International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 13 (2018) pp. 11140-11150

Modeling Contagion Financial Risk using Entropy Algorithm and Round by Round Model

Saâd Benbachir
Director of the Center of the Strategic Studies in Law, Economics and Management
FSJES-Agdal, University of Mohamed 5 of Rabat-Morocco.

Mohammed El Haddad
PhD, Director of the Laboratoy of the Studies and Researches in Management Sciences
FSJES-Agdal, University of Mohamed 5 of Rabat-Morocco.

Younes Benbachir
PhD Researcher, Laboratoy of the Studies and Researches in Management Sciences
FSJES-Agdal, University of Mohamed 5 of Rabat-Morocco.

Abstract
The purpose of this paper is to investigate the systemic risk through the contagion channel by first applying the maximum entropy algorithm in order to estimate the exposure matrix of the Moroccan interbank market and secondly by using the round by round model to simulate contagion process after stressing each bank in turn by idiosyncratic chock. The data used are aggregated loans and debts and the Tier-one of two years 2015 and 2016, extracted from the balance sheets of eight major Moroccan banks. The results obtained suggest that during the two years 2015 and 2016, the Moroccan banking system is quite resilient for pessimistic scenarios corresponding to very high loss given rates. The method used could be applied by the central bank with real bilateral exposure matrices. This would give more reliable results and provide a decision-making tool in the context of financial stability policy.

Keywords: Interbank market, Systemic risk, Contagion, Idiosyncratic shock, Resilience
JEL Classification Code: G10, G19, G21, G38, G39

INTRODUCTION

The concept of financial contagion is generally used to describe the spillover of the effects of shocks from one financial institution to others financial institutions. One of the prominent channel of contagion is the channel stemming from interbank markets. The insolvency of a bank due to exogenous shock may trigger multiple bank failures. Therefore, interbank exposures are considered as a channel of contagion where a shock on one bank may spread to other banks through domino effects. Recently, great attention has been paid to contagion through interbank markets, both by academics and policymakers.

Recently, Allen and Gale (2000) have proved that the propagation of shocks in the interbank market depends on the precise pattern of banks' linkages. They argue that complete structures, where there exists symmetrical linkages between all banks, are more resilient to contagion risk than incomplete structures, where banks are partially related to each other. Freixas et al. (2000) argue as well that complete structures improve the resilience of banks to contagion, while incomplete structures increase financial fragility of banks.

Many researchers have tried to study the impact of financial contagion on interbank markets. One of the main constraints encountered by researchers is the unavailability of bilateral interbank exposures. In order to bypass this problem several researches have adopted an approach based on the maximization of entropy of the matrix of bilateral lending (e.g. Upper and Worms, 2004, Blavarg and Nimander, 2002; Wells, 2004; Degryse and Nguyen, 2007; van Lelyveld and Liedorp, 2006) in order to estimate the exposure matrix of the interbank market. This approach assumes that interbank market has complete structure as defined by Allen and Gale (2000), i.e. each bank lends to all the other banks. Obviously this hypothesis can be in contradiction with the actual structure of the market. After obtaining an estimate of the exposure matrix describing the linkages between banks, most studies apply a simulation methodology which analyze the effects of contagion on the system after stressing each bank in turn by idiosyncratic chock and identifying the potential risk contagion on other banks linked to that bank after two rounds. This method is well known in literature as the round by round model.

In this paper, we investigate the contagion risk through Moroccan interbank market first by applying the maximum entropy algorithm in order to estimate the exposure matrix and secondly by using the round by round model to simulate contagion process in three rounds after stressing each bank in turn by idiosyncratic chock.

The rest of the paper is organized as follows. The next section gives a brief review of empirical literature concerning the maximum entropy algorithm and the round by round simulation model. Section three describes and characterizes the Moroccan interbank market and presents the data used in this work. In section four we explain first the methodology of maximization of the entropy of the exposure matrix and secondly we describe in detail the round by round simulation model. In section five, we present, interpret and compare the results obtained in our paper. The final section concludes our paper and summarizes the main findings.

EMPIRICAL LITERATURE REVIEW

Recently, several empirical studies have analyzed the risk of contagion in interbank markets in different countries.

Sheldon and Maurer (1998) try to assess the level of systemic risk in a Switzerland banking system on the basis of interbank loan structures. They construct a matrix of interbank loans for Switzerland based on known marginal loan distributions and the principle of entropy maximization. Their results suggest that the latent systemic risk associated with the interbank loan structure existing among Swiss banks in 1987-1995 posed little threat to the stability of the Swiss banking system.

Blavarg and Nimander (2002) analyze the Swedish interbank market during the period September 1999-September 2001. They deduce that contagion is rarely spread to the largest banks and the interbank contagion risk imported from abroad mainly stems from exchange rate exposures.

Furfine (2003) examines the likelihood that failure of one bank would cause the subsequent collapse of a large number of other banks in the federal funds interbank market. They use unique data on interbank payment flows to quantify the magnitude of bilateral federal funds exposures. After that, they simulate the impact of various failure scenarios and find that the risk of contagion is found to be economically small.

Upper and Worms (2004) use balance sheet information in order to estimate the matrix of bilateral credit relationships for the German banking system and they test whether the breakdown of a single bank can lead to contagion. They find that the financial safety net (institutional guarantees for saving banks and cooperative banks) considerably reduces, but does not eliminate, the danger of contagion.

Wells (2004) uses data on loans and deposits between UK-resident banks to estimate the distribution of bilateral exposures. He examines the potential for contagion by assuming the sudden failure of each individual bank and estimating the losses incurred to other banks as a result of the initial shock. His study suggests that, while a single bank failure is rarely sufficient to trigger the outright failure of other banks, it does have the potential to weaken substantially the capital holdings of the banking system. He also conclude that the severity of contagion risk depends greatly on the maintained assumptions about the distribution of interbank loans and the level of loss given default.

Amundsen and Arnt (2005) use records of payments in the Danish large value payment system to compute a data set on bilateral exposures between banks. Using this data set, they analyze subsequently the risk of contagion in the Danish interbank market. They find that the risk of financial contagion due to an unexpected failure of a major bank is very limited. This is true even when the loss given default rate is assumed equal to 100 per cent. They find also that when contagion is identified it affects only smaller banks.

Mistrulli (2005,2011) analyzes how contagion propagates within the Italian interbank market using a unique data set including actual bilateral exposures. He shows that the maximum entropy method, which estimates the exposure bilateral matrix of the interbank market, tends to underestimate the extent of contagion. He also proves that under certain circumstances, depending on the structure of the interbank linkages, the recovery rates of interbank exposures and banks' capitalization, the maximum entropy approach overestimates the scope for contagion.

Van Lelyveld and Liedorp (2006) investigate interlinkages and contagion risks in the Dutch interbank market. They first estimate the exposures in the interbank market at bank level. Next, they perform a scenario analysis to measure contagion risks. They find that the bankruptcy of one of the large banks will put a considerable burden on the other banks but will not lead to a complete collapse of the interbank market.

Degryse and Nguyen (2007, 2011) investigate the evolution and determinants of contagion risk for the Belgian banking system over the period 1993–2002 using detailed information on aggregate interbank exposures of individual banks, large bilateral interbank exposures, and cross-border interbank exposures. They find that the "structure" of the interbank market affects contagion risk. They show that a change from a complete structure (where all banks have symmetric links) toward a "multiple money-center" structure (where money centers are symmetrically linked to otherwise disconnected banks) has decreased the risk and impact of contagion.

Toivanen (2009) applies the maximum entropy method in order to estimate the exposures matrix of the Finnish interbank market in 2005-2007. He analyzes the existence of contagion during a Finnish banking crisis. The results suggest that five of ten deposit banks are possible starting points for contagious effects. The author finds that the magnitude of contagion is conditional on the first failing bank. He also finds that middle-sized banks and large commercial banks cause damaging domino effects. However, he concludes that the contagion is currently a low probability event in the Finnish interbank market.

Krznar (2009) explores systemic risk in the Croatian interbank market. He describes the Croatian interbank market as a small market behaving as a multiple money center structure, with bilateral exposures concentrated on a few big banks. In order to assess contagion risk in the Croatian banking system, the author performs simulations of idiosyncratic bank failures and macroeconomic shocks. His conclusion leads to the unlikelihood of the occurrence of bank contagion due to an idiosyncratic failure according to the hard definition of insolvency. He also obtains that bank contagion stemming from macroeconomic shocks could only materialize in highly improbable scenarios.

MOROCCAN INTERBANK MARKET AND DATA

Moroccan interbank market

The interbank market is a market where banks exchange short-term assets (borrowing or lending), and where the central bank also intervenes to provide or regain liquidity. It is therefore also the market that allows the Central Bank to compensate the balance sheet of banks in the event of a liquidity crisis.

The interbank market allows banks to balance their cash flow by offsetting deficits or surpluses among themselves or by borrowing the additional sums they need from the various specialized banks.

But before using the interbank market for their cash flow needs, banks generally trade off the different refinancing options available to them in order to optimize the use of the financial resources provided by these options, starting with those that offer the lowest interest rates. Prior to the refinancing reform of

June 1995, Moroccan banks were able to obtain advances either on the interbank market or by re-discounting from Bank Al Maghrib preferential loans at low interest rates or by the various advances that banks made to them on the money market.

Since the introduction of this banking reform, only recourse to the money market has remained, where interbank transactions have developed because they offer more advantageous rates than those of Bank Al Maghrib, while following, both upwards and downwards, the evolution of the key interest rates of the central bank. The interbank market has three functions:

(a) The redistribution among banks themselves of their liquidity deficits and surpluses;

(b) The adjustments of the treasury structures between banks;

(c) The regulation of bank liquidity by the central bank.

The Moroccan banking sector consists of 19 banks divided into two subsectors, the first include 14 general banks (commercial banks) and the second is composed by 5 specialized banks. The State is the majority shareholder in the capital of 6 banks since 2009 and of 4 specialized banks (public banks), while foreign shareholder holds the majority in 7 institutions (since 2008). The following table shows the 8 main retail banks in Morocco.

Table 1: 8 Major Banks of Morocco

Bank Name	Location	Owner
Attijariwafa Bank	Casablanca	SNI Holding - Morocco
Banque Populaire du Maroc	Casablanca	BCP - Morocco
BMCE Bank	Casablanca	Finance Com - Morocco
Société Générale Maroc	Casablanca	Société Générale - France
BMCI	Casablanca	BNP Paribas - France
Crédit Agricole du Maroc	Rabat	State of Morocco
Crédit du Maroc	Casablanca	Crédit agricole - France
CIH Bank	Casablanca	CDG Group - Morocco

Following numerous privatizations during the 1990s, the Moroccan financial ecosystem underwent a major transformation, which resulted in an increase in the rate of banking, a multiplication of bank branches and the rapid development of bank payment methods. In addition, the advent of a highly capillary postal bank, the revitalization of the role of agricultural credit and the emergence of regional banks since the advanced regionalization process have made it possible to reach the most modest urban and rural social strata.

The merger of two private banks Wafa Bank and Attijari Bank in 2003 gave birth to a large Moroccan banking group called AttijariWafa Bank (AWB) which has been able to conquer the African continent for the last ten years. The latter group was joined by the two Moroccan banks Banque Centrale Populaire (BCP) and BMCE Bank. Since the beginning of the last decade, these last three banks together account for about two thirds of market share in Morocco. AttijariWafa Bank is present in 14 African countries and continues to proliferate.

On the other hand, the evolution of the economy and financial jurisdiction in Morocco has earned the confidence of several foreign investors who have taken shares in the capital of some Moroccan banks. On the African continent as a whole, Moroccan banks are particularly dynamic in their efforts to increase the banking rate of local populations. Since 2006, the first three Moroccan banks have opened more than half of all bank branches created in the UEMOA (West African Economic and Monetary Union). At the same time as the emergence of free zones during the last two decades, offshore banks have emerged and find themselves in a valuable support for the development of offshore industries, especially in Tangiers. Since the launch of Casablanca Finance City (CFC), a large number of new offshore banks have directed their African activities within this financial center.

Data

In this paper, we apply the RAS algorithm and the Round by Round model to estimate first the exposures interbank matrices for the 8 Moroccan major banks within two years 2015 and 2016.

As the bilateral exposure interlinkages between the 8 banks are unavailable for confidentiality reasons, we extract only aggregated loans and debts from the balance sheet of the 8 banks. We also calculate the Tier one capital for the 8 banks using the definition given by the central bank (Bank al Maghrib) concerning the Bank's Tier one (Circular of Bank Al Maghrib relating to the credit institutions' own funds, August 2013). We calculate the tier one of the 8 major banks using the following formula :

$$Tier\ one = Elements\ to\ add - Elements\ to\ substract$$

The elements to add in the calculation of the tier one, given in the article 8 of the circular of Bank Al Maghrib relating to the credit institutions' own funds, are:

- The share premium, the capital contribution and merger premium;
- The reserves;
- The credit retained earnings;

The elements to substract in the calculation of the tier one, given in the article 9 of the circular of Bank Al Maghrib relating to the credit institutions' own funds, are:

- The formation expenses and intangible assets net of amortization and provisions for depreciation;
- The debit retained earnings;
- The amount of equity participation held in the form of equity instruments issued by credit institutions or similar institutions and insurance or reinsurance companies;
- The amount of specific shares held in securitization collective funds;

The tables below show the loans, the debts and the tier one of the 8 banks studied in this paper through the years 2015 and 2016. The data are given in thousands of dirhams.

Table 2: Aggregated loans of the 8 major Moroccan Banks in 2015 and 2016

	B1	B2	B3	B4	B5	B6	B7	B8
Loans 2015	4594160	12389704	2539075	1293707	3367920	775827	1976062	3154193
Loans 2016	7722593	4627122	6557474	164041	1241572	545357	129900	103653

Table 3: Aggregated debts of the 8 major Moroccan Banks in years 2015 and 2016

	B1	B2	B3	B4	B5	B6	B7	B8
Debts 2015	8235242	3271277	12753126	259760	2332143	780326	2252507	206268
Debts 2016	724495	7382993	9700705	306113	1860348	451691	585939	79428

Table 4: Tier-one of the 8 major Moroccan Banks in 2015 and 2016

	B1	B2	B3	B4	B5	B6	B7	B8
Tier-1 2015	16046794	3205538	6386829	7464449	3574027	2723178	5408922	5990873
Tier-1 2016	20637127	3139551	6727575	7313829	3693470	2619853	5255437	6371924

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 13 (2018) pp. 11140-11150
© Research India Publications. http://www.ripublication.com

METHODOLOGY

Description of the RAS algorithm

Let $X = (x_{ij})_{\substack{1\le i\le 8\\1\le j\le 8}}$ be the bilateral exposures matrix of order 8 representing the lending linkages between the 8 banks.

$$
\begin{array}{c|c|c}
 & & a_i = \sum_{j=1}^{8} x_{ij} \\
\hline
 & X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{17} & x_{18} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{27} & x_{28} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{i7} & x_{i8} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ x_{71} & x_{72} & \cdots & x_{7j} & \cdots & x_{77} & x_{78} \\ x_{81} & x_{82} & \cdots & x_{8j} & \cdots & x_{87} & x_{88} \end{pmatrix} & \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_i \\ \vdots \\ a_7 \\ a_8 \end{matrix} \\
\hline
l_j = \sum_{i=1}^{8} x_{ij} & \begin{matrix} l_1 & l_2 & \cdots & l_j & \cdots & l_7 & l_8 \end{matrix} &
\end{array}
$$

The element x_{ij} denotes the lending of bank i towards bank j (or borrowing of bank j from bank i). The sum a_i across the row i represents the total value of interbank assets of bank i and the sum l_j down the column j represents the total value of interbank liabilities of bank j. The bank's aggregated interbank assets and liabilities are observable from the balance sheet of each bank. We have to estimate the elements x_{ij} of the interbank exposure matrix X under the constraints:

$$
\begin{cases} \sum_{j=1}^{8} x_{ij} = a_i \\ \sum_{i=1}^{8} x_{ij} = l_j \\ x_{ij} \ge 0 \end{cases} \tag{1}
$$

The condition $x_{ii} = 0$ must be added as a bank i does not lend to itself.

One of the most widely used methods to estimate the matrix X is the entropy maximization approach whose purpose is to minimize the uncertainty on the bilateral interlinkages between banks. This method assumes that the assets and liabilities among banks are proportionally distributed depending on bank size, which is consistent with the complete interbank structure of the market proposed by Allen and Gale (2000).

If we choose the normalization:

$$
\sum_{i=1}^{8} a_i = \sum_{j=1}^{8} l_j = 1 \tag{2}
$$

then the solution:

$$
x_{ij} = a_i \times l_j \tag{3}
$$

satisfy the constraints (1), reflecting the relative importance of each bank in the interbank market .

As each bank is never exposed to itself, the process of the estimation of the exposure matrix X begin with the initialization matrix $X^0 = (x_{ij}^0)_{\substack{1\le i\le 8\\1\le j\le 8}}$ defined by:

$$
x_{ij}^0 = \begin{cases} 0 & if \quad i = j \\ a_i \times l_j & otherwise \end{cases} \tag{4}
$$

It is clear that this matrix X violates the two first conditions of (1). The entropy maximization method consists to solve the optimization problem:

$$
\min_{x_{ij}} \sum_{i=1}^{8} \sum_{j=1}^{8} x_{ij} ln\left(\frac{x_{ij}}{x_{ij}^0}\right) \tag{5}
$$

subject to

$$
\begin{cases} \sum_{j=1}^{8} x_{ij} = a_i \\ \sum_{i=1}^{8} x_{ij} = l_j \\ x_{ij} \ge 0 \end{cases} \tag{6}
$$

This problem can be solved by an iterative method known in the economic field as RAS algorithm (see Censor and Zenios (1998)). This algorithm has different names in other fields: the iterative proportional fitting procedure (IPFP, also known as bi-proportional fitting) in statistics, and matrix raking or matrix scaling in computer science.

An exhaustive treatment of the algorithm and its mathematical foundations can be found in the book of Bishop et al. (1975). The first general proof of convergence, built on non-trivial measure theoretic theorems and entropy minimization, is due to Csiszár (1975). It is well known that the equality $\sum_{i=1}^{8} a_i = \sum_{j=1}^{8} l_j$ is a necessary condition for a convergence of the RAS algorithm.

The RAS algorithm follows the following iterative steps.

▪ *Initial step : Initialization*

We set for $k = 0$

$$
X^0 = (x_{ij}^0)_{\substack{1\le i\le 8\\1\le j\le 8}} \tag{7}
$$

with

$$
x_{ij}^0 = \begin{cases} 0 & if \quad i = j \\ a_i \times l_j & otherwise \end{cases} \tag{8}
$$

We set $k = 1$.

▪ *Step 1 : (Row fitting)* For $1 \le i \le 8$ we define :

$$
r_i^k = \frac{a_i}{\sum_{j=1}^{8} x_{ij}^k} \tag{9}
$$

We define the matrix $Y = (y_{ij})_{\substack{1\le i\le m\\1\le j\le n}}$ by : for $1 \le i \le 8$ and $1 \le j \le 8$

$$
y_{ij} = r_i^k x_{ij}^k \tag{10}
$$

▪ *Step 2 : (Column fitting)* For $1 \le j \le 8$ we define :

$$
s_j^k = \frac{l}{\sum_{i=1}^{m} x_{ij}^k} \tag{11}
$$

We define the matrix $X^{k+1} = (x_{ij}^{k+1})_{\substack{1\le i\le 8\\1\le j\le 8}}$ by : for $1 \le i \le 8$ and $1 \le j \le 8$

$$
x_{ij}^{k+1} = s_j^k y_{ij} \tag{12}
$$

▪ *Step 3 : Stopping test (convergence*)

If $\max_{\substack{1\le i\le 8\\1\le j\le 8}} \left|x_{ij}^{k+1} - x_{ij}^k\right| < 10^{-6}$ then X^{k+1} is taken as a solution. Otherwise the counter is incremented $k = k + 1$ and the iteration is restarted from step 1.

Matricial formulation of the RAS algorithm *:* We can show

$$
X^{k+1} = R^k A S^k \tag{13}
$$

where :

$$
A = X^0 \tag{14}
$$

$$R^k = diag(R_1^k, R_2^k, \dots, R_m^k) = \begin{pmatrix} R_1^k & 0 & \cdots & 0 \\ 0 & R_2^k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & R_m^k \end{pmatrix} \quad (15)$$

$$S^k = diag(S_1^k, S_2^k, \dots, S_n^k) = \begin{pmatrix} S_1^k & 0 & \cdots & 0 \\ 0 & S_2^k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & S_n^k \end{pmatrix} \quad (16)$$

with

$$R_i^k = \prod_{l=1}^{k} r_i^l \quad (17)$$

and

$$S_j^k = \prod_{l=1}^{k} s_j^l \quad (18)$$

The formula $X^{k+1} = R^k A S^k$ justify the name RAS given to the algorithm.

Presentation of the Round by Round model

The round by round model consists to explore the effects of the failure of a single bank on the rest of the banking system. The initial failure is supposed due to an idiosyncratic shock. We examine whether the failure of the single bank would lead to subsequent collapses of other banks through three rounds.

This simulation model is founded on assumptions:

1) Contagion doesn't depend on macroeconomic shocks;
2) Portfolio holdings and prices are constant during the contagion process;
3) Central bank does not intervene, acting as the "lender of last resort";
4) Banks do not recourse to recapitalization.

If the initial failure of a bank does not lead the collapse of a bank, there is no contagion phenomenon. On the contrary, if one bank or several banks fall into default in the first round then we continue the simulation by examining the impact of the failure of defaulting banks on other banks on the following round. We pursue the process until the third round.

During the process, the contagion occurs when at least one bank suffers a loss which exceeds the bank's tier one capital.

Description of the Round by Round Algorithm

Let $B = \{B_1, B_2, \dots, B_8\}$ be the set of the 8 major Moroccan banks. We suppose we have already obtained the bilateral exposure interbank matrix $X = (x_{ij})_{\substack{1\le i\le 8\\ 1\le j\le 8}}$, where x_{ij} is the amount of lending of the bank B_i towards bank B_j. We denote by c_i the initial tier one capital of the bank B_i.

To illustrate how the algorithm works, we suppose the failure of the bank B_{j_0} , for any $1 \le j_0 \le 8$, due to an idiosyncratic shock.

- *First Round:*

A bank $B_i \neq B_{j_0}$ fails at the first round if its exposure x_{ij_0} to the bank B_{j_0} multiplied by a percentage representing the loss rate α (Loss Given Default, LGD) exceeds the tier one capital of the bank B_i:

$$\alpha \times x_{ij_0} > c_i \quad (19)$$

The new capital of the bank B_i at the first round becomes:

$$c_{ij_0}^1 = c_{ij_0}^0 - \alpha \times x_{ij_0} \quad (20)$$

where $c_{ij_0}^0 = c_i$ is the initial capital of the bank B_i.

Let $Df_{B_{j_0}}^1$ be the set of banks defaulting at the first round after the failure of the bank B_{j_0}:

$$Df_{B_{j_0}}^1 = \{B_i \neq B_{j_0} / c_{ij_0}^1 = c_{ij_0}^0 - \alpha \times x_{ij_0} < 0\} \quad (21)$$

Let $Nf_{B_{j_0}}^1$ be the set of banks not defaulting at the first round after the failure of the bank B_{j_0} defined by:

$$Nf_{B_{j_0}}^1 = \{B_i \neq B_{j_0} / c_{ij_0}^1 = c_{ij_0}^0 - \alpha \times x_{ij_0} \ge 0\} \quad (22)$$

- *Second round:*

The bank B_{j_0} and the banks B_i belonging to the set $Df_{B_{j_0}}^1$ are in distress. At this second step, we're looking for the banks belonging to $Nf_{B_{j_0}}^1$ which will default.

A bank $B_i \in Nf_{B_{j_0}}^1$ fails at the second round if the sum $\sum_{B_k \in Df_{B_{j_0}}^1} x_{ik}$ of its exposures (to the banks $B_k \in Df_{B_{j_0}}^1$) multiplied by the same loss rate α (Loss Given Default, LGD) exceeds the capital of the bank B_i:

$$\alpha \times \sum_{B_k \in Df_{B_{j_0}}^1} x_{ik} > c_{ij_0}^1 \quad (23)$$

The new capital of the bank B_i at the second round becomes:

$$c_{ij_0}^2 = c_{ij_0}^1 - \alpha \times \sum_{B_k \in Df_{B_{j_0}}^1} x_{ik} \quad (24)$$

Let $Df_{B_{j_0}}^2$ be the set of banks defaulting at the second round defined by:

$$Df_{B_{j_0}}^2 = \left\{ B_k \in Nf_{B_{j_0}}^1 \middle/ c_{ij_0}^2 = c_{ij_0}^1 - \alpha \times \sum_{B_k \in Df_{B_{j_0}}^1} x_{ik} < 0 \right\} \quad (25)$$

Let $Nf_{B_{j_0}}^2$ be the set of banks not defaulting at the second round defined by:

$$Nf_{B_{j_0}}^2 = \left\{ B_k \in Nf_{B_{j_0}}^1 \middle/ c_{ij_0}^2 = c_{ij_0}^1 - \alpha \times \sum_{B_k \in Df_{B_{j_0}}^1} x_{ik} \ge 0 \right\} \quad (26)$$

We continue the iterative process until there is no failure. The following table illustrates the process of the algorithm until it stops.

Table 5: Round by round Algorithm

Round	Defaulting banks
First	$Df_{B_{j_0}}^1 = \{B_i \neq B_{j_0} / c_{ij_0}^1 = c_{ij_0}^0 - \alpha \times x_{ij_0} < 0\}$
Second	$Df_{B_{j_0}}^2 = \left\{ B_k \in Nf_{B_{j_0}}^1 \middle/ c_{ij_0}^2 = c_{ij_0}^1 - \alpha \times \sum_{B_k \in Df_{B_{j_0}}^1} x_{ik} < 0 \right\}$
	$\vdots$
n-th	$Df_{B_{j_0}}^n = \left\{ B_k \in Nf_{B_{j_0}}^{n-1} \middle/ c_{ij_0}^n = c_{ij_0}^{n-1} - \alpha \times \sum_{B_k \in Df_{B_{j_0}}^{n-1}} x_{ik} < 0 \right\}$
$\vdots$	$\vdots$
N-th	$Df_{B_{j_0}}^N = \emptyset$

Round	Non-Defaulting banks
First	$Nf^1_{B_{j_0}} = \{B_i \neq B_{j_0} / c^1_{ij_0} = c^0_{ij_0} - \alpha \times x_{ij_0} \geq 0\}$
Second	$Nf^2_{B_{j_0}} = \left\{ B_k \in Nf^1_{B_{j_0}} \Big/ c^2_{ij_0} = c^1_{ij_0} - \alpha \times \sum_{B_k \in Df^1_{B_{j_0}}} x_{ik} \geq 0 \right\}$
	⋮
n-th	$Nf^n_{B_{j_0}} = \left\{ B_k \in Nf^{n-1}_{B_{j_0}} \Big/ c^n_{ij_0} = c^{n-1}_{ij_0} - \alpha \times \sum_{B_k \in Df^{n-1}_{B_{j_0}}} x_{ik} \geq 0 \right\}$
⋮	⋮
N-th	$Nf^N_{B_{j_0}} = \left\{ B_k \in Nf^{N-1}_{B_{j_0}} \Big/ c^N_{ij_0} = c^{N-1}_{ij_0} - \alpha \times \sum_{B_k \in Df^{N-1}_{B_{j_0}}} x_{ik} \geq 0 \right\}$

In this paper we run the round by round algorithm three times. The loss given default α is chosen exogenously and we give it different values $0 < \alpha \leq 100\%$.

Results and comparison

The RAS algorithm assumes there exists a bilateral linkages between all banks showing a complete structure as described in Allen and Gale (2000). The following figure illustrate the symmetrical relationships existing between the 8 banks studied in the two years 2015 and 2016.

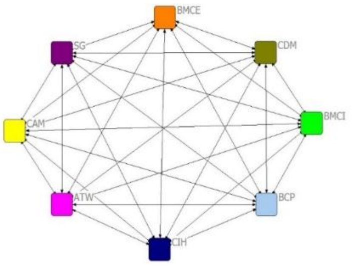

Figure 1: Bilateral interlinkages between 8 Moroccan major banks

Analysis of the 2015's results

Bilateral exposure matrix of 2015

The running of the RAS algorithm gives the following bilateral exposure matrix.

Table 6: Bilateral exposure matrix of the year 2015

2015	B1	B2	B3	B4	B5	B6	B7	B8	Loans
B1	0	961658	2531741	47249	454486	139891	419489	39645	**4594160**
B2	4029816	0	5826584	108739	1045961	321948	965418	91239	**12389704**
B3	1165869	640295	0	31459	302608	93143	279306	26396	**2539075**
B4	359690	197542	520065	0	93360	28736	86171	8144	**1293707**
B5	1001120	549815	1447488	27014	0	79981	239837	22666	**3367920**
B6	218924	120233	316536	5907	56823	0	52447	4957	**775827**
B7	583919	320688	844270	15756	151559	46650	0	13221	**1976062**
B8	875905	481047	1266443	23635	227346	69977	209839	0	**3154193**
Debts	**8235242**	**3271277**	**12753126**	**259760**	**2332143**	**780326**	**2252507**	**206268**	30090649

The following table shows the data concerning the aggregated debts and loans and the tier one of the 8 banks for the year 2015. We deduce from these data the percentages of debts, loans, tier one as well as the rates Tier one/Debts and Tier one/Loans.

Table 7: Percentage of Debts, of Loans, of Tier-1, Tier-1/Debts, Tier1/Loans of the year 2015

2015	B1	B2	B3	B4	B5	B6	B7	B8	Total
% Debts	**27,37**	**10,87**	**42,38**	0,86	7,75	2,59	7,49	0,69	100%
% Loans	**15,27**	**41,17**	8,44	4,30	11,19	2,58	6,57	10,48	100%
%Tier 1	31,59	6,31	12,57	14,69	7,04	5,36	10,65	11,79	100%
Tier 1/Debts	1,9486	0,9799	0,5008	28,7359	1,5325	3,4898	2,4013	29,0441	
Tier 1/Loans	3,4929	**0,2587**	2,5154	5,7698	1,0612	3,5100	2,7372	1,8993	

We can especially notice that the debts of the three first banks B1, B2 and B3 represent more than 80% of the total debts, the loans of the second bank B2 represents more than 41% of the total loans and the two first banks B1 and B2 constitute more that 56% of the aggregated loans. We remark also that the rate of the Tier one of the bank B2 relative to its loans represents the lowest one.

Results of the Round by Round algorithm: Year 2015

We run the Round by Round algorithm on the bilateral exposure matrix of 2015 for three times. We simulate an idiosyncratic shock on each bank at turn for different values of the loss given default $0 < \alpha \leq 100\%$ and we observe the impact of the contagion on the other banks throughout three rounds. We summarize below the results obtained.

1) For $0 < \alpha \leq 55{,}01\%$:

For this interval, when any bank is shocked then no bank is defaulting at the first round and therefore at the next rounds after.

Consequently, the Moroccan banking system represented by the eight major banks is strongly resilient when the LGD belongs to $]0\%, 55{,}01\%]$. This is illustrated in the following binary matrix.

Table 8: Resilience of the banks in the First Round for $0 < \alpha \leq 55{,}01\%$ (Year 2015)

	B1	B2	B3	B4	B5	B6	B7	B8
B1	1	0	0	0	0	0	0	0
B2	0	1	0	0	0	0	0	0
B3	0	0	1	0	0	0	0	0
B4	0	0	0	1	0	0	0	0
B5	0	0	0	0	1	0	0	0
B6	0	0	0	0	0	1	0	0
B7	0	0	0	0	0	0	1	0
B8	0	0	0	0	0	0	0	1

The diagonal of the binary matrix is filled with 1, meaning that at each row only one bank is stressed by an idiosyncratic shock. We can notice that the non-diagonal elements are equal to 0, which means that no banks is defaulting.

2) For $55{,}01\% < \alpha \leq 79{,}54\%$:

For this interval, when the bank B3 at the third row is shocked at the first round then only the bank B2 is defaulting. When the other banks are chocked at the first round then no bank fails. We notice that there is no contagion effect in the other rounds. This is illustrated in the following binary matrices.

Table 9: Contagion through three rounds for $55{,}01\% < \alpha \leq 79{,}54\%$ (Year 2015)

	Round 1							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	1	0	0	0	0	0	0	0
B2	0	1	0	0	0	0	0	0
B3	0	**1**	**1**	0	0	0	0	0
B4	0	0	0	1	0	0	0	0
B5	0	0	0	0	1	0	0	0
B6	0	0	0	0	0	1	0	0
B7	0	0	0	0	0	0	1	0
B8	0	0	0	0	0	0	0	1

	Round 2							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

	Round 3							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

The graph bellow shows the contagion impact when bank B3 is shocked for all values of the LGD in the interval $]55{,}01\%, 79{,}54\%\ ]$.

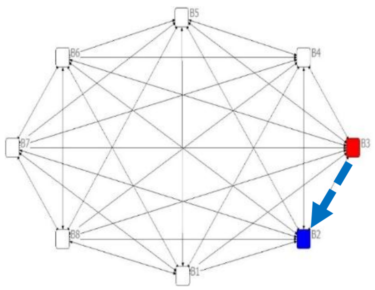

Figure 2: Defaulting banks after idiosyncratic shock of Bank B3 (Year 2015)
: Chocked bank in the 1st round: Bank B3 : Defaulting banks in the 1st round: Bank B2

3) For $79{,}54\% < \alpha \leq 100\%$:

For this interval, when the bank B1 in the first row is shocked at the first round then only the bank B2 is defaulting. When the bank B3 in the third row is shocked at the first round then only the bank B2 is defaulting. When the other banks are chocked at the first round then no bank fails. We notice that there is no contagion effect in the other rounds. This is illustrated in the following binary matrices.

Table 10: Contagion through three rounds for $79{,}54\% < \alpha \leq 100\%$ (Year 2015)

	Round 1							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	**1**	**1**	0	0	0	0	0	0
B2	0	1	0	0	0	0	0	0
B3	0	**1**	**1**	0	0	0	0	0
B4	0	0	0	1	0	0	0	0
B5	0	0	0	0	1	0	0	0
B6	0	0	0	0	0	1	0	0
B7	0	0	0	0	0	0	1	0
B8	0	0	0	0	0	0	0	1

	Round 2							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

	Round 3							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

The graphs bellow show the contagion impact when bank B1 or bank B3 are shocked for all values of the LGD in the interval $]79{,}54\ \%, 100\%]$.
The graphs bellow show the contagion impact when bank B1 or bank B3 are shocked for all values of the LGD in the interval $]79{,}54\ \%, 100\%]$.

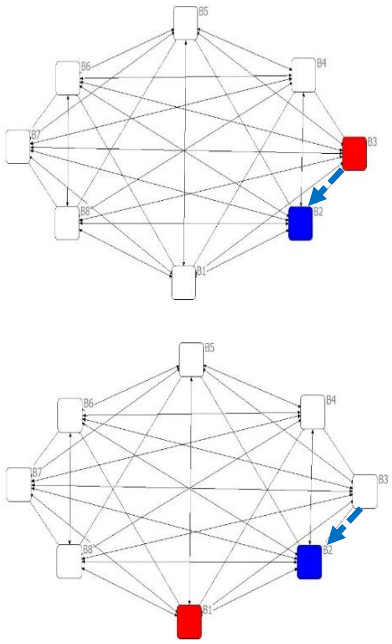

Figure 3: Defaulting banks after idiosyncratic shock of Banks B1 and B3 (Year 2015)
: Chocked bank in the 1st round : Defaulting banks in the 1st round

Interpretation: Year 2015

The table below show that the bank B2 is the most exposed in the interbank market as its loans represent more than 41% of the total loans.

Table 11: Year 2015

2015	B1	B2	B3	B4	B5	B6	B7	B8	Total
Loans	4594160	12389704	2539075	1293707	3367920	775827	1976062	3154193	30090649
% Loans	15,27	**41,17**	8,44	4,30	11,19	2,58	6,57	10,48	100%

We can also notice in the table below that the rate of Tier one to the Loans of the bank B2 is the lowest one.

Table 12: Year 2015

2015	B1	B2	B3	B4	B5	B6	B7	B8	Total
Loans2015	4594160	12389704	2539075	1293707	3367920	775827	1976062	3154193	30090649
Tier 1 2015	16046794	3205538	6386829	7464449	3574027	2723178	5408922	5990873	50800610
Tier 1/Loans	3,4929	**0,2587**	2,5154	5,7698	1,0612	3,5100	2,7372	1,8993	

The two information cited above justify why the bank B2 is the weakest bank and it is the only bank that fails after an idiosyncratic shock.

The table below presents the degree of exposure of the bank B2 to other banks.

Table 13: Degree of exposure of Bank 2 to other banks (Year 2015)

2015	B1	B3	B4	B5	B6	B7	B8	Loans
B2	4029816	5826584	108739	1045961	321948	965418	91239	**12389704**
B2	**32,53%**	**47,03%**	0,88%	8,44%	2,60%	7,79%	0,74%	100,00%

We can remark that Bank B2 is more exposed first to B3 then to B1 with exposure rates of 47,03% and 32,53% respectively. This explains why the bank B2 fails when the bank B3 is shocked for $\alpha > 55{,}01\%$, whereas the bank B2 fails when the bank B1 is shocked for $\alpha > 79{,}54\% > 55{,}01\%$.

We conclude that the Moroccan banking system in 2015 can be considered strongly resilient when the LGD is less than 55,01%. When the LGD is greater than 55,01%, the bank B2 is the only bank that fails.

Analysis of the 2016's results

Bilateral exposure matrix of 2016

The running of the RAS algorithm gives the following bilateral exposure matrix.

Table 14: Bilateral exposure matrix of the year 2016

2016	B1	B2	B3	B4	B5	B6	B7	B8	Loans
B1	0	2271640	4620461	75717	477294	113089	144783	19609	**7722593**
B2	192394	0	3759065	61601	388312	92005	117791	15953	**4627122**
B3	464481	4461798	0	148719	937469	222121	284373	38515	**6557474**
B4	4921	47269	96145	0	9932	2353	3013	408	**164041**
B5	39243	376970	766747	12565	0	18767	24026	3254	**1241572**
B6	16437	157897	321158	5263	33176	0	10064	1363	**545357**
B7	3931	37762	76808	1259	7934	1880	0	326	**129900**
B8	3087	29657	60322	989	6231	1476	1890	0	**103653**
Total Debts	**724495**	**7382993**	**9700705**	**306113**	**1860348**	**451691**	**585939**	**79428**	**21091712**

The following table shows the data concerning the aggregated debts and loans and the tier one of the 8 banks for the year 2016. We deduce from these data the percentages of debts, loans, tier one as well as the rates Tier one/Debts and Tier one/Loans.

Table 15: % of Debts, of Loans, of Tier-1, Tier-1/Debts, Tier1/Loans of the year 2016

2016	B1	B2	B3	B4	B5	B6	B7	B8	Total
% Debts	3,44	**35,00**	**45,99**	1,45	8,82	2,14	2,78	0,38	100%
% Loans	**36,61**	**21,94**	**31,09**	0,78	5,89	2,59	0,62	0,49	100%
% Tier 1	37,01	5,63	12,07	13,12	6,624	4,70	9,43	11,43	100%
Tier 1/Debts	8,4848	0,4252	0,6935	23,8926	1,9854	5,8001	8,9693	0,2226	
Tier 1/Loans	2,6723	**0,6785**	1,0259	44,5856	2,9748	4,8039	40,4574	1,4736	

We can especially notice that the debts of the two banks B2 and B3 represent more than 70% of the total debts, the loans of the second bank B2 represents more than 21% of the total loans and the three first banks B1, B2 and B3 constitute more that 89% of the aggregated loans. We remark also that the rate of the Tier one of the bank B2 relative to its loans represents the lowest one.

Results of the Round by Round algorithm: Year 2016

We run the Round by Round algorithm on the bilateral exposure matrix of 2016 for three times. We simulate an idiosyncratic shock on each bank at turn for different values of the loss given default $0 < \alpha \leq 100\%$ and we observe the impact of the contagion on the other banks throughout three rounds. We summarize below the results obtained.

1) For $0 < \alpha \leq 83{,}51\%$:

For this interval, when any bank is shocked then no bank is defaulting at the first round and therefore at the next rounds after.

Consequently, the Moroccan banking system represented by the eight major banks is strongly resilient when the LGD belongs to $]0\%, 83{,}51\%]$. This is illustrated in the following binary matrix.

Table 16: Resilience of the banks in the First Round for $0 < \alpha \leq 83{,}51\%$ (Year 2016)

	B1	B2	B3	B4	B5	B6	B7	B8
B1	1	0	0	0	0	0	0	0
B2	0	1	0	0	0	0	0	0
B3	0	0	1	0	0	0	0	0
B4	0	0	0	1	0	0	0	0
B5	0	0	0	0	1	0	0	0
B6	0	0	0	0	0	1	0	0
B7	0	0	0	0	0	0	1	0
B8	0	0	0	0	0	0	0	1

The diagonal of the binary matrix is filled with 1, meaning that at each row only one bank is stressed by an idiosyncratic shock. We can notice that the non-diagonal elements are equal to 0, which means that no banks is defaulting.

2) For $83{,}51\% < \alpha \leq 100\%$:

For this interval, when the bank B3 at the third row is shocked at the first round then only the bank B2 is defaulting. When the other banks are chocked at the first round then no bank fails. We notice that there is no contagion effect in the other rounds. This is illustrated in the following binary matrices.

Table 17: Contagion through three rounds for $83{,}51\% < \alpha \leq 100\%$ (Year 2016)

	Round 1							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	1	0	0	0	0	0	0	0
B2	0	1	0	0	0	0	0	0
B3	0	1	1	0	0	0	0	0
B4	0	0	0	1	0	0	0	0
B5	0	0	0	0	1	0	0	0
B6	0	0	0	0	0	1	0	0
B7	0	0	0	0	0	0	1	0
B8	0	0	0	0	0	0	0	1

	Round 2							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

	Round 3							
	B1	B2	B3	B4	B5	B6	B7	B8
B1	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0
B5	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0
B7	0	0	0	0	0	0	0	0
B8	0	0	0	0	0	0	0	0

The graph bellow shows the contagion impact when bank B3 is shocked for all values of the LGD in the interval $]83{,}51\%, 100\%]$.

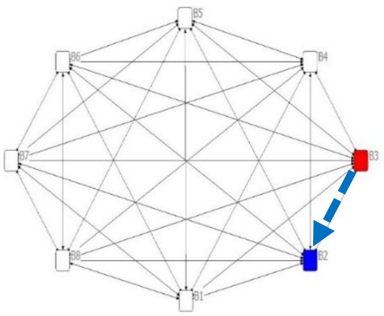

Figure 4: Defaulting banks after idiosyncratic shock of Banks B3 (Year 2016)
: Chocked bank in the 1st round: Bank B3 : Defaulting banks in the 1st round: Bank B2

Interpretation: Year 2016

The table below show that the bank B2 is well exposed in the interbank market as its loans of 2016 represent more than 21% of the total loans. It is less exposed that in the year 2015.

Table 18: Year 2016

2016	B1	B2	B3	B4	B5	B6	B7	B8	Total
Loans	7722593	4627122	6557474	164040	1241572	545357	129900	103653	**21091712**
% Loans	**36,61**	**21,94**	**31,09**	0,78	5,89	2,59	0,62	0,49	100%

We can also notice in the table below that the rate of Tier one to the Loans of the bank B2 in 2016 is the lowest one, although it has improved compared to the year 2015.

Table 19: Year 2016

2016	B1	B2	B3	B4	B5	B6	B7	B8	Total
Loans	7722593	4627122	6557474	164040	1241572	545357	129900	103653	**21091712**
Tier 1	20637127	3139551	6727575	7313829	3693470	2619853	5255437	6371924	**55758766**
Tier 1/Loans	2,6723	**0,6785**	1,0259	44,5856	2,9748	4,8039	40,4574	61,4736	

The two information cited above justify why the bank B2 is the weakest bank and it is the only bank that fails after an idiosyncratic shock.

The table below presents the degree of exposure of the bank B2 to other banks.

Table 20: Degree of exposure of Bank B2 (Year 2016)

2016	B1	B3	B4	B5	B6	B7	B8	Total Loans
B2	192394	3759065	61601	388312	92005	117791	15953	**4627122**
B2	4,16%	**81,24%**	1,33%	8,39%	1,99%	2,55%	0,34%	100,00%

We can remark that Bank B2 is only exposed to the bank B3 with an exposure rate of 81,24%. This explains why the bank B2 fails only when the bank B3 is shocked in 2016.

We conclude that banking system in 2016 can be considered strongly resilient when the LGD is less than 83,51%. When the LGD is greater than 83,51%, the bank B2 is the only bank that fails after shocking the bank B3. The idiosyncratic shocking of bank B1 no longer causes the failure of the bank B2 as in 2015.

Comparison between the two years 2015 and 2016

We can remark in the table below that the Tier one/Loans rate of the Bank B2 has slightly improved from 0,2587 to 0,6785 and its exposure has decreased from 41,17% to 21,94%.

Table 21: Evolution of Debts, Loans and Tier-1/Loans between 2015 and 2016

Bank B2	2015	2016
Percentage of Debts	10,87 %	35,00 %
Percentage of Loans	41,17 %	21,94 %
Tier one/Loans	**0,2587**	**0,6785**

This improvement explains why the stressing of bank B1 in 2016 no longer causes the failure of the bank B2 as in 2015, and B3 is the only bank which triggers the failure of bank B2 in 2016.

We conclude that the resilience of the Moroccan banking system has been slightly improved in 2016 compared to 2015.

ROBUSTNESS OF THE MOROCCAN INTERBANK MARKET

To measure the degree of resilience of the Moroccan banking system to idiosyncratic shocks, we estimate the Tier One thresholds of the 8 banks below which these banks would fail after a shock for a loss rate $\alpha = 10\%$ representing an optimistic scenario. For this purpose, we fix the loss rate $\alpha = 10\%$ and we look for the amount a_i by which we must divide the initial Tier one of a bank B_i causing the failure of this bank after any idiosyncratic shock of another bank $B_k \neq B_i$.

Moroccan banking market's robustness in 2015

The following table presents the New Tier one of the 8 banks, in the 2015, below which the banks would fail for a loss rate $\alpha = 10\%$.

Table 22: Tier-1's Thresholds causing failure of banks for α = 10% in 2015

2015	B1	B2	B3	B4	B5	B6	B7	B8
a_i	63,3825	5,5016	54,7818	143,5292	24,6913	86,0307	64,0663	47,3048
Initial Tier 1	16046794	3205538	6386829	7464449	3574027	2723178	5408922	5990873
New Tier 1	253174	582656	116587	52006	144748	31654	84427	126644

In 2015, Bank B4 is the strongest in terms of Tier one as we must divide the initial Tier one of this bank by more than 143 before it fails for the loss rate $\alpha = 10\%$, whereas Bank B2 is the weakest bank. The table below ranks banks from the weakest to the strongest one.

Table 23: Ranking of banks from the weakest to the strongest in terms of tier-one in 2015

2015	**B4**	B6	B7	B1	B3	B8	B5	**B2**
a_i	**143,5292**	86,0307	64,0663	63,3825	54,7818	47,3048	24,6913	**5,5016**

Moroccan banking market's robustness in 2016

The following table presents the New Tier one of the 8 banks, in the 2016, below which the banks would fail for a loss rate $\alpha = 10\%$.

Table 24: Tier-1's Thresholds causing failure of banks for α = 10% in 2016

2016	B1	B2	B3	B4	B5	B6	B7	B8
a_i	44,6647	8,3520	15,0782	760,7109	48,1707	81,5753	684,2311	1056,3181
Initial Tier 1	20637127	3139551	6727575	7313829	3693470	2619853	5255437	6371924
New Tier 1	462046	375904	446179	9614	76675	32116	7681	6032

In 2016, Bank B8 is the strongest in terms of Tier one as we must divide the initial Tier one of this bank by more than 1056 before it fails for the loss rate $\alpha = 10\%$, whereas Bank B2 is the weakest bank. The table below ranks banks from the weakest to the strongest one.

Table 25: Ranking of banks from the weakest to the strongest in terms of tier-one in 2016

2016	**B8**	B4	B7	B6	B5	B1	B3	**B2**
a_i	**1056,3181**	760,7109	684,2311	81,5753	48,1707	44,6647	15,0782	**8,3520**

CONCLUSION

To our knowledge, this work is the first study that has been devoted to contagion risk in the Moroccan banking market by combining the RAS algorithm which estimates the bilateral

exposure matrix and the Round By Round model which models the contagion impact after idiosyncratic shocks.
The data used are the aggregate lending and borrowings for the two years 2015 and 2016 extracted from the balance sheets of the eight major Moroccan banks. We also estimated the Tier one of the eight banks based on the definition of the core capital provided by the circular BAM (Circular of Bank Al Maghrib relating to the credit institutions' own funds, August 2013).
We have first applied the entropy maximization approach (RAS algorithm) in order to estimate the bilateral exposure matrices of the Moroccan interbank market for the two years 2015 and 2016. Secondly, we apply the Round By Round method to model the risk of contagion due to idiosyncratic shocks.
In summary, our results suggest that the Moroccan interbank market represented by the 8 major banks is quite resilient to contagion risk in the two years 2015 and 2016. The situation has been slightly improved in 2016 compared to 2015.

REFERENCES

[1] Allen, F. and Gale, D. (2000), "Financial Contagion, Journal of Political Economy". 108 (1), 1-33.

[2] Amundsen, E. and Arnt, H. (2005), "Contagion Risk in the Danish Interbank Market". Danmarks Nationalbank, Working Paper 25.

[3] Arinaminpathy, N., Kapadia, S. and May, R. (2012), "Size and Complexity in Model Financial Systems". Proceedings of the National Academy of Sciences of the United States of America 109 (45), 18338-43.

[4] Blavarg, M. and Nimander P. (2002), "Interbank Exposures and Systemic Risk". Sveriges Riksbank Economic Review 2, 19-45.

[5] Bishop, Y. M. M., Feinberg, S. E. and Holland P. W. (1975), "Discrete Multivariate Analysis- Theory and Practice". MIT Press, Cambridge, Mass. 575 p.

[6] Censor, Y. and Zenios, S. (1998), "Parallel Optimization: Theory, Algorithms, and Applications". Oxford University Press, New York.

[7] Csiszár, I. (1975), "I-Divergence of Probability Distributions and Minimization Problems". Annals of Probability. 3 (1): 146–158.

[8] Cocco, J., Gomes, F. and Martins, N. (2009), "Lending Relationships in the Interbank Market". Journal of Financial Intermediation 18 (1), 24-48.

[9] Craig, B. and von Peter, G. (2010), "Interbank Tiering and Money Center Banks". Deutsche Bundesbank, Discussion Paper 12/2010.

[10] Degryse, H. and Nguyen, G. (2007), "Interbank Exposures: An Empirical Examination of Contagion Risk in the Belgian Banking System". International Journal of Central Banking 3 (2), 123-171.

[11] Elsinger, H., Lehar, A. and Summer, M. (2006), "Risk Assessment for Banking Systems". Management Science 52 (9), 1301-1314.

[12] Farinha, L. and Gaspar, V. (2008), "Portuguese Banks in the Euro Area Market for Daily Funds". European Central Bank Working Paper 985.

[13] Freixas, X., Parigi, B. and Rochet, C. (2000), "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank". Journal of Money, Credit and Banking 32 (3), 611–38.

[14] Furfine, C. (1999), "The Microstructure of the Federal Funds Market". Financial Markets, Institutions and Instruments 8 (5), 24-44.

[15] Furfine, C. (2001), "Banks as Monitors of other Banks, Evidence from the Overnight Federal Funds Market". The Journal of Business 74 (1), 33-57.

[16] Furfine, C. (2003), "Interbank Exposures, Quantifying the Risk of Contagion". Journal of Money, Credit and Banking 35 (1), 111-128.

[17] Gai, P. and Kapadia, S. (2010), "Contagion in Financial Networks". Proceedings of the Royal Society A, 466 (2120) 2401-23.

[18] Georg, C., (2011), "The Effect of the Interbank Network Structure on Contagion and Common Shocks". Reserve Bank of New Zealand Working Paper.

[19] Iori, G., Jafarey, S. and Padilla, F. (2006), "Systemic Risk on the Interbank Market". Journal of Economic Behaviour & Organization 6, 525-542.

[20] James, C. (1991), "The Losses Realized in Bank Failures". The Journal of Finance 46 (4), 1223–1242.

[21] Kapadia, S., Drehmann, M., Elliott, J. and Sterne, G. (2012), "Liquidity Risk, Cash Flow Constraints, and Systemic Feedbacks". Forthcoming in Haubrich, J.G. and Lo, A.W. (eds), Quantifying Systemic Risk, University of Chicago Press.

[22] Kaufman, G. (1994), "Bank Contagion, A Review of the Theory and Evidence". Journal of Financial Services Research 8 (2), 123-150.

[23] Krznar, M. (2009), "Contagion Risk in the Croatian Banking System". Croatian National Bank Working Papers 20.

[24] Lelyveld, van I. and Liedorp, F. (2006), "Interbank Contagion in the Dutch Banking Sector: A Sensitivity Analysis". International Journal of Central Banking 2 (2), 99-133.

[25] Millard, S. and Polenghi, M. (2004), "The Relationship Between the Overnight Interbank Unsecured Loan Market and the CHAPS Sterling System". Bank of England Quarterly Bulletin, 42-47.

[26] Mistrulli, P. (2005), "Interbank Lending Patterns and Financial Contagion". Banca D'Italia, Working Paper 641.

[27] Mistrulli, P. (2011), "Assessing Financial Contagion in the Interbank Market: Maximum Entropy vs. Observed Interbank Lending Patterns". Journal of Banking and Finance 35, 1114-1127.

[28] Müller, J. (2006a), "Interbank Credit Lines as a Channel of Contagion". Journal of Financial Services Research 29 (1), 37-60.

[29] Müller, J. (2006b), "The Potential for Contagion in the Swiss Interbank Market". Mimeo.

[30] Nier, E., Yang, J., Yorulmazer, T. and Alentorn, A. (2007), "Network Models and Financial Stability".

Journal of Economic Dynamics and Control 31, 2033-2060.

[31] Sheldon, G. and Maurer, M. (1998), "Interbank Lending and Systemic Risk: An Empirical Analysis for Switzerland". Swiss Journal of Economics and Statistics 134 (4.2), 685-704.

[32] Soramäki, K., Bech, M., Arnold, J., Glass, R. and Beyeler, W. (2007), "The Topology of Interbank Payment Flows". Physica A 379, 317-333.

[33] Toivanen, M. (2009), "Financial Interlinkages and Risk of Contagion in the Finnish Interbank Market". Bank of Finland Research Discussion Papers 6.

[34] Upper, C. and Worms, A. (2004), "Estimating Bilateral Exposures in the German Interbank Market: Is There a Danger of Contagion". European Economic Review 48 (4), 827-849.

[35] Wells, S. (2004), "Financial Interlinkages in the United Kingdom's Interbank Market and the Risk of Contagion". Bank of England Working Paper 230.