

A Note on Intuitionistic Fuzzy Set on the Basis of Reference Function

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Abstract

In this paper, an attempt has been made to present intuitionistic fuzzy set on the basis of reference function. It has been tried to see characteristic of intuitionistic fuzzy set by applying reference function.

Keywords: Membership Function, Reference function, Membership value.

INTRODUCTION

Fuzzy set theory was discovered by Zadeh [1] in 1965. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. Atanassov [2] added a new direction to the fuzzy algebra by introducing the concept of intuitionistic fuzzy set. Many authors have given different definitions on fuzzy set. But unfortunately it has been accepted that for fuzzy set A and its complement A^c , neither $A \cap A^c$ is empty set nor $A \cup A^c$ is the universal set. Whereas the operations of union and intersection of crisp sets are indeed special cases of the corresponding operation of two fuzzy sets, they end up giving peculiar results while defining $A \cap A^c$ and $A \cup A^c$.

In this regard, Baruah [3], forwarded an extended definition of complement of fuzzy sets which enable us to define complement of fuzzy sets in a way that give us $A \cap A^c$ is empty and $A \cup A^c$ is universal set.

It is intended to discuss fuzzy set on the basis of reference function in the later sections.

PRELIMINARIES

Intuitionistic fuzzy sets: Let a set X be fixed. An IFS A in X is an object of the following form :

$$A = \{ x, \mu_A(x), \gamma_A(x) ; x \in X \}$$

When $\gamma_A(x) = 1 - \mu_A(x)$ for all $x \in X$ is ordinary fuzzy set.

Also for each IFS A in X, if $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$, Then $\pi_A(x)$ is called the degree of indeterminacy of x to A, or called the degree of hesitancy of x to A.

Some basic operation on Intuitionistic Fuzzy sets

- i. $A \leq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$
- ii. $A = B$ iff $A \subseteq B$ and $B \supseteq A$
- iii. $A^c = \{ x, \gamma_A(x), \mu_A(x); x \in X \}$
- iv. $A \cup B = \{ x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\} \}$
- v. $A \cap B = \{ x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\} \}$

Now let us discuss some preliminaries of fuzzy set on the basis of reference function

Definition

Baruah [3] has given the definition of fuzzy set from a new perspective. According to this definition, to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x); x \in X\}$, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

Some basic operations on fuzzy sets on the basis of reference function

Let $A = \{x, \mu_1(x), \mu_2(x); x \in X\}$ and $B = \{x, \mu_3(x), \mu_4(x); x \in X\}$ be two fuzzy sets defined over the same universe X.

- a. $A \subseteq B$ iff $\mu_1(x) \leq \mu_3(x)$ and $\mu_4(x) \leq \mu_2(x)$ for all $x \in X$.
- b. $A \cup B = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x))\}$ for all $x \in X$.
- c. $A \cap B = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x))\}$ for all $x \in X$.

If for some $x \in U$, $\min(\mu_1(x), \mu_3(x)) \leq \max(\mu_2(x), \mu_4(x))$, then our conclusion will be $A \cap B = \phi$.

- d. $A^c = \{x, \mu_1(x), \mu_2(x); x \in X\}^c$
 $= \{x, \mu_2(x), 0; x \in X\} \cup \{x, 1, \mu_1(x); x \in X\}$

- e. If $D = \{x, \mu(x), 0; x \in X\}$ then $D^C = \{x, 1, \mu(x); x \in X\}$ for all $x \in X$.

Now, the intuitionistic fuzzy set may be defined on the basis of reference function in the following way.

DEFINITION

Let X be a fixed set. An intuitionistic fuzzy set A on the basis of reference function is a form

$A = \langle x, (\mu_A(x), 0), (1, \mu_A(x)); x \in X \rangle$, Where $0 \leq \mu_A(x) \leq 1$, $\mu(x) = \mu_A(x) - 0$ is membership value which is taken from 0 to $\mu_A(x)$ here $\mu_A(x)$ is known as membership function and 0 is reference function of the membership value and similarly $\gamma(x) = 1 - \mu_A(x)$ is non membership value which is taken from $\mu_A(x)$ to 1.

APPLICATION OF INTUITIONISTIC FUZZY SET ON THE BASIS OF REFERENCE FUNCTION

Let $A = \langle (\mu_A(x), 0), (1, \mu_A(x)) \rangle$ and $B = \langle (1, \mu_A(x)), (\mu_A(x), 0) \rangle$ be two intuitionistic fuzzy set on the basis of reference function.

Now let us see

$$\begin{aligned} A \cap B &= \langle (\mu_A(x), 0), (1, \mu_A(x)) \rangle \cap \langle (1, \mu_A(x)), (\mu_A(x), 0) \rangle \\ &= \langle (\min\{\mu_A(x), 0\}, (1, \mu_A(x))), \max\{(1, \mu_A(x)), (\mu_A(x), 0)\} \rangle \\ &= \langle (\mu_A(x), \mu_A(x)), (1, 0) \rangle \end{aligned}$$

Which is nothing but empty set.

Now

$$\begin{aligned} A \cup B &= \langle (\mu_A(x), 0), (1, \mu_A(x)) \rangle \cup \langle (1, \mu_A(x)), (\mu_A(x), 0) \rangle \\ &= \langle (\max\{\mu_A(x), 0\}, (1, \mu_A(x))), \min\{(1, \mu_A(x)), (\mu_A(x), 0)\} \rangle \\ &= \langle (1, 0), (\mu_A(x), \mu_A(x)) \rangle \end{aligned}$$

Which is universal set.

From the intersection and union of two intuitionistic fuzzy set A and B on the basis of reference function it is clearly seen that B is complement of A and hence $B = A^C$.

CONCLUSION

It may be concluded that for an intuitionistic fuzzy set A , $A \cap A^C$ is empty and $A \cup A^C$ is universal set when we use intuitionistic fuzzy set on the basis of reference function. Here $\mu(x)$ is taken from 0 to $\mu_A(x)$ and $\mu_A(x)$ is known as membership function and 0 is reference function and their difference is known as membership value. The actual meaning of this representation is that membership value is taken from 0 to $\mu_A(x)$. Similarly for non membership value that is $\gamma(x)$ is taken from $\mu_A(x)$ to 1 and their difference is known as non membership value. The authors expect that this paper would throw some new light on the development of intuitionistic fuzzy set.

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