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Stereographic *l* – Axial Half Logistic Distribution

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Abstract

Following Phani et al (2017) an attempt is made to develop the Stereographic-l-axial Half Logistic distribution by extending the Stereographic Semicircular Half Logistic distribution for modeling axial data. Stereographic l - axial Half Logistic distribution over any arc of length l has possible applications in Quality Control, weather studies and pollution control experiments. We consider the asymptotic behavior of the Stereographic Semicircular Half Logistic distribution, derive the first two trigonometric moments for proposed model, we state and prove some theorems that characterize the Stereographic Semicircular Half Logistic model. MATLAB is used to plot graphs and compute population characteristics.

Keywords: Circular model, Characteristic function, l-axial data, Stereographic projection, Trigonometric moments.

INTRODUCTION

Kantam and Rosaiah (1998) applied Half Logistic distribution in acceptance sampling based on life tests. Several recurrence relations satisfied by the single and product moments of order statistics from a generalized Half Logistic distribution enable one to compute the single and product moments of all order statistics. For all sample sizes in a simple recursive manner, this may be done for any choice of the shape parameter. These moments can then be used to determine the best linear unbiased estimators of location and scale parameters from complete as well as Type-II censored samples.

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004) and Byoung et al (2008). Phani et al (2013) constructed some semicircular distributions by applying Inverse Stereographic projection.

Taking this as a cue, on the lines of Phani et al (2017) an attempt is made to develop the Stereographic- l-axial Half Logistic distribution by extending the Stereographic Semicircular Half Logistic distribution for modeling axial data. Stereographic l - axial Half Logistic distribution over l – arc has possible applications in Quality Control, weather studies and pollution control experiments. We consider the asymptotic behaviour of the Stereographic Semicircular Half Logistic distribution, derive the first two trigonometric moments for proposed model, we state and prove some theorems that characterize the Stereographic Semicircular Half Logistic model. MATLAB is used to plot graphs and compute population characteristics.

METHODOLOGY OF MODIFIED INVERSE STEREOGRAPHIC PROJECTION FOR

Semicircular models

Methodology of modified inverse stereographic projection for construction of circular models is well explained by Phani et al (2013).

If a linear random variable X has a support on R, then θ has a support on $(-\pi, \pi)$ and if X has a support on R^+, then θ has a support on $(0, \pi)$. These means that, after the Inverse stereographic projection is applied, we can deal circular data if the support of X is on R and we can handle semicircular data if the support of X is on R^+.

STEREOGRAPHIC SEMICIRCULAR HALF LOGISTIC DISTRIBUTION [Phani et al (2017)]

A random variable X on the real line is said to have an Half Logistic distribution with location parameter α and scale parameter $\beta > 0,$ if the probability density function and probability distribution function of X are given respectively by

$$f(x) = \frac{2}{\beta}\left[1+\exp\left(\frac{-(x-\alpha)}{\beta}\right)\right]^{-2} \exp\left(\frac{-(x-\alpha)}{\beta}\right),$$
$$0 < x < \infty,\ \beta > 0. \qquad (3.1)$$

$$F(x) = \frac{1-\exp\left(\frac{-(x-\alpha)}{\beta}\right)}{1+\exp\left(\frac{-(x-\alpha)}{\beta}\right)},\ 0 < x < \infty \qquad (3.2)$$

Then by applying modified inverse stereographic projection defined by a one to one mapping $x = v\tan\left(\frac{\theta}{2}\right), v \in \mathbf{R}^+,$ this leads to a semicircular model on unit semicircle. We call this model as Stereographic Semicircular Half Logistic Distribution.

Definition:

A random variable X_{SC} on the semicircle is said to have the Stereographic Semicircular Half Logistic distribution with location parameter μ scale parameter $\sigma > 0$ denoted by **SSCHLD** (σ, μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta) = \frac{1}{\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right), \tag{3.3}$$

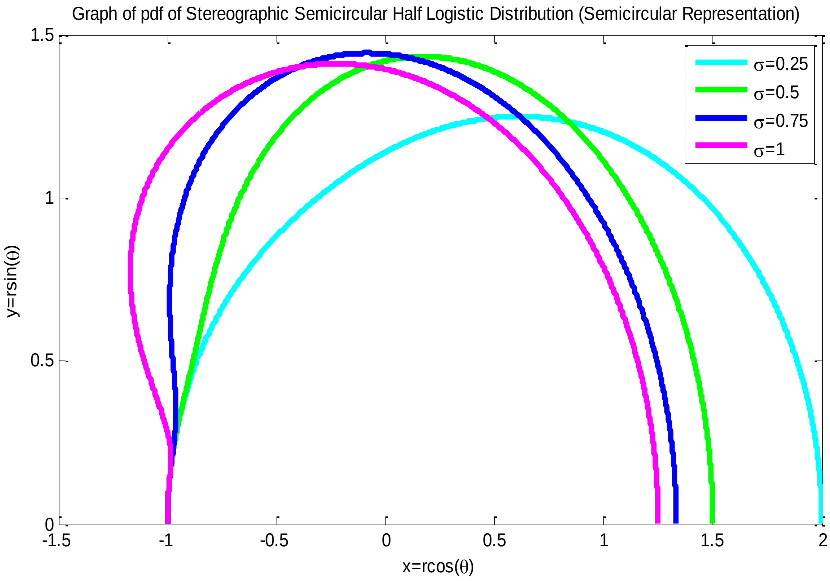
$$0<\theta<\pi,\ \sigma=\frac{\beta}{\nu}>0,\ \mu=\frac{\alpha}{\nu}$$

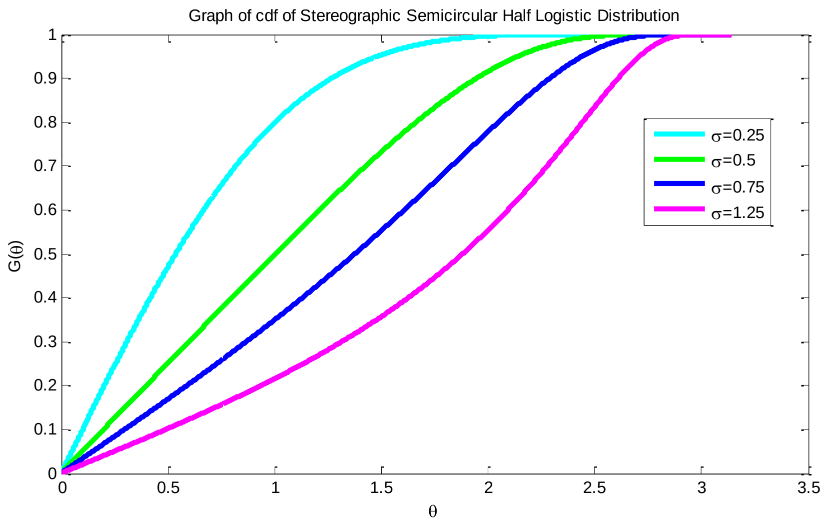
$$G(\theta) = \left[1-\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1},$$

$$0<\theta<\pi,\ \sigma=\frac{\beta}{\nu}>0,\ \mu=\frac{\alpha}{\nu} \tag{3.4}$$

Hence the proposed new model **SSCHLD** (σ, μ) is a semicircular model.

The graphs of probability density function, cumulative distribution function are plotted here.





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CHARACTERISTIC FUNCTION OF STEREOGRAPHIC SEMICIRCULAR HALF

Logistic Distribution

The characteristic function of a Semicircular model with probability density function $g(\theta)$ is defined as

$$\varphi_p(\theta) = \int_0^{\pi} e^{ip\theta} g(\theta)\, d\theta \,,\, p \in \mathbb{Z} \,.$$

If $G(\theta)$ and $g(\theta)$ are the cdf and the pdf of the Stereographic Semicircular model and $F(x)$ and $f(x)$ are the cdf and the pdf of the respective linear model, then the characteristic function of Stereographic Semicircular model is

$$\varphi_{X_{SC}}(p) = \varphi_{2\tan^{-1}\left(\frac{x}{v}\right)}(p), p \in \mathbb{Z}$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function. Numerical integration of Weddle's rule is used for the computation of the values of the characteristic functions of both the Stereographic Semicircular models.

The characteristic function of Stereographic Semicircular Half Logistic Model is

$$\varphi_{X_{SC}}(p) = \int_0^{\pi} e^{ip\theta} g(\theta)\, d\theta$$

$$= \int_0^{\pi} e^{ip\theta} \frac{1}{\sigma} \sec^2\left(\frac{\theta}{2}\right) \left[1 + \exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right) - \mu\right)}{\sigma}\right)\right]^{-2} \exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right) - \mu}{\sigma}\right)\right) d\theta$$

$$0 < \theta < \pi,\ \sigma > 0,\ \mu < \tan\left(\frac{\theta}{2}\right) \qquad (4.1)$$

The graph of the characteristic function for various values of parameter are plotted here

The expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are available in Mardia and Jupp (2000). These characteristics for the Stereographic Semicircular Half Logistic distribution are also based on their respective trigonometric moments and can be expressed in terms of trigonometric moments α_p and β_p which are presented here.

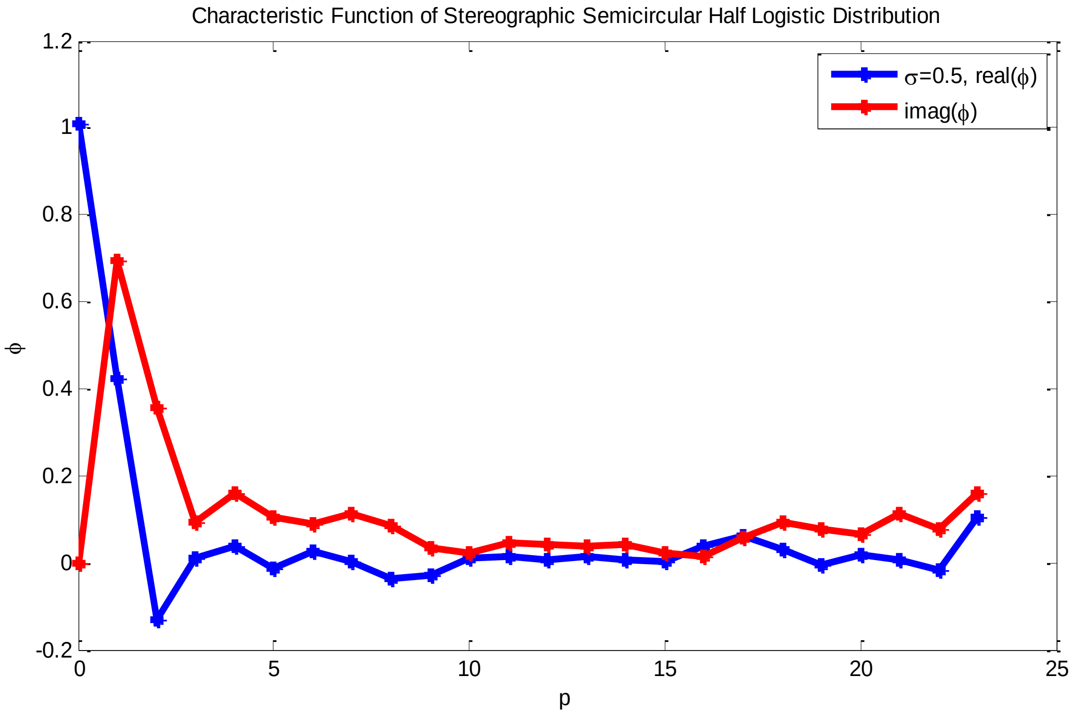


Table 4.1. Population Characteristics of SSCHLD

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	0.7962	0.2571	0.1003	-0.1801
α_2	0.4310	-0.4301	-0.3403	-0.1897
β_1	0.3774	0.8297	0.7931	0.7390
β_2	0.4235	0.3395	0.0737	-0.2489
Resultant Length				
ρ_1	0.8811	0.8686	0.7994	0.7607
ρ_2	0.6042	0.5479	0.3482	0.3129
Mean Direction				
μ_0	0.4426	1.2703	1.4450	1.8099
Circular Variance				
v_0	0.1189	0.1314	0.2006	0.2393
Circular Standard Deviation				
σ_0	0.5032	0.5308	0.6692	0.7397
	1.0038	1.0969	1.4526	1.5243
Central Trigonometric Moments				
α_1^*	0.8811	0.8686	0.7994	0.7607
α_2^*	0.6007	0.5467	0.3479	0.2830
β_1^*	0	0	0	0
β_2^*	-0.0655	-0.0367	0.0134	0.1337
Skewness γ_1^0	-1.5967	-0.7713	0.1491	1.1416
Kurtosis γ_2^0	-0.1408	-1.3058	-1.5006	-0.9052

SOME CHARACTERIZATION THEOREMS OF STEREOGRAPHIC SEMICIRCULAR

Half Logistic Distribution

Here we recall a probability density and distribution functions of Stereographic Semicircular Exponential (Phani et al (2013)) and Circular uniform distributions that will involve in the characterization theorems of Stereographic Semicircular Half Logistic Distribution.

a) Stereographic Semicircular Exponential Distribution(Phani et al(2013))

A random variable X_{SC} on unit semicircle is said to have Stereographic Semicircular Exponential distribution with scale parameter $\sigma > 0$, denoted by SSEXP (σ), if the probability density and cumulative distribution functions are given by

$$g(\theta) = \frac{\sigma}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\sigma\tan\left(\frac{\theta}{2}\right)\right),$$

$$\text{for } 0 \le \theta < \pi \text{ and } \sigma > 0 \qquad (5.1)$$

$$G(\theta) = 1 - e^{-\sigma\tan\left(\frac{\theta}{2}\right)} \qquad (5.2)$$

b) Semicircular Uniform Distribution

A random variable X_C on unit semicircle is said to have Semicircular Uniform distribution, if the probability density function is given by

$$g(\theta) = \frac{1}{\pi}, \quad 0 \le \theta < \pi \qquad (5.3)$$

Theorem 5.1 Let θ be a continuous circular random variable with probability density function $g(\theta)$. Then the random variable $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$ follows Stereographic Semicircular Half Logistic distribution if and only if θ follows Stereographic Semicircular Exponential distribution by Phani et al (2013) with scale parameter $\sigma = 1$.

Proof:

Suppose θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$,

$$g(\theta) = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\tan\left(\frac{\theta}{2}\right)\right), \quad 0 \le \theta < \pi$$

Let $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$, this implies that

$\theta = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right)\right)$ and

$$h(\phi) = |J|\, g_\theta(\phi)$$

$h(\phi) = \dfrac{\sec^2\left(\frac{\phi}{2}\right)e^{-\tan\left(\frac{\phi}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\phi}{2}\right)}\right)^2}, \; 0 \le \phi < \pi$, which is the density function of Stereographic Semicircular Half Logistic distribution.

Conversely, suppose that the random variable $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$ follows Stereographic Semicircular Half Logistic distribution, then the distribution function of θ is

$$G_\theta(\theta_0) = \Pr(\theta \le \theta_0)$$

$$= \Pr\left(2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right)\right) \le \theta_0\right)$$

$$= \Pr\left(\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right) \le e^{\tan\left(\frac{\theta_0}{2}\right)}\right)$$

$$= 1 - \Pr\left(\phi \le 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta_0}{2}\right)}}{1-e^{-\tan\left(\frac{\theta_0}{2}\right)}}\right)\right)\right)$$

$G_\theta(\theta_0) = 1 - e^{-\tan\left(\frac{\theta_0}{2}\right)}$, which is the distribution function of the Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$.

Hence the theorem.

Theorem 5.2 Let θ be a continuous circular random variable with probability density function $g(\theta)$. Then the random variable $\phi = 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)} - 1\right)\right)$ follows Stereographic Semicircular Half Logistic distribution if and only if θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$.

Proof:

Suppose θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$,

$$g(\theta) = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\tan\left(\frac{\theta}{2}\right)\right), \quad 0 \le \theta < \pi$$

Let $\phi = 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)} - 1\right)\right)$, this implies that

$\theta = 2\tan^{-1}\left(\ln\left(\frac{1+e^{\tan\left(\frac{\phi}{2}\right)}}{2}\right)\right)$ and the Jacobian of this transformation is $|J| = \dfrac{\sec^2\left(\frac{\phi}{2}\right)e^{\tan\left(\frac{\phi}{2}\right)}}{2\left(1+e^{\tan\left(\frac{\phi}{2}\right)}\right)}$.

Therefore, $h(\phi) = |J|\, g_\theta(\phi)$

$$h(\phi)=\frac{\sec^2\left(\frac{\phi}{2}\right)e^{-\tan\left(\frac{\phi}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\phi}{2}\right)}\right)^2},\ 0\le\phi<\pi$$, which is the density function of Stereographic Semicircular Half Logistic distribution.

Conversely, suppose that the random variable $\phi=2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)}-1\right)\right)$ follows Stereographic Semicircular Half Logistic distribution, then the distribution function of θ is

$$G_\theta(\theta_0)=\Pr(\theta\le\theta_0)$$

$$=\Pr\left(2\tan^{-1}\left(\ln\left(\frac{1+e^{\tan\left(\frac{\phi}{2}\right)}}{2}\right)\right)\le\theta_0\right)$$

$$=\Pr\left(\phi\le 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta_0}{2}\right)}-1\right)\right)\right)$$

$$=1-e^{\tan\left(\frac{\theta_0}{2}\right)}$$, which is the distribution function of the Stereographic Semicircular Exponential distribution with scale parameter $\sigma=1$.

Hence the theorem.

Theorem 5.3 Let θ be a circular random variable which follows Semicircular Uniform distribution, then the random variable $\phi=2\tan^{-1}\left(\ln\left(\frac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)$ has Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ if and only if $p=\pi$.

Proof:

Suppose θ follows Semicircular Uniform distribution with probability density function

$$g(\theta)=\frac{1}{\pi},\ 0\le\theta<\pi.$$

Let $\phi=2\tan^{-1}\left(\ln\left(\frac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)$, this implies $\theta=2\tan^{-1}\left(\frac{p}{1+e^{\tan\left(\frac{\theta}{2}\right)}}\right)$ and the Jacobian of the transformation is $|J|=\frac{p\sec^2\left(\frac{\theta}{2}\right)e^{\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)^2}$.

Thus the density function of ϕ is $h(\phi)=|J|\,g_\theta(\phi)$

$$h(\phi)=\frac{p\sec^2\left(\frac{\theta}{2}\right)e^{\tan\left(\frac{\theta}{2}\right)}}{\pi\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)^2}$$, which is the density function of Stereographic Semicircular Half Logistic distribution if $p=\pi$.

Conversely, suppose ϕ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$, then

$$G_\theta(\theta_0)=Pr(\theta\le\theta_0)$$

$$=Pr\left(\theta\le 2\tan^{-1}\left(\frac{p}{1+e^{\tan\left(\frac{\theta}{2}\right)}}\right)\right)$$

$$=1-Pr\left(\phi\le 2\tan^{-1}\left(\ln\left(\frac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)\right)=\frac{1}{\pi}$$ for $p=\pi$.

Hence the theorem.

Theorem 5.4 The circular random variable θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ with density function $g(\theta)$ in (3.3) if and only if $g(\theta)$ satisfies the initial value problem $\frac{dg}{d\theta}-\left(\tan\left(\frac{\theta}{2}\right)+\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\right)g=\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)g^2$, with $g(0)=\frac{1}{4}$.

Proof:

Consider the differential equation

$$\frac{dg}{d\theta}-\left(\tan\left(\frac{\theta}{2}\right)+\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\right)g=\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)g^2,\ \text{with g(0)}=\frac{1}{4}. \quad (5.4)$$

Suppose θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Its density function is

$$g(\theta)=\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(-\left(\tan\left(\frac{\theta}{2}\right)\right)\right)\right]^{-2}\exp\left(-\left(\tan\left(\frac{\theta}{2}\right)\right)\right), \quad (5.5)$$

It is easily shown that $g(\theta)$ satisfies the initial value problem (5.4).

Conversely, assume that $g(\theta)$ satisfies the initial value problem (4.6.4), which is a non linear differential equation (in particular Bernoulli's equation), by solving this equation, we have

$$g(\theta)=\sec^2\left(\frac{\theta}{2}\right)\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)^{-2}e^{-\tan\left(\frac{\theta}{2}\right)}$$

, which is the probability density function of Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Theorem 5.5 The circular random variable θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ with distribution function $\mathrm{G}(\theta)$ in (3.4)if and only if $\mathrm{G}(\theta)$ satisfies the initial value problem

$$\frac{dG}{d\theta}-\left(\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)}\right)G=-\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)},\ \text{with}\ G(\pi)=1.$$

Proof:

Consider the differential equation

$$\frac{dG}{d\theta}-\left(\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)}\right)G=-\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)},\ \text{with}\ G(\pi)=1. \quad (5.6)$$

Suppose θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Its distribution function is

$$G(\theta)=\left[1-\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]\left[1+\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]^{-1}, \quad (5.7)$$

It is easily shown that $G(\theta)$ satisfies the initial value problem (4.6.6).

Conversely, assume that $\mathrm{G}(\theta)$ satisfies the initial value problem (4.6.6), which is a non linear differential equation, by solving this equation, we have

$$G(\theta)=\left[1-\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]\left[1+\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]^{-1},$$

which is the distribution function of Stereographic Circular Half Logistic distribution with scale parameter $\sigma=1$.

Hence the theorem.

STEREOGRAPHIC - l -AXIAL HALF LOGISTIC DISTRIBUTION

We extend the above Stereographic Semicircular Half Logistic model to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,\ldots,$ so it is desirable to extend the Stereographic Semicircular Half logistic distribution to construct the Stereographic- l -axial Half Logistic distribution, we consider the density function of Stereographic Semicircular Half Logistic distribution and use the transformation $\phi=\frac{2\theta}{l}$, $l=1,2,\ldots,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{l}{2\sigma}\sec^2\left(\frac{l\phi}{4}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{l\phi}{4}\right)\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{l\phi}{4}\right)}{\sigma}\right)\right),$$

$$0<\phi<\frac{2\pi}{l},\ \sigma>0\ \text{and}\ l=1,2,.. \quad (6.1)$$

We call it as **Stereographic-l-axial Half Logistic distribution**

Case (1) When $l=1$, in the probability density function (5.1), we get the density function

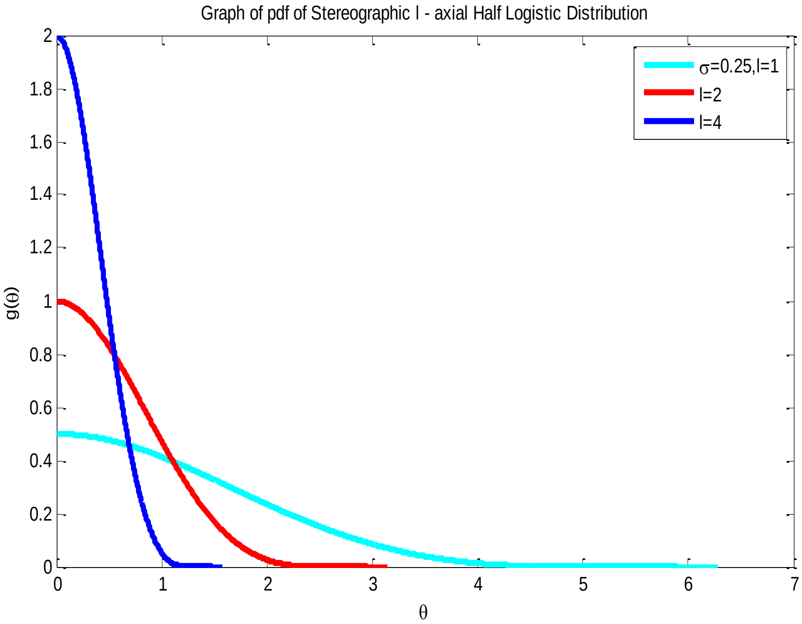
$$g(\phi)=\frac{1}{2\sigma}\sec^2\left(\frac{\phi}{4}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\phi}{4}\right)\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\phi}{4}\right)}{\sigma}\right)\right),$$

$$0<\theta<2\pi\ \text{and}\ \sigma>0 \quad (6.2)$$

We call it as **Stereographic Circular Half Logistic distribution**.

Case (2) When $l = 2$, the probability density function (6.1) is the same as that of **Stereographic Semicircular Half Logistic Distribution**

The graph of the pdf of **Stereographic *l* - axial Half Logistic Distribution** for various values of l is plotted.



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