$Y_6 = -\pi^2 \alpha Y_1,$

$Y_7 = \pi^2 \alpha N_5 - \pi^2 \alpha Y_2,$

$Y_8 = -\frac{Y_3}{[1+\varepsilon Cos[\Omega t]]},$

$Y_9 = \frac{-Y_4 - (k^2 - Ri)Y_3}{[1+\varepsilon Cos[\Omega t]]},$

$Y_{10} = \frac{-Y_3 \varepsilon \Omega}{[1+\varepsilon Cos[\Omega t]]^2},$

$Y_{11} = \frac{-Y_5 - (k^2 - Ri)Y_4}{[1+\varepsilon Cos[\Omega t]]},$

$Y_{12} = \frac{-Y_4 \varepsilon \Omega}{[1+\varepsilon Cos[\Omega t]]^2},$

$Y_{13} = \frac{\partial T_0}{\partial z} N_5 \pi \alpha + \frac{\partial T_0}{\partial z} \pi \alpha Y_1,$

$Y_{14} = \frac{-Y_5 \varepsilon \Omega}{[1+\varepsilon Cos[\Omega t]]^2},$

$Y_{15} = \frac{-(k^2 - Ri)Y_5}{[1+\varepsilon Cos[\Omega t]]},$

$Y_{16} = -\frac{\partial T_0}{\partial z} N_5 \pi \alpha + \frac{\partial T_0}{\partial z} \pi \alpha Y_2$

Substituting equations (38), (41) and (42) in equation (37), we get a third order equation in B after neglecting the terms of the type $\frac{\partial^4 B}{\partial t^4}, \frac{\partial^3 B}{\partial t^3}, \frac{\partial^2 B}{\partial t^2}, B\left(\frac{\partial B}{\partial t}\right)^2, \left(\frac{\partial B}{\partial t}\right)^2, B^2 \frac{\partial B}{\partial t}, B\frac{\partial^3 B}{\partial t^3}, B\frac{\partial^2 B}{\partial t^2}.$

$$P_1 \frac{\partial B}{\partial t} = P_2 B - P_3 B^3, \tag{43}$$

where , $P_1 = M1 + M4 + M5 + M8 + M9$,

$M1 = [1 + \varepsilon \text{Cos}[\Omega t]\ [\text{M2}\ Cos[\Omega t] + \text{M3 Sin}[\Omega t]],$

$M2 = 2Y_{14}Y_6\varepsilon + 2Y_{12}Y_7\Omega$,

$M3 = 2Y_{11}Y_7\Omega\varepsilon + 2(4\pi^2 - Ri)Y_{14}Y_6 + 2(4\pi^2 - Ri)Y_{12}Y_7,$

$M4 = 2(4\pi^2 - Ri)[1 + \varepsilon\,\text{Cos}(\Omega t)]^2\ [Y_{15}Y_6 + Y_{16}Y_6]$,

$M5 = [1 + \varepsilon\,\text{Cos}(\Omega t)]^3\ [M6\text{Sin}[\Omega t] + 2(4\pi^2 - Ri)\text{M7}]$,

$\text{M6} = 2Y_{15}Y_6\varepsilon\Omega$,

$M7 = Y_{16}Y_6 + Y_{13}Y_7,$

$\text{M8} = 2(4\pi^2 - Ri)[Y_{15}Y_6 + Y_{11}Y_7][1 + \varepsilon\,\text{Cos}(\Omega t)]^2$,

$\text{M9} = [4Y_{14}Y_6\varepsilon\Omega + 4Y_{12}Y_7\varepsilon\Omega]\text{Sin}^2[\Omega t]$,

$P_2 = -2(4\pi^2 - Ri)[Y_{15}Y_7[1 + \varepsilon\,\text{Cos}(\Omega t)]^2 + Y_{16}Y_7[1 + \varepsilon\,\text{Cos}(\Omega t)]^3] - 4Y_{14}Y_7\varepsilon\Omega\text{Sin}^2[\Omega t] + M10$,

$M10 = [1 + \varepsilon\,\text{Cos}(\Omega t)][-2(4\pi^2 - Ri)Y_{14}Y_7 Sin[\Omega t] - 2Y_{14}Y_7\Omega Cos[\Omega t] - 2Y_{15}Y_7\varepsilon\Omega\text{Sin}[\Omega t],$

$P_3 = [Y_2 - N_5]\pi^2\alpha Y_5 Y_7^2[1 + \varepsilon\,\text{Cos}(\Omega t)]^2]$.

Equation (43) is obviously the Ginzburg-Landau equation for non-linear convection in a micropolar fluid with vertical oscillation and heat source. By using equation (43) in equation (42), we get *F*.

Heat Transport

The influence of gravity modulation with internal heat source on heat transport which is quantified in terms of Nusselt number (Nu) is defined as follows:

$$Nu = \frac{Heat\ transport\ by\ (conduction + convection)}{Heat\ transport\ by\ (conduction)}$$

$$= \frac{\left[\frac{k}{2\pi}\int_0^{2\pi/k}(1 - z + T)_z\,dx\right]_{z=0}}{\left[\frac{k}{2\pi}\int_0^{2\pi/k}(1 - z)_z\,dx\right]_{z=0}} , \tag{44}$$

where, subscript in the integrand denotes the derivative with respect to z.

Substituting equation (33) in equation (44) and completing the integration, we get

$Nu = 1 - 2\pi F(t).$

RESULTS AND DISCUSSIONS

In the study of thermal instability in a fluid layer, external regulation of convection is important. In this paper, time depended vertical oscillation (gravity modulation) is considered as an external force for convection. The non-linear analysis is carried out in order to study the effects of vertical oscillations and internal heating on heat transport in a micropolar fluid. The fourth ordered non-autonomous Lorenz model is obtained by using truncated Fourier series. Ginzburg-Landau equation is then derived from Lorenz model. The effect of Coupling Parameter N_1, Inertia Parameter N_2, Couple Stress Parameter N_3, Micropolar Heat Conduction Parameter N_5, Prandtl number Pr , internal Rayleigh number Ri, amplitude ε and frequency Ω of the gravity modulation are analyzed and the results are depicted in figures (2)-(9).

From the figures, we oberve that for small time, Nusselt number, Nu is less than one, which indicates that the heat transfer is due to conduction, as time increases, Nu also increases and becomes greater than one, which shows that convective region is in place. Further increase in time, the graphs remains oscillatory which means that early chaos is precipated.

From figures (2)- (9), we observe that

(i) Increase in internal Rayleigh number increases the amount of heat transfer.

(ii) increase in N_1 , decrease the rate of heat transfer and thus stabilizes the system. This is on account of the presence of suspended particles, the critical Rayleigh number increases and hence the heat transport decreases.

(iii) increase in N_2, increases inertia of the fluid due to the suspended particles and then increases the amount of heat transfer.

(iv) increase in N_3 increases couple stress of the fluid and decreases gyrational velocities.

So, increases the measure of heat transfer.

(v) When N_5 increases, the heat induced into the fluid, due to this microelements, also increase, thus reduces the heat transfer.

(vi) Pr reduces the measure of heat transfer.

(vii) amplitude ε of the gravity modulation increase the measure of heat transfer

(viii) frequency Ω of the modulation decrease the measure of heat transfer

CONCLUSION

(i) By adding suspended particles into Newtonian fluids, heat transport decreases and thus stabilizes the system.

(ii) N_1, Ω and Pr decreases the rate of heat transfer.

(iii) $N_2, N_3, N_5, \varepsilon$ and Ri increases the rate of heat transfer.

(iv) Heat transport can be controlled effectively by gravity modulation.

ACKNOWLEDGEMENTS

Authors, S Pranesh and Vasudha Yekasi, would like to acknowledge the management of Christ University for their support in completing the work. Author, Shahnaz Bathul, would like to acknowledge the management of Jawaharlal Technological University for their support in completing the work.

REFERENCES

[1] Lukaszewicz G (1999) Micropolar fluids—theory and application, Birkhauser, Basle.

[2] Eringen AC (1966) Theory of micropolar fluids. Math Mech 16:1–18

[3] Sheremet MA, Pop I, Ishak A (2017) Time-dependent natural convection of micropolar fluid in a wavy triangular cavity. Int J Heat Mass Transf 105:610–622

[4] Miroshnichenko IV, Sheremet MA, Pop I (2017) Natural convection in a trapezoidal cavity filled with a micropolar fluid under the effect of a local heat source. Int J Mech Sci 120:182–189

[5] Dupuy D, Panasenko GP, Stavre R (2004) Asymptotic methods for micropolar fluids in a tube structure. Math Models Methods ApplSci 14:735–758

[6] Gibanov NS, Sheremet MA, Pop I (2016b) Natural convection of micropolar fluid in a wavy differentially heated cavity. J Mol Liq 221:518–525

[7] Boukrouche M, Paoli L (2012) Asymptotic analysis of a micropolar fluid flow in a thin domain with a free and rough boundary. SIAM J Math Anal 44:1211–1256

[8] Benes M, Pazanin I (2016) Effective flow of incompressible micropolar fluid through a system of thin pipes. Acta Appl Math 143:29–43

[9] P.G. Siddheshwar and S. Pranesh. "Effect of temperature/gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum." Int. J. Magn. Magn. Mater, 192 (1999), 159-176.

[10] PG Siddheshwar, S Pranesh," **Linear and weakly non-Linear analyses of convection in a micropolar fluid", Hydrodynamics VI-Theory and Applications, 2005.**

[11] S.Pranesh and Riya Baby, "Effect of Non-Uniform Temperature Gradient on the Onset of Rayleigh-Bénard Electro Convection in a Micropolar Fluid", Applied Mathematics, 2012, 3, 442-450

[12] S.Pranesh, Sameena Tarannum and T V Joseph "Linear and Weakly Non-Linear Analyses of Gravity Modulation and Electric Field on the Onset of Rayleigh-Bénard Convection in a Micropolar Fluid ",Journal of Advances in Mathematics Vol 9, No 3,(2014).

[13] S Pranesh, Vasudha Satri and Shahnaz Bathul (2016) Effects of Gravity Modulation and Internal Heat Generation on the onset of Rayleigh-Bénard convection in a Micropolar Fluid , Journal of Advances in Mathematics, 12, 6270-6285.

[14] S Pranesh, Vasudha Satri and Shahnaz Bathul (2017) Effects of Temperature Modulation and Internal Heat Generation on the onset of Rayleigh-Bénard convection in a Micropolar Fluid, Global Journal of Pure and Applied Mathematics, 13(6), 2411-2437

[15] E. S. Nelson, "An examination of anticipation of g-jitter on space station and its effects on materials processes," NASA Tech. Memo. 103775 (1991).

[16] M. Wadih and B. Roux, "Natural convection in a long vertical cylinder under gravity modulation, J. Fluid Mech. 193, 391 (1988).

[17] M. Wadih, N. Zahibo, and B. Roux, "Effect of gravity jitter on natural convection in a vertical cylinder," in Low Gravity Fluid Dynamics and Transport Phenomena, edited by J. N. Koster and R. L. Sani (AIAA, New York, 1990), pp. 309–354.

[18] G. Z. Gershuni, E. M. Zhukhovitskii, and I. S. Iurkov, "On convective stability in the presence of periodically varying parameter," J. Appl. Math. Mech. 34, 442 (1970).

[19] P. M. Gresho and R. L. Sani, "The effect of gravity modulation on the stability of a heated fluid layer," J. Fluid Mech. 40, 783 (1970).

[20] P.G. Siddheshwa, G N Sekhar and Jayalatha, "Effect of time-periodic vertical oscillations of the Rayleigh–Bénard system on nonlinear convection in viscoelastic liquids", J. Non-Newtonian Fluid Mech. 165 (2010) 1412–1418

[21] B.S. Bhadauria , I.Hashim , P.G.Siddheshwar ," Effect of internal-heating on weakly non-linear stability analysis of Rayleigh–Bénard convection under g-jitter", International Journal of Non-Linear Mechanics,54,(2013),35–42

[22] Tatyana Lyubimova and Kseniya Kovalevskaya "Gravity modulation effect on the onset of thermal buoyancy convection in a horizontal layer of the Oldroyd fluid", The Japan Society of Fluid Mechanics, Fluid Dyn. Res. 48 (2016) 061419 (14pp)

[23] Maria Thomas and Sangeetha George, "Effect of Gravity Modulation and Internal Heat Generation on Rayleigh-Bénard Convection in Couple Stress Fluid with Maxwell-Cattaneo Law", International Journal of Applied Engineering Research, Volume 13, Number 5 (2018) pp. 2688–2693

[24] D.L.Turcotte and G. Schubert(eds.), *Geodynamics* (Cambridge University Press, Cambridge, 2001).

[25] K. A. Emanuel (ed.), *Atmospheric Convection* (Oxford University Press New York, 1994).

[26] E. Bodenschatz, W. Pesch, and G. Ahlers, Annu. Rev. Fluid Mech. **32**, 709 (2000).

[27] S.P. Bhattacharya and S.K. Jena, "Thermal instability of a horizontal layer of micropolar fluid with heat source," Proc. Indian Acad. Sci., vol. 93(1), pp. 13–26,(1984).

[28] M. Takashima, "The stability of natural convection in an inclined fluid layer with internal heat generation," Journal of the Physical Society of Japan, vol.58, pp. 4431–4440, (1989).

[29] S Pranesh and Ritu Bawa " Effects of Magnetic Field and Internal Heat Generation on the onset of Rayleigh-Bénard Convection in Nanofluid", International Journal of Engineering Research & Technology (IJERT) ,Vol. 4 Issue 05, May-(2015).

[30] H J Priyanka, S. Pranesh," Effect of Internal Heat Generation on the Onset of Rayleigh-Bènard Chandrashekar Convection in a Micropolar Fluid with Maxwell Cattàneo Law", Recent Trends in Fluid Mechanics, Vol 5, No 1, (2018)

[31] P.G. Siddheshwar , R.K. Vanishree," Lorenz and Ginzburg-Landau equations for thermal convection in a high-porosity medium with heat source", Ain Shams Eng J (2016)

Figures:

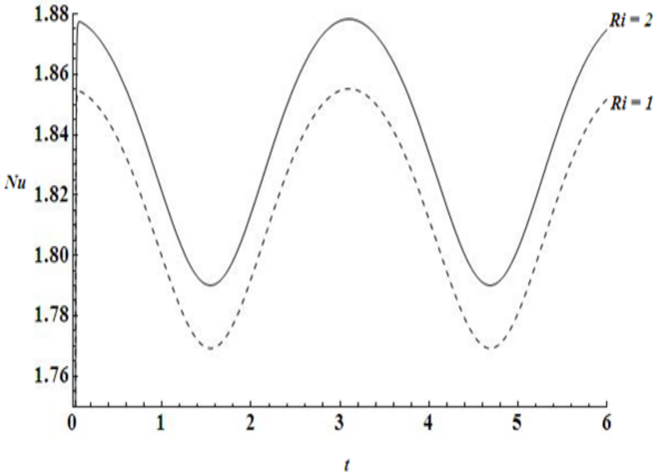

Figure 2: Plot of Nusselt number Nu versus time *t* for various values of internal Rayleigh number *Ri* for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2.$

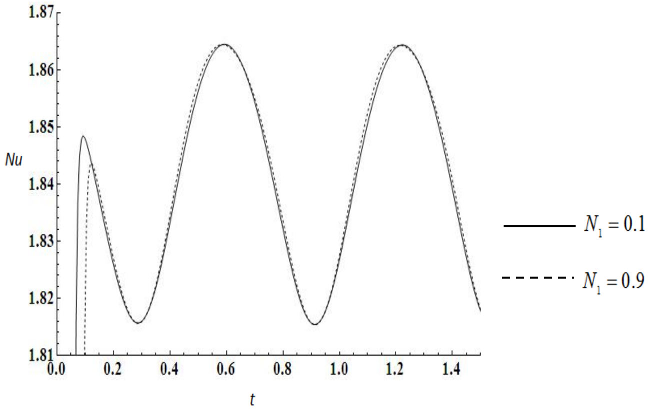

Figure 3: Plot of Nusselt number Nu versus time *t* for various values of Coupling Parameter N_1 for $N_2 = 0.5, N_3 = 2, N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2.$

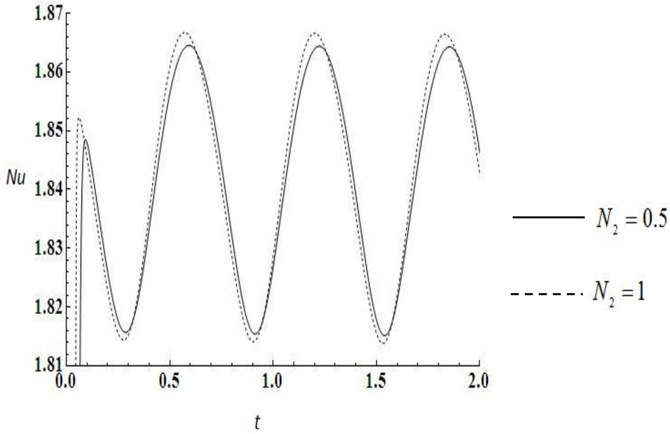

Figure 4: Plot of Nusselt number Nu versus time *t* for various values of Inertia Parameter N_2 for $N_1 = 0.1, N_3 = 2, N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2.$

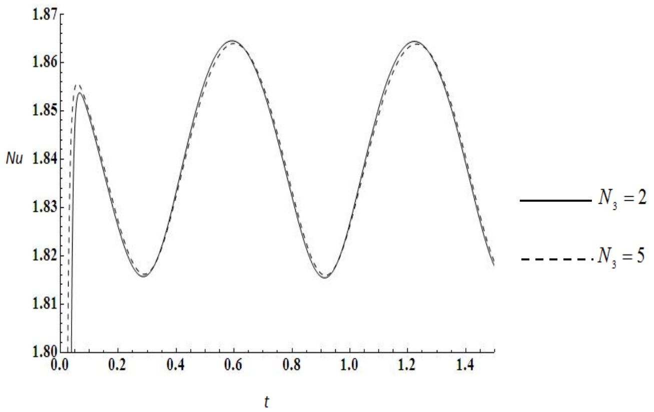

Figure 5: Plot of Nusselt number Nu versus time t for various values of Couple Stress Parameter N_3 for $N_1 = 0.1, N_2 = 0.5, N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

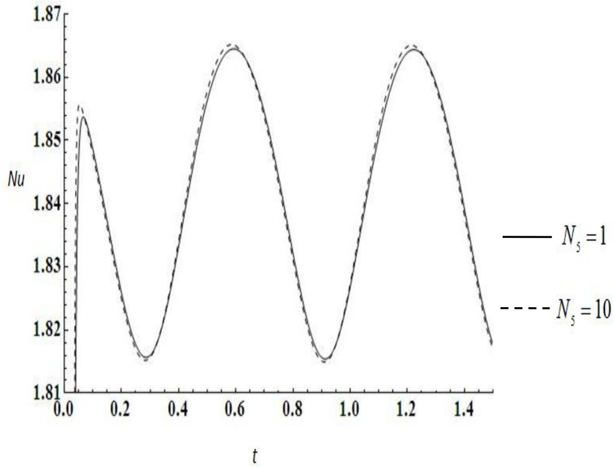

Figure 6: Plot of Nusselt number Nu versus time t for various values of Micropolar Heat Conduction Parameter N_5 for $N_1 = 0.1, N_3 = 2, N_2 = 0.5, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

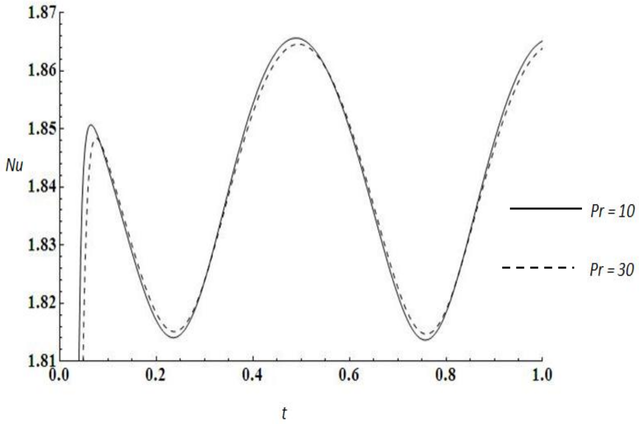

Figure 7: Plot of Nusselt number Nu versus time t for various values of Prandtl number Pr and internal Rayleigh number Ri for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

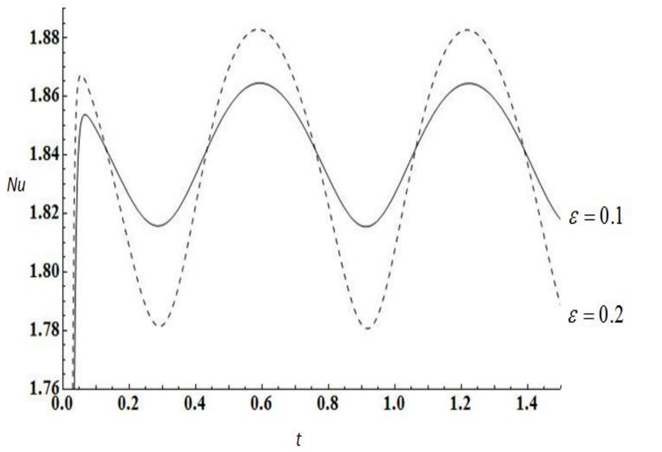

Figure 8: Plot of Nusselt number Nu versus time t for various values of amplitude of modulation ε for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, Pr = 10, \Omega = 2, Ri = 2$.

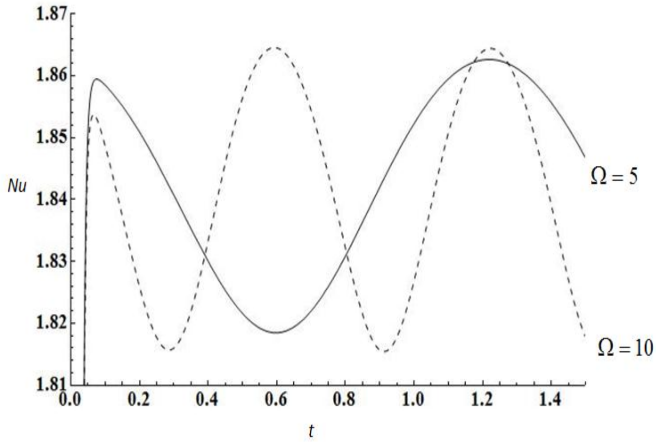

Figure 9: Plot of Nusselt number Nu versus time t for various values of frequency of modulation Ω for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, \varepsilon = 0.2, Ri = 2, , Pr = 10$.