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A Characterization for Topologically Integer Additive Set-Indexers of Graphs II

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Abstract
The objective of this paper is to extend the specific labeling of topological integer additive set-indexers for some graph classes with minimum cardinality. Here, we also prevail the result for subset of a Top-IASI need not forms a Top-IASI. In this paper ,we obtain Top-IASI for star graph, double star graph, path graph, cycle graph, pan graph, tadpole graph, wheel graph and $P_2 + \overline{K_m}$ graph.

Keywords: Integer additive set-labeling of graphs (IASL), integer additive set-indexers of graphs (IASI), topological integer additive set-labeling (Top-IASL), topological integer additive set-indexers of graphs(Top-IASI).

INTRODUCTION

Before seventeenth century none was familiar with the name of graph labeling. The word graph labeling came into existence when in 1736, Euler[11] solved the bridge problem. Graph labeling became the shining star in sixties when Rosa[1] gave the concept of β- valuation of graphs. Now the era of graph labeling started. Many authors introduced various graph labelings and obtained thousands of new results. Research in graph labeling took a new turn when Acharya [3,4] initiated the concept of set-valuation of graphs. In 1986, Acharya[5] established a link between graph theory and point set-topology by introducing the concept of topological set-indexers. In 2012, Germina and Anandevally[9] labeled the elements of the graph as the subset of non-negative integers by introducing the concept of integer additive set-indexers of graph and provided the new momentum in this field. Later, Sudev and Germina[13] gave a new enthusiasm in this research. If along the edges this mapping become injective then the labeling is called IASI. Sudev and Germina [12] imposed the concept of topology over integer additive set-indexer and named it as topological integer additive set-indexers (Top-IASI) of graphs. Later in 2016, Mehra and Puneet[14] extended this special labeling over different graph types and produced the Top- IASI function with minimum cardinality for those graph.

Theorem 1.1 [4]:- Every graph has a set-indexer.

Let N_0 denote the set of all non-negative integers and A, B ⊆ N_0 , the sum of these sets is denoted by A+B and is defined as A+B= $\{ a + b: a\epsilon\ A,\ b\epsilon\ B\}$.

Theorem 1.2 [13]:- Every graph has a integer additive set-labeling.

Theorem 1.3 [13]:- Every graph has a integer additive set-indexer.

Motivated from the concept of introducing the topological integer additive set-indexed function over graphs, we further intensify this existing mapping over some other graph classes like star graph, double star graph, cycle graph, wheel graph, $P_2+ \overline{K_m}$ graph, $P_n* S_m$ graph, $H_{n,n}$ graph and bi-partite graph. Here, we also verify a subset of Top-IASI of a graph need not become Top-IASI.

PRELIMINARIES

All the terms and definitions of graph and graph classes are not covered in this paper, for this we refer to West [6], Harary [7] and Bonday and Murty [8] and for topological concepts we refer to Joshy [10]. All the graphs considered in this paper are finite, simple, non-trivial and connected.

Definition 2.1[12] Topological integer additive set-labeling (Top-IASL):-

Let f: V (G) → P(X)-{ϕ} be an IASL of the graph G with non-empty set $X \subseteq N_0$. Then, f is said to be Top-IASL if f (V(G)) ∪ {ϕ} forms a topology on the set X and the graph G is called Top- IASL graph.

Definition 2.2[12] Topological integer additive set-Indexer (Top-IASI):- A topological integer additive set-labeling f is called a topological integer additive set-indexer (Top-IASI) if the associated function f^+: E (G) → P(X) – {0, ϕ} defined by $f^+(uv) = f(u) + f(v)$ ∀ uv ∈ E (G), is also injective. A graph G which admits topological integer additive set- indexer is called a topological integer additive set-indexed graph (Top- IASI graph).

Proposition 2.3[12]:-Let G be a topological IASL graph then minimum cardinality of the ground set X is known as the topological set-indexing number of that graph.

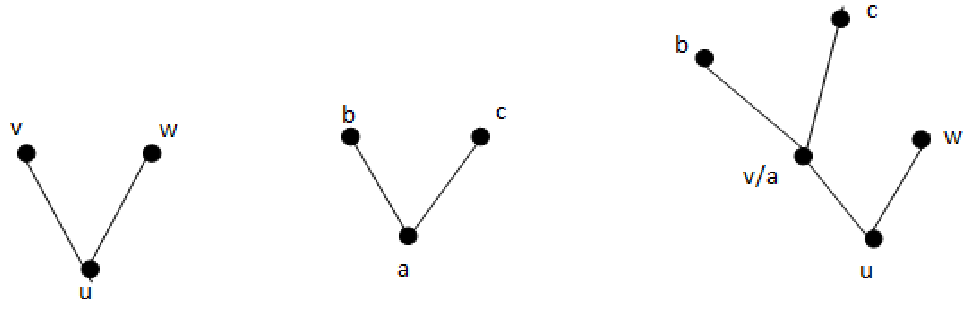
Definition 2.4[12]:- A Top-IASI f on a graph G must have at least one pendant vertex.

Proposition 2.5[12]:- Let f: V (G) → P(X) - {ϕ} is a Top-IASL of a graph. Then, the set-labels containing the maximal element of the ground set X be assigned to the pendant vertices of the graph those are adjacent to that vertex whose set-label is {0}.

Proposition 2.6[12]:- Let f be Top-IASL on a graph G with ground set X. Then, an element x_r of X can be assign in the labeling of any vertex v of G if and only if $x_r + x_s \leq l$, where x_s is any element in the set-label of adjacent vertex of u in G and l is the maximal element in X.

Proposition 2.7[12]:- If G has a Top-IASL f, with single pendant vertex then the only set-label of the vertex of G containing the maximal element of X is X itself.

Definition 2.8[2]:- A merge graph $G_1 * G_2$ can be formed from two graphs G_1 and G_2 by merging a node of G_1 with a node of G_2. As an example, let us consider T_3, a tree with three vertices and S_2, a star on three vertices then $T_3 * S_2$ is formed as follows:



T_3 S_2 $T_3 * S_2$

Definition 2.9[15]:- $H_{n,n}$ is the graph with vertex set $V(H_{n,n}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ and the edge set $E(H_{n,n}) = \{ v_i u_j : 1 \leq i \leq n, n-i+1 \leq j \leq n\}$.

Definition 2.10:- A graph G with vertices set V(G) is said to be bipartite graph if its vertices set is partitioned into two non- empty subsets V_1 and V_2 such that every edge of G has one end point in V_1 and another in V_2.

MAIN RESULTS

This paper is the advancement of the results obtained by Sudev and Germina [12]. In this paper, we produce the special type of graph labeling as Top-IASL for some other graph classes. Here, during generating of the function the concept of minimum cardinality is also keep in mind.

Theorem 3.1:- Every star graph $K_{1,m}$ reveals a topological integer additive set-indexing.

Proof:- Let v be a vertex of order m in the star graph $G = K_{1,m}$ and $\{u_1, u_2, \ldots, u_m\}$are m adjacent vertices to the vertex v. Here, X is the ground set containing the non-negative integers. Without loss of generality, we can assume that {0} is assigned to the vertex v. The labeling of the vertices adjacent to v will be defined by the function as follows:

Let $f: V(G) \to P(X) - \{\emptyset\}$ be defined as

$f(u_1) = \{1\}$

Case 1: If m is even.

- $$f(u_i) = \begin{cases} \{0,1,2,\ldots,i-1\}, & 2 \leq i \leq \frac{m+2}{2} \\ \left\{1,2,\ldots, \quad i-\frac{m}{2}\right\}, & \frac{m+4}{2} \leq i \leq m \end{cases}$$

Where $X = \{0,1,2,3,\ldots,\frac{m}{2}\}$ and

Case 2: If m is odd.

- $$f(u_i) = \begin{cases} \{0,1,2,\ldots,i-1\}, & 2 \leq i \leq \frac{m+3}{2} \\ \left\{1,2,\ldots, \quad i-(\frac{m+1}{2})\right\}, & \frac{m+5}{2} \leq i \leq m,\ m \neq 3 \end{cases}$$

Where $X = \{0,1,2,3,\ldots,\frac{m+1}{2}\}$.

Let $f^{+}: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as

$f^{+}(u_i u_j) = f(u_i) + f(u_j)$ for all $u_i u_j \in E(G)$

Then $\tau = f(V(G)) \cup \{\phi\}$ forms a topology. Hence, the star graph admits a topological IASI.

Illustration 3.2:- Show that $K_{1,7}$ admits a Top- IASI.

Solution:- Let v be a vertex of order 7 in the star graph $G = K_{1,7}$ and $\{u_1, u_2, \ldots, u_7\}$ are the vertices set adjacent vertices to the vertex v. Here, X is non-empty finite ground set containing the non-negative integers. Here, n=7 is odd so the labeling according to the Theorem 3.1, will be as follows:

Let $f: V(K_{1,7}) \to P(X) - \{\emptyset\}$ be defined as

$f(v) = \{0\}$, $f(u_1) = \{1\}$

$$f(u_i) = \begin{cases} \{0,1,2,\ldots,i-1\}, & 2 \leq i \leq 5 \\ \{1,2,\ldots, \quad i-4\}, & 6 \leq i \leq 7 \end{cases}$$

Where $X = \{0,1,2,3,\ldots,4\}$ and labeling corresponding to the edges are :

Let $f^{+}: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as

$f^{+}(u_i u_j) = f(u_i) + f(u_j)$ for all $u_i u_j \in E(G)$. Then $\tau = f(V(G)) \cup \{\phi\}$ forms a topology.

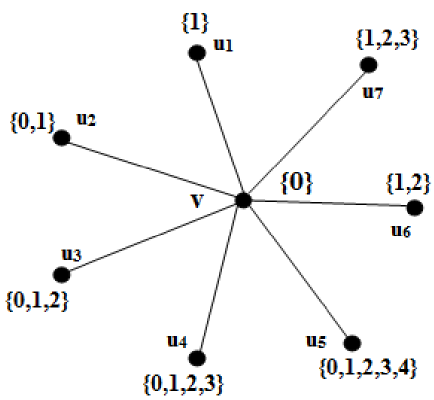


Figure 1: Depicts Top-IASI for $K_{1,7}$ star graph.

Theorem 3.3:- A double star graph $S_{p,q}$ reveals a Top- IASI.

Proof:- Let G = $S_{p,q}$ be a double star graph and { $u_1, u_2, \ldots, u_p$ } are p vertices of star S_p with central vertex u_1 labeled as {1}. Let X be the ground set containing the non-negative integer. Here, we restrict our labeling over p and q as $p < q$.

The labeling for remaining vertices of S_p are:

Case 1:- If q is even.

Let $f: V(S_p) \to P(X) - \{\emptyset\}$ then

$f(u_j) = \{ \{0,1,2,\ldots,j-1\},\ 2 \le j \le p \}$

Let S_q be another star graph with central vertex v_1 labeled as {0} and vertices $\{v_2, v_3, \ldots, v_q\}$ are adjacent to v_1.

The labeling for the vertices adjacent to v_1 will be defined by the function as follows:

Let $f: V(S_q) \to P(X) - \{\emptyset\}$ be the function then

$$f(v_i) = \begin{cases} \{0,1,2,\ldots,i+p-2\}, & 2 \le i \le \frac{q}{2} \\ \left\{1,2,\ldots,\ i - (\frac{q-2p+4}{2})\right\}, & \frac{q+2}{2} \le i \le q \end{cases}$$

Where X = $\{0,1,2,\ldots,\frac{q+2p-4}{2}\}$.

Case 2:- If q is odd.

Let $f: V(S_m) \to P(X) - \{\emptyset\}$ then

$f(u_j) = \{ \{0,1,2,\ldots,j-1\},\ 2 \le j \le p \}$

For S_q labeling is defined as:

Let $f: V(S_q) \to P(X) - \{\emptyset\}$ be the function then

$f(v_1) = \{0\}$

$$f(v_i) = \begin{cases} \{0,1,2,\ldots,i+p-2\}, & 2 \le i \le \frac{q-1}{2} \\ \left\{1,2,\ldots,i - (\frac{q-3p+3}{2})\right\}, & \frac{q+1}{2} \le i \le q, q \neq 3 \end{cases}$$

Where X = $\{0,1,2,\ldots,\frac{q+2p-1}{2}\}$ and v_1 is the adjacent vertex to u_1 of S_p.

And the labeling for edges of the graph are:

Let $f^+: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as

$f^+(u_i u_j) = f(u_i) + f(u_j)$, for all $u_i u_j \in E(G)$

Then $\tau = f(V(G)) \cup \{\phi\}$ forms a topology. Hence, the double star graph admits a Top- IASI.

Illustration 3.4:- Show that $S_{5,6}$ admits a Top- IASI.

Solution:- After comparing it with $S_{p,q}$, we form p=5 and q=6.

Here, $p < q$ and q is even. According to theorem 3.3 case 1, the labeling will be as:

Let $f: V(S_5) \to P(X) - \{\emptyset\}$ be defined as ,

$f(u_1) = \{1\}$ and $f(u_j) = \{ \{0,1,2,\ldots,j-1\},\ 2 \le j \le 5 \}$

For S_6 the labeling is as follows:

Let $f: V(S_6) \to P(X) - \{\emptyset\}$ be the function then

$f(v_1) = \{0\}$ and $f(v_i) = \begin{cases} \{0,1,2,\ldots,i+3\}, & 2 \le i \le 3 \\ \{1,2,\ldots,\ i\}, & 3 < i \le 6 \end{cases}$

Where X = {0,1,2,…, 6}.

Hence, $S_{5,6}$ admits a topological IASI.

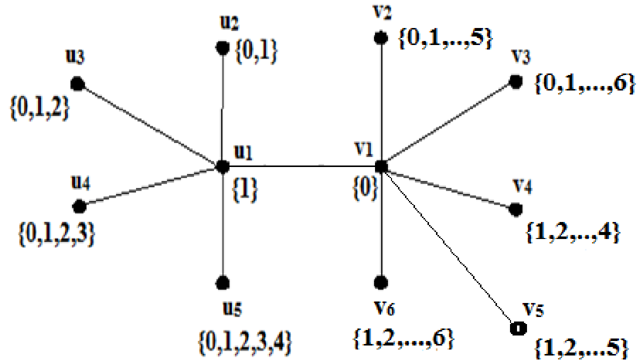


Figure 2: Depicts Top-IASI for $S_{5,6}$ double star graph.

Theorem 3.5: A cycle graph does not reveals a topological integer additive set-indexing.

Proof:- Let C_n be a cycle graph with vertices set{ $u_1, u_2, \ldots, u_n$}. For topological integer additive set-indexing, the graph must have one pendant vertex. But due to closeness of cycle graph C_n, it does not have any pendant vertex. Hence, the ground set X cannot be assigned to any of the vertex among the graph. So, its vertices does not form topology.

Proposition 3.6:- Let G_1 be a Top-IASI graph. If H is the subset of G_1 then H may not admit Top-IASI.

Proof:- If f is any IASI in graph G_1. Let f* be the restriction of f to V (H) and f^{+*} be the corresponding restriction of f^+ to E (H). Now, f* and f^{+*} forms an IASI on H because the subset of a IASI graph is also IASI. Here, f (V (G)) $\cup\{\emptyset\}$ also forms a topology on X where $X \subseteq N_0$. Now, f* (V (H)) $\subseteq$ f (V (G)) and it is not necessary that f* (V (H)) $\cup\{\emptyset\}$ forms a topology because the subset of a topological space need not form a topology. Hence the proof.

Theorem 3.7:- If P_m and S_n are two Top-IASI then their merging as G= $P_m * S_n$ also admits topological integer additive set-indexing (Top-IASI).

Proof:- Let $\{u_1, u_2, \ldots, u_m\}$ and $\{v_1, v_2, \ldots, v_m, v\}$ are the vertices set of P_n and S_m graphs respectively. Without loss of generality, we can assume that u_m vertex of P_m is merge with the central vertex v of S_n. If m=1,2 then $P_1 * S_n$ and $P_2 * S_n$ becomes S_n and S_{n+1} star graphs respectively for any arbitrary value of n. Hence, the labeling will be same as in Theorem 3.1. The labeling corresponding to the remaining values of m will be defined as:

Let f : $V(P_m * S_n) \to P(X) - \{\emptyset\}$ be defined as:-

$f(u_1) = X$, $f(u_2) = \{0\}$, $f(u_3) = \{1\}$

Case 1:- If m=3 and n has any arbitrary value.

$$f(v_j) = \begin{cases} \{0,1,\ldots,j\}, & 1 \le j \le \left\lceil \frac{n}{2} \right\rceil + 1. \\ \left\{1,2,\ldots,\left(j-\frac{n}{2}\right)\right\}, & \left\lceil \frac{n}{2} \right\rceil + 1 \le j \le n \text{ and } n \text{ is even.} \\ \left\{1,2,\ldots,\left(j-\frac{n-1}{2}\right)\right\}, & \left\lceil \frac{n}{2} \right\rceil + 1 \le j \le n \text{ and } n \text{ is odd.} \end{cases}$$

Case 2:- If m ≥ 4 and n has any arbitrary value.

$f(u_i) = \{0,1,\ldots,i-3\}$, $4 \le i \le m$.

Case (a):- If m is even.

$$f(v_j) = \begin{cases} \{1,2,\ldots,j+1\} & 1 \le j \le \left\lceil \frac{n+m}{2} \right\rceil - 2. \\ \left\{0,1,2,\ldots,\left(j-\left\lceil\frac{n}{2}\right\rceil+\left(\frac{m-1}{2}\right)\right)\right\} & \left\lceil \frac{n+m}{2} \right\rceil - 1 \le j \le n. \end{cases}$$

Case (b):- If m is odd.

$$f(v_j) = \begin{cases} \{1,\ldots,j+1\}, & 1 \le j \le \left\lceil \frac{n+m}{2} \right\rceil - 2. \\ \left\{0,1,2,\ldots,\left(j+\left\lceil\frac{m-n}{2}\right\rceil\right)\right\}, & \left\lceil \frac{n+m}{2} \right\rceil - 1 \le j \le n \text{ and } n \text{ is even.} \\ \left\{0,1,2,\ldots,\left(j+\left\lceil\frac{m-n}{2}\right\rceil-1\right)\right\}, & \left\lceil \frac{n+m}{2} \right\rceil - 1 \le j \le n \text{ and } n \text{ is odd.} \end{cases}$$

with corresponding ground set X and the labeling for edges of the graph are:

Let $f^+: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as

$f^+(u_i u_j) = f(u_i) + f(u_j)$, for all $u_i u_j \in E(G)$. Then τ= f (V (G)) ∪ {ϕ } forms a topology. Hence, $P_m * S_n$ admits topological integer additive set-indexing (Top-IASI).

Illustration 3.8:- Show $P_3 * S_7$ admits Topological Integer Additive Set-Indexing.

Solution:- After comparing it with Theorem 3.7, m=3 and n=7. The labeling for the vertices of P_3 will be as: $f(u_1) = X$, $f(u_2) = \{0\}$, $f(u_3) = \{1\}$ and for labeling to the vertices of S_7, we will take case 2(b) of above Theorem 3.7. The labeling for vertices of S_7 are as:

If $f : V(S_7) \to P(X) - \{\emptyset\}$ then

$$f(v_j) = \begin{cases} \{1,\ldots,j+1\}, & 1 \le j \le 3. \\ \{0,1,2,\ldots,(j-3)\}, & 4 \le j \le 7\ . \end{cases}$$

Hence, labeling defined in this way with the ground set X forms a topology. Hence, the graph $P_3 * S_7$ admits Top-IASI.

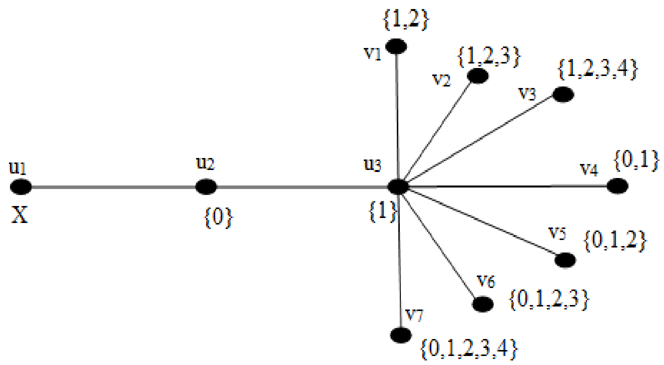


Figure 3: Shows $P_3 * S_7$ as Top-IASI.

Theorem3.9:- $H_{n,n}$ graph forms a topological integer additive set-indexer.

Proof:- Let (X, Y) be the vertices set of G= $H_{n,n}$ with { $v_1, v_2,\ldots v_n$ } and { $u_1, u_2,\ldots u_n$ } vertices respectively. Here, X and Y sets are disjoint and degree of corresponding vertices are increase by 1. So, $deg(v_1)= deg(u_1)= 1$ followed by $deg(v_2) = deg(u_2)= 2,..,deg(v_n) = deg(u_n)= n$. If n=1 then it is a disconnected graph with two isolated vertex. we can assign one vertex as {0} and other as X*={0,1}. If n=2 then assign $u_1= \{0\}$, $u_2=\{1\}$, $v_1= \{0,1\}$ and $v_2=$ X* to the vertices of $H_{2,2}$. For n=3 the labeling will be as follows:

Now, W.L.O. generality we can assume the ground set X*={0,1,2,..,n} is assign to pendant vertex v_n. Now, the labeling for the remaining vertices will be as follows:

Let $f : V(H_{n,n}) \to P(X^*) - \{\emptyset\}$ be defined as:

$f(v_1) = \{1\}$, $f(v_2) = \{2\}$

$$f(v_i) = \begin{cases} \{0,1,\ldots,i-1\}, & 3 \le i \le n-1. \\ \{0,1,2,\ldots,i\}, & i = n\ . \end{cases}$$

and for the remaining vertices labeling are as:

$f(u_1) = \{0\}$, $f(u_2) = \{0,1\}$, $f(u_3)= \{0,2\}$

$f(u_i) = \{1,2,\ldots,n-2\}$ $4 \le i \le n$ and the labeling corresponding to the edges are as:

Let $f^+: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as

$f^+(u_i u_j) = f(u_i) + f(u_j)$ for all $u_i u_j \in E(G)$. Then τ= f (V (G)) ∪ {ϕ } forms a topology.

Hence, graph $H_{n,n}$ admits Top- IASI.

Illustration 3.10:- Show $H_{5,5}$ admits Top-IASI.

Solution:- After comparing it with Theorem 3.9, n=4. So, X*= {0,1,2,3,4} will be taken as the ground set and remaining labeling will be as follows:

Let $f : V(H_{4,4}) \to P(X^*) - \{\emptyset\}$ be defined as:

$f(v_1) = \{1\}$, $f(v_2) = \{2\}$, $f(u_1) = \{0\}$, $f(u_2) = \{0,1\}$, $f(u_3)= \{0,2\}$

$$f(v_i) = \begin{cases} \{0,1,\ldots,i-1\}, & i = 3 \\ \{0,1,2,\ldots,i\}, & i = 4. \end{cases}$$ and $f(u_i) = \{1,2,\ldots,n-2\}$ $i = 4$

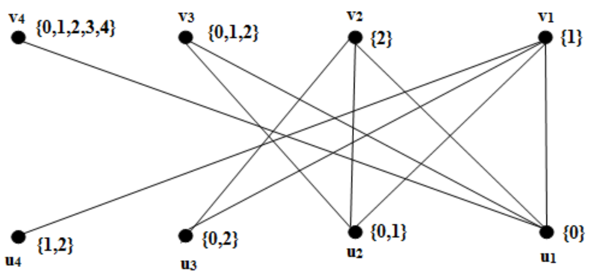


Figure 4: Shows the admissibility of Top-IASI for the graph $H_{4,4}$.

Theorem 3.11:- Every bipartite graph $B_{2,m}$ admits topological integer additive set-indexer.

Proof:- Let G= $B_{2,m}$ be a bipartite graph with two non-empty set of vertices X and Y where $\{u_1, u_2\}$ is the vertex set of X and $\{v_1, v_2, \ldots, v_m\}$ is the vertex set of Y. Since it is a bipartite graph so all the vertices of set X is not adjacent with all the vertices of set Y. So, there exists at least on vertex in the set Y having the degree 1. So, the ground set X* should be assigned to this vertex say u_n and {0} is assigned to the adjacent vertex of u_1 and the labeling corresponding to the vertices are as:

Let f : V($B_{2,m}$) → P(X*)- {∅} be defined as:

$f(u_1) = \{0\}$, $f(u_2) = \{1\}$, $f(v_1)= \{0,1\}$

$$f(v_i) = \begin{cases} \{1,2,\ldots,i\} & 2 \le i \le \left[\frac{m}{2}\right] \\ \left\{0,1,\ldots,\left(i-\left[\frac{m}{2}\right]+1\right)\right\} & \left[\frac{m}{2}\right] < i \le m \end{cases}$$

Here, ground set X* $=\{0,1,2,\ldots,\frac{m+2}{2}\}$ as m is even and X* $=\{0,1,2,\ldots,\frac{m+3}{2}\}$ as m is odd. The labeling corresponding to the edges $f^{+}: E(G) \to P(X) - \{\emptyset, \{0\}\}$ be defined as $f^{+}(u_i u_j) = f(u_i) + f(u_j)$ for all $u_i u_j \in$ E(G). Then τ= f (V (G)) ∪ {ϕ } forms a topology. Hence, graph $B_{2,m}$ admits Top-IASI.

Illustration 3.12:-. $B_{2,5}$ reveals Top-IASI.

Solution:- If we replace m by 5 in Theorem 3.11 then that becomes the solution for this. Here, m=5 (odd) so, the ground set X*= {0,1,2,3,4} and the labeling will be as follows:

f : V($B_{2,5}$) → P(X*)- {∅} be defined as:

$f(u_1) = \{0\}$, $f(u_2) = \{1\}$, $f(v_1)= \{0,1\}$

$$f(v_i) = \begin{cases} \{1,2,\ldots,i\} & i = 2 \\ \{0,1,\ldots,(i-1)\} & 2 < i \le 5 \end{cases}$$

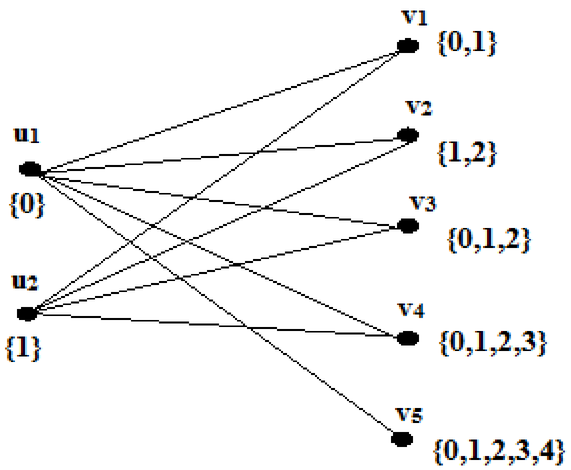


Figure 5: Shows the admissibility of Top-SIASI for the bipartite graph $B_{2,6}$.

Theorem 3.13:- A wheel graph W_n does not reveals a Top-IASI.

Proof: Let W_n be a wheel graph with vertices set{ $u_1, u_2, \ldots, u_n$}. Here, no vertex of W_n is pendant. Hence ground set X cannot be assigned to any vertex . For topological integer additive set-indexing, f(V(W_n)) ∪ {ϕ } forms a topology with ground set X*. But here, it is not possible. Hence , W_n does not reveals a Top-IASI.

Theorem 3.14:- A P_2+ $\overline{K_m}$ graph does not reveals a topological integer additive set- indexing.

Proof: Let P_2+ $\overline{K_m}$ be a graph with vertices set{ u_1, u_2} and { $v_1, v_2, \ldots, u_m$} respectively. For topological integer additive set-indexing, the ground set X must be assigned as the set label to any of the vertex. But it does not have any pendant vertex. So, the ground set X cannot be assigned to any of the, vertex among the graph. So, its vertices does not form topology.

CONCLUSION

In this paper, we defined the Top-IASI for some graph classes. Here, we also characterized that a subset of a topological integer additive set-indexed graph need not prevail the same characteristic. During defining the Top-IASI for the graph, the concept of minimum cardinality is also kept in mind. Some area which needs further study are:

a) To find the Top-IASI for graph operations.

b) To find the necessary and sufficient conditions for various graphs those have Top-IASI.

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