. The secondary grades $\mu_{\tilde{A}}(x,u)$ is set to 1 for interval type-2 fuzzy sets and hence interval type-2 fuzzy sets are characterized as shown in equation (2) [18]. Interval type 2 triangular membership function is as shown in figure 1.

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x \subseteq [0,1]} 1/(x,u) = \int_{x \in X} \left[\int_{u \in J_x \subseteq [0,1]} 1/u\right]/x \quad (2)$$

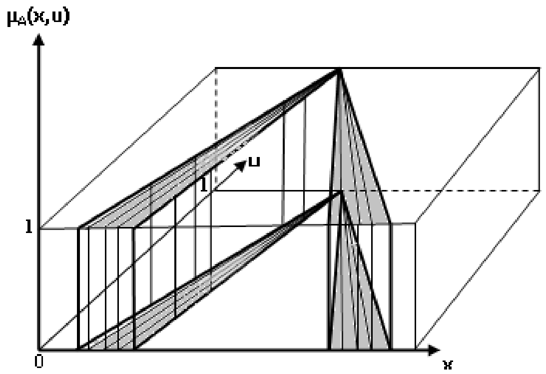

Figure 1. Interval Type 2 Triangular MF

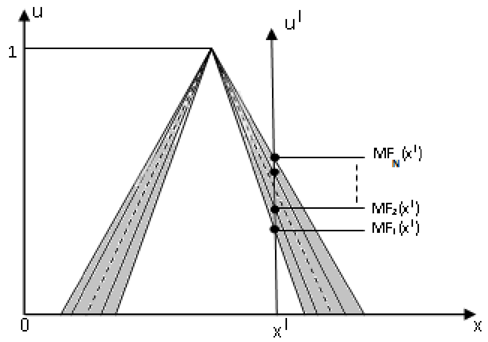

Figure 2. FOU of Type 2 Triangular Membership Function

The union of all the primary memberships is called footprint of uncertainty (FOU). An IT2FS is characterized by its 2-d FOU.

$$\text{FOU}(\tilde{A}) = \cup_{\forall x \in X} J_x \quad (3)$$

In figure 2, the shaded region is called the FOU. Figure 3 shows the FOU of Gaussian primary membership function. It is bounded by a lower member function ($\underline{\mu}_{\tilde{A}}$(x)) and an upper membership function ($\overline{\mu}_{\tilde{A}}$ (x)) which are two type-1 membership functions [19]. The footprint of uncertainty is the area between $\underline{\mu}_{\tilde{A}}$(x) and $\overline{\mu}_{\tilde{A}}$(x). Using eq.(3),

$$\text{FOU}(\tilde{A}) = \cup_{\forall x \in X} \left[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)\right] \quad (4)$$

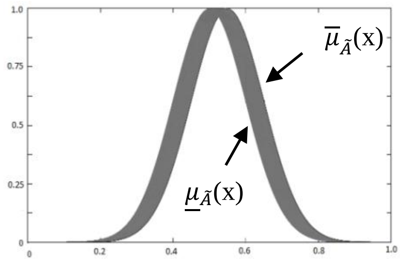

Figure 3. FOU of the Gaussian primary membership function

T2FLS has five components (i)Fuzzifier (ii) Rules (iii) Inference Engine (iv) Type Reducer (v) Defuzzifier. Mamdani T2FLS uses three fuzzifiers, namely, singleton, non-singleton type 1 and non singleton type 2 [20]. Sugeno T2FLS uses only singleton fuzzifier.

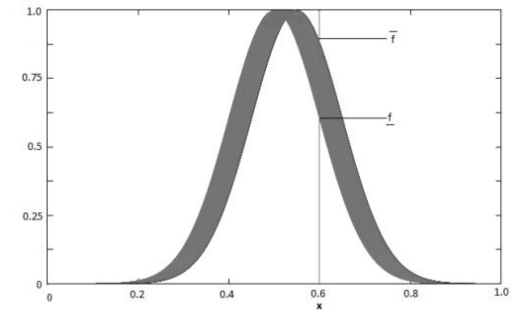

Figure 4. Singleton Fuzzifier

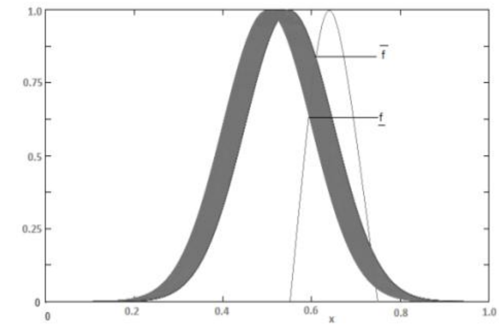

Figure 5. Non-Singleton Type 1 Fuzzifier

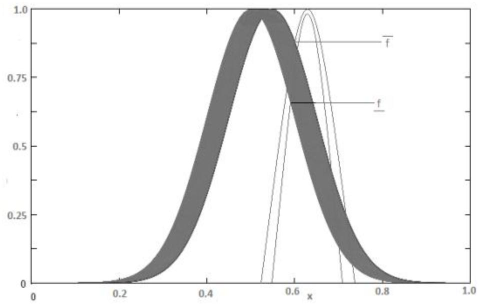

Figure 6. Non-Singleton Type 2 Fuzzifier

Singleton fuzzifier, non-singleton type 1 fuzzifier and non-singleton type 2 fuzzifier are illustrated in figure 4, figure 5 and figure 6 respectively. The fuzzified inputs $\bar{f}$ and $\underline{f}$ are found at the intersection of singleton at 'x', Gaussian function whose mean is at ‘x’ and set of Gaussian functions whose centres are at 'x' with the lower membership and upper membership function which lead to the singleton fuzzifier, non-singleton type 1 fuzzifier and non-singleton type 2 fuzzifier respectively. Though singleton fuzzifier is the most commonly used fuzzifier, it is not suitable in the presence of noise [21].

In T1FLS, both the antecedents and consequents of the rules are all described T1 fuzzy sets. In T2FLS, some or all of the antecedents and consequents are described by the membership functions of T2FS.

Inference Engine uses the rule base to convert the input fuzzy sets to output fuzzy sets. Using the fuzzy operators, the antecedents of the fired rules are combined.

Type reducer generates type-1 fuzzy set from type 2 fuzzy set. The commonly used type reducers are centre of sets andcentroid.

Defuzzification is the last step of a fuzzy logic system.The output fuzzy set is converted into a crisp number in this step. There are various defuzzifiers like centroid, mean of maxima, maximum, etc.

PROPOSED ALGORITHM

Step 1: Input medical images.

Step 2: Apply segmentation technique to obtain soft tissue region from MRI image and bone tissue region from CT image.

Step 3: Copy the segmented soft tissue region and bone tissue region to a new intermediate fused image using the maximum rule.

Step 4: Save the locations in the intermediate image that do not contain the segmented regions.

Step 5: Copy the pixels at saved locations from MRI image into image I_1 and from CT image into image I_2 .

Step 6: Apply discrete wavelet transform upto desired level L.

Step 7: At each level, use maximum rule to fuse detailed coefficients. Approximation coefficients are fused as follows:

Step 8: Form two column vectors using the pixel values in I_1 and I_2. These column vectors will act as input to the type 2 fuzzy logic system.

Step 9: Set up the fuzzy logic system using appropriate input and output membership functions, fuzzy operator, aggregation method and defuzzification method.

Step 10: The output of the fuzzy logic system is copied to the intermediate fused image at each level

Step 11: The final inverse discrete wavelet transform values are restored at the saved locations.

Step 12: Compute the various performance metrics of the fused image using the input images and reference image if available.

Fuzzy Logic System

Type 2 fuzzy logic system used in the proposed approach is as shown in figure 7. The images are fuzzified using Gaussian membership functions as shown in figure 8. Experiments are performed using eight membership functions. Both Mamdani type and Sugeno type fuzzy inference systems are experimented to evaluate the rules and produce the fuzzified output. For Mamdani type, the following settings are used. AND method is performed using MIN. OR method is performed using MAX. Aggregation is performed using MAX. ‘Centre of sets’ is used for type reduction. Implication method used is MIN. Centroid is used for defuzzification. For Sugeno type, AND method is performed using MIN. OR method is performed using MAX. Defuzzification is done using weighted average.

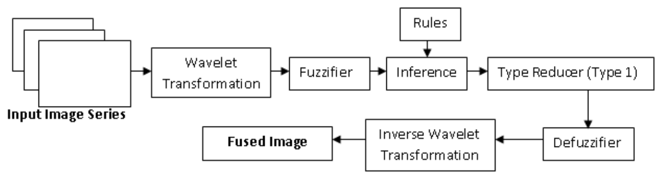

Figure 7. Fuzzy Logic System used for image fusion

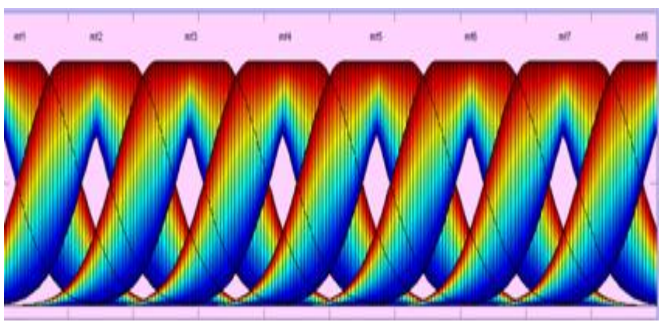

Figure 8. Eight membership functions

Fuzzy Rules

The rules used to fuse CT and MRI images are shown in Table 1. The brighter shade of the two images are selected for the fused image. VD represents 'very dark', MD represents 'medium dark', D represents 'dark', L represents 'light', VL represents 'very light', B represents 'bright', MB represents 'medium bright' and VB represents 'very bright'. Table 1 shows the fusion rules for eight membership functions.

Table 1. Fusion rules for 8 membership functions

Input Image 1 \ Input Image 2	**VD**	**MD**	**D**	**L**	**VL**	**B**	**MB**	**VB**
VD	VD	MD	D	L	VL	B	MB	VB
MD	MD	MD	D	L	VL	B	MB	VB
D	D	D	D	L	VL	B	MB	VB
L	L	L	L	L	VL	B	MB	VB
VL	VL	VL	VL	VL	VL	B	MB	VB
B	B	B	B	B	B	B	MB	VB
MB	MB	MB	MB	MB	MB	MB	MB	VB
VB	VB	VB	VB	VB	VB	VB	VB	VB

EXPERIMENTAL RESULTS AND ANALYSIS

The images are obtained from online database "ATLAS". Two sets of images are used. Data set 1 does not contain reference image whereas data set 2 has a reference image.

There are two types of metrics that are computed for evaluate the performance of image fusion. One set of metrics are used when the reference image is available. They compare the fused image with the reference image. Another set of metrics are used when reference image is not available. They use only the features of the fused image. These metrics however can be computed when reference image is not available as well. The details of these metrics is discussed in [22].

Table 2 shows the values of these metrics for data set 1 where reference image is not available. Table 3 and Table 4 show the metrics for data set 2 where reference image is available. For every metric, the best value obtained is highlighted. Metrics like Entropy (ENT), Standard Deviation (SD), Spatial Frequency (SF), Fusion factor (FF), Fusion Quality Index (FQI), Fusion Similarity Metric (FSM), Correlation (CORR), Signal to Noise Ratio (SNR), Peak Signal to Noise Ratio (PSNR), Mutual Information (MI), Structural Similarity Metric (SSM) and Universal Quality Index (QI) must be high whereas Cross Entropy (CE), Fusion Symmetry (FS), Root Mean Square Error (RMSE), Percentage Fit Error (PFE), Mean Absolute Error (MAE) must be low for indicating good fusion.

A. Results obtained for Data Set 1

Table 2 Evaluation Metrics without using reference image for data set 1

Metric	FL TYPE 1	T2M	T1NST2M	T2NST2M	T2S
ENT	6.9179	6.7504	6.7546	6.7374	**6.9619**
SD	**0.292**	0.2823	0.2710	0.2636	0.2895
CE	1.1716	1.0150	1.2179	1.2334	**0.8429**
SF	0.1592	0.1526	0.1470	0.1434	**0.1657**
FF	2.4922	2.4887	2.3819	2.3435	**2.4985**
FQI	0.5423	**0.5903**	0.5795	0.5813	0.5812
FSM	0.8391	0.8471	0.8145	0.8111	**0.8496**
FS	0.0401	0.0388	0.0297	**0.0264**	0.0388

Figure9a. Original CT Image

Figure9b. Original MRI Image

Figure 9c. Segmented CT image

Figure9d. Segmented MRI image

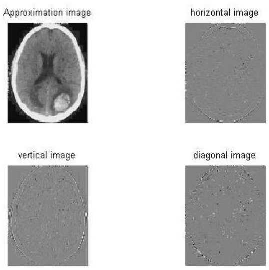

Figure9e.DWT of CT image

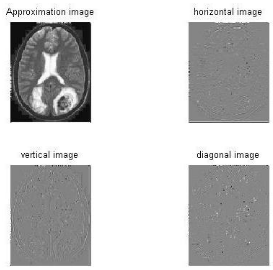

Figure9f.DWT of MRI image

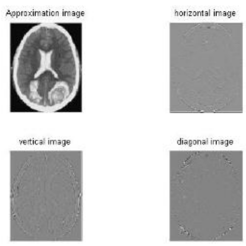

Figure9g.DWT of fused images

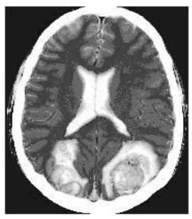

Figure9h. Fused CT and MRI images

Fig.9a and Fig.9b show original CT image and MRI image respectively. Fig.9c and fig.9d show segmented bone tissue in CT image and segmented soft tissue from MRI image. Fig.9e and fig. 9f show the wavelet coefficient of CT and MRI images. Fig.9g contains the DWT of superimposed partially fused image. Fig.9h shows the final fused image.

B. Results obtained for Data Set 2

The input original CT and MRI images are shown in fig.10a and fig. 10brespectively. Segmented CT and MRI images are shown in fig 10c and fig. 10drespectively. DWT coefficients are shown in fig. 10e and fig.10f respectively. fig.10g shows DWT of the fused image. Fig.10h shows the final fused image using the proposed approach.

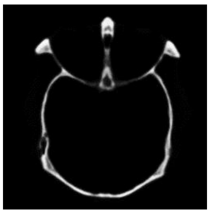

Figure 10a. Original CT Image

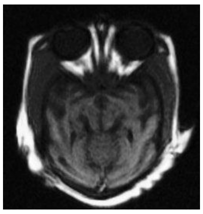

Figure 10b. Original MRI Image

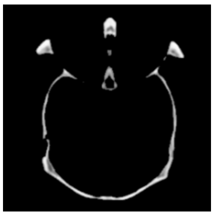

Figure10c. Segmented CT image

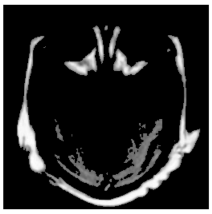

Figure 10d. Segmented MRI image

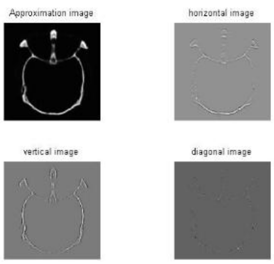

Figure 10e.DWT of CT image

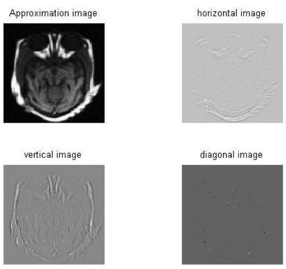

Figure 10f.DWT of MRI image

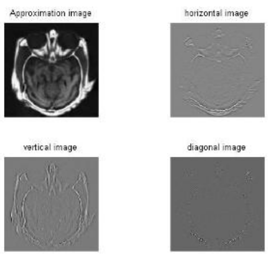

Figure 10g. Reference fused image

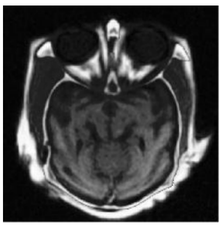

Figure 10h. Fused CT and MRI images using proposed approach

Table 3. Evaluation Metrics using reference image

Metric	FL TYPE 1	T2M	T1NST2M	T2NST2M	T2S
RMSE	**0.0500**	0.0919	0.0748	0.0947	0.0579
PFE	13.7419	16.5875	16.4348	17.1233	**12.7597**
MAE	0.0263	0.0443	0.0335	0.0315	**0.0247**
CORR	**0.9895**	0.9861	0.9774	0.9646	**0.9895**
SNR	16.0588	15.6044	13.3776	11.3331	**16.8490**
PSNR	60.7648	60.5375	59.4241	58.4019	**60.8599**
MI	1.6765	1.6628	1.6628	1.6465	**1.6765**
QI	0.8977	0.7961	0.7507	0.7996	**0.8985**
SSIM	**0.9997**	0.9996	0.9993	0.9989	**0.9997**

Table 4. Evaluation Metrics without using reference image

Metric	FL TYPE 1	T2M	T1NST2M	T2NST2M	T2S
ENT	5.7949	6.2163	6.3317	6.1926	**6.6534**
SD	0.2367	0.2275	0.2149	0.2038	**0.2377**
CE	**5.4416**	6.2664	5.9224	5.9443	6.6302
SF	0.0803	0.0704	0.0676	0.0707	**0.0876**
FF	2.6960	2.7228	2.7228	2.6960	**2.7960**
FQI	0.8331	0.8065	0.7574	0.7087	**0.9146**
FSM	0.8003	0.8119	0.8008	0.7932	**0.8159**
FS	**0.0672**	0.0721	0.0721	0.0672	**0.0672**

Results obtained are shown in table 2 through table 4. FLTYPE1 column represents Sugeno type 1 fuzzy logic. T2M column represents Mamdani singleton fuzzifier. T1NST2M represents Mamdani nonsingleton type 1 fuzzifier. T2NST2M represents Mamdani nonsingleton type 2 fuzzifier. T2S represents type 2 Sugeno. The best value for each metric is highlighted. It shows that Sugeno FIS gives better results compared to other methods for fusion of CT and MRI images.

CONCLUSION

CT and MRI image fusion is an important application in image processing. Both the bone tissue and soft tissue are visible in the same image. This helps in treatment planning. The prerequisite is that both the images must be aligned. The proposed approach segments hard tissue from CT image and soft tissue from MRI image using Otsu's segmentation technique. The other regions are fused using interval type 2 fuzzy logic. The performance metrics obtained are compared with type 1 fuzzy logic. Experiments are conducted for different fuzzifiers of type 2 fuzzy logic and found that the results obtained for Sugeno type 2 fuzzy logic are promising. The subjective analysis also shows that the fused images contain most of the information present in the original images and are clear from visual perspective also. Fusion of PET images with CT and MRI images shall be experimented in our future work. Color images also can be considered.

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